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CONTENTS

PAGE
NUMBER 278.—On the rotation of some planetary nebulae. W. W. Campbell and J. H. Moore 1
NUMBER 279.—The wave-lengths of the green nebular lines N_1 and N_2. W. W. Campbell and J. H. Moore 6
The wave-lengths of the nebular lines N_1 and N_2. W. H. Wright 9
NUMBER 280.—Elements and ephemeris of Comet *a* 1916 (Neujmin). Jessica M. Young, H. M. Jeffers, and T. A. Pierce 11
NUMBER 281.—Elements and ephemeris of Comet *e* 1915 (Taylor). H. M. Jeffers and F. J. Neubauer 12
Note on the return of Comet *e* 1909 (Daniel). S. Einarsson and Margaret Harwood 14
NUMBER 282.—Elements and ephemeris of Comet *b* 1916 (Wolf). R. T. Crawford and D. Alter 15
NUMBER 283.—Modern positions of Astronomische Gesellschaft Zone stars. R. H. Tucker 17
NUMBER 284.—On the radial velocity of β *Ursae Majoris.* W. W. Campbell and Adelaide M. Hobe 22
On systematic errors in the measured interval between close lines. W. W. Campbell 28
Wolf-Rayet bands in the nuclei of three nebulae: N.G.C. 6826, N.G.C. 418_2, N.G.C. 40. G. F. Paddock 29
An error in radial velocity observations arising from non-uniform slit illumination. W. W. Campbell 30
NUMBER 285.—On the orbit of *Aethra.* D. Alter 32
NUMBER 286.—Ephemeris of Comet *b* 1916 (Wolf). R. T. Crawford 35
NUMBER 287.—Six stars whose radial velocities vary. J. H. Moore 37
Four stars whose radial velocities are variable. R. E. Wilson 38
RS *Sagittarii,* Algol variable and spectroscopic binary. G. F. Paddock 39
Notes on the spectrum of γ *Argus.* G. F. Paddock 40
NUMBER 288.—The spectroscopic binary h 4 *Centauri.* G. F. Paddock 42
Note on the identification of two more lines in the spectrum of ν_4 *Eridani.* G. F. Paddock 47
NUMBER 289.—Ephemeris of Comet *b* 1916 (Wolf). R. T. Crawford 48
NUMBER 290.—Elements and ephemeris of Comet *a* 1917 (Mellish). H. M. Jeffers and Elizabeth J. Easton 50
NUMBER 291.—The quartz spectrograph of the Lick Observatory and some observations recently secured with it. W. H. Wright 52
NUMBER 292.—The refraction at Mount Hamilton. R. H. Tucker 56
NUMBER 293.—Measures of comets in 1915 and in 1916. R. G. Aitken 64
Photographic measures of satellites and of comets. R. F. Sanford 65
Observations of comets. W. K. Green 67
Photographic positions of comets. W. K. Green 67
NUMBER 294.—The radial velocities of eleven southern light-variables. G. F. Paddock 68
NUMBER 295.—Second elements and ephemeris of Comet *b* 1916 (Wolf). R. T. Crawford and D. Alter 74
NUMBER 296.—Search ephemeris for Faye's Comet. H. M. Jeffers 78
Elements and ephemeris of Comet *b* 1917 (Schaumasse). Jessica M. Young and H. M. Jeffers 78
NUMBER 297.—On some relations of the spiral nebulae to the Milky Way. R. F. Sanford 80
NUMBER 298.—A study of certain nebulae. W. K. Green 92
NUMBER 299.—Parallax of the Ring Nebula in *Lyra.* B. L. Newkirk 100
NUMBER 300.—Three novae in spiral nebulae. H. D. Curtis 108
NUMBER 301.—Orbit and perturbations of (716) *Berkeley.* F. J. Neubauer 111
NUMBER 302.—On the orbit of (718) *Erida.* C. S. Mundt 115
NUMBER 303.—The orbit of the spectrographic binary γ *Phoenicis.* R. E. Wilson 116
On the orbit of the spectroscopic binary σ *Puppis.* R. E. Wilson 117
NUMBER 304.—A spectrographic study of κ *Pegasi.* F. Henroteau 120
NUMBER 305.—On finite velocity of gravitation as a possible factor in stellar evolution. D. Alter 128
NUMBER 306.—One hundred new double stars. Twenty-fourth list. R. G. Aitken 132
NUMBER 307.—The magnitude equation and the visual scale. R. H. Tucker 136
NUMBER 308.—Miscellaneous Meridian Circle results:
Observed corrections to the right ascension of the Moon. Observed correction to the right ascension of *Venus.* Meridian Circle observations of *Nova Monocerotis.* Comparison stars for Asteroid (21). Observations of XZ *Cygni.* The diurnal variation in the atmospheric refraction. Observed errors in the Harvard A.G. Zone. R. H. Tucker 144
NUMBER 309.—Elements and ephemeris of Wolf's Object, 1918 DB. H. M. Jeffers 147
NUMBER 310.—On variation of velocity of waves due to motion of the source. D. Alter 151
NUMBER 311.—A spectroscopic study of β *Canis Majoris.* F. Henroteau 155
NUMBER 312.—The visual magnitude scale and the counts of stars. R. H. Tucker 162
NUMBER 313.—*Nova Monocerotis.* G. F. Paddock 170
NUMBER 314.—A spectrographic study of σ *Scorpii.* F. Henroteau 173
The radial velocities of the stars ε and γ *Orionis.* F. Henroteau 178
NUMBER 315.—The orbit of the spectroscopic binary p *Velorum.* R. F. Sanford 181
NUMBER 316.—The orbit of *Sirius.* R. G. Aitken 184
NUMBER 317.—The orbit of the binary star A 417 = 83 *Aquarii.* R. G. Aitken 191

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PLATES

Plate I. (*a*) The planetary nebula N.G.C. 7009.
(*b*) Spectrogram of N.G.C. 7009.
(*c*) Spectrogram of N.G.C. 6543 4

Plate. II. Figs. 1, 2, 3, 4. The quartz spectrograph of the Lick Observatory.
Fig. 5. Prismatic photograph of nebula N.G.C. 7662 53

Plate III. Fig. 1. The spiral nebula N.G.C. 4527.
Fig. 2. The spiral nebula N.G.C. 4321 109

ERRATA

Page 22, col. 2, line 11, *for* Kuthnick *read* Guthnick.
Page 133, line 1, *for* 0h *read* 3h.
Page 134, line 25, *for* 2978 *read* 2979.
Page 134, A 2993, *for* $19^h 41^m 13^s$ *read* $19^h 42^m 13^s$.
Page 135, line 1, *for* 0^h *read* 20^h.
Bulletin 312. The position of four tables were changed by the printer in the make-up of the pages, following the reading of the galley proof. In all references to Table III in the text adjoining page 164, *for* below *read* above; in the text following Table VI, page 166, *for* following *read* preceding.

Volume I.
Page 140, *Titan* and *Rhea*, Aug. 28, *for* $p = 117.^\circ 3$ *read* $p = 177.^\circ 3$.
Page 143, *Rhea* and *Dione*, *for* 1901 June 13 *read* 1901 July 13.
Page 147, *Tethys* and *Enceladus*, Sept. 16, *for* $s = 35.''95$ *read* $s = 50.''01$.
Page 148, *Saturn* and *Titan*, Aug. 6, *for* $p = 54.^\circ 7$ *read* $p = 234.^\circ 7$.
Aug. 29, *for* $s = 124.''91$ *read* $s = 75.''45$.
Sept. 18, *for* $p = 280.^\circ 8$ *read* $p = 100.^\circ 8$.

Volume II.
Page 37, *Titan–Tethys*, Oct. 2, *for* $p = 226.^\circ 02$ *read* $p = 221.^\circ 02$.
Page 40, *Rhea–Tethys*, Sept. 15, *for* $s = 32.''65$ *read* $s = 31.''65$.
Page 40, *Rhea–Enceladus*, Aug. 24, *for* $s = 36.''65$ *read* $s = 34.''65$.
Page 42, *Dione–Tethys*, Sept. 12, *for* $s = 88.''22$ *read* $s = 90.''13$.
Page 44, *Enceladus–Mimas*, June 18, *for* $p = 248.^\circ 88$ *read* $p = 68.^\circ 88$.

Volume VI.
Page 77, col. 1, Ol 30, *for* Decl. —29° 11′ *read* —29° 19′.
Page 79, col. 2, Σ 321 AB, mag., *for* 9.0–9.6 *read* 7.0–8.4.
Page 80, col. 1, Σ 321 CD, mag., *for* 7.8–8.4 *read* 9.0–9.6.
Page 82, col. 1, Hu 461, *for* 6.°72 etc. *read* 67.°2 etc.
Page 184, Σ 152, measures given belong to some other star.
Page 185, col. 2, Σ 376, *for* 351.°3 *read* 251.°3.
Page 188, col. 2, Σ 1503, *for* 11.″42 *read* 11.″52.
Page 189, col. 1, Σ 1537, *for* 356.°1 *read* 355.°1.
Page 190, col. 2, Σ 2085, *for* 6.″48 *read* 6.″38.
Page 190, col. 2, *for* Σ 2042 *read* Σ 2142.
Page 190, col. 2, Σ 2185, *for* 1896.59 *read* 1896.56.
Page 190, col. 2, Σ 2209, *for* 22.″82 *read* 29.″58.
Page 191, col. 1, Σ 2232, *for* 6.″74 *read* 6.″71.
Page 191, col. 2, Σ 2396, *for* 31.″31 *read* 31.″38.
Page 192, col. 1, *for* Σ 2501 *read* Σ 2509.
Page 193, col. 2, Σ 2880, *for* 353.°3 *read* 353.°0.
Page 194, col. 1, Σ 2924, *for* 1900.01 *read* 1900.67.
Page 195, col. 2, Σ 3089, *for* 20.″33 *read* 24.″73. Measures given belong to some other star.

Volume VIII.
Page 72, line —6, *b Velorum*, *for* $\delta = -46^\circ\,7'$ *read* $\delta = -46^\circ\,18'$.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 278

ON THE ROTATION OF SOME PLANETARY NEBULAE

We have recently secured observations of a few of the brighter planetary nebulae with a spectrograph of high dispersion in order to test these objects for Doppler-Fizeau effects as evidences of rotation. Indications of relative motion within several nebulae have been found, but the present paper is confined to the results for two planetaries which show undoubted rotational effects.

In the case of a planetary whose form is approximately elliptical, it was natural to assume that the maximum rotational effect would be observed along the major axis of the ellipse. The slit was set upon the longer axis of the nebula and the slit-length made slightly greater than the longer diameter of the visible image. The nebula was then carefully held central on the slit during the entire exposure.

If the bright lines composing the nebular spectrum are found to be inclined to the zero direction, as given by the comparison spectrum, it is necessary to consider that this inclination may be due to (*a*) rotation of the nebula, (*b*) non-uniform illumination of the slit, (*c*) instrumental causes, or (*d*) to two or three of these combined. A little consideration will show that a comparison of two spectrograms obtained with the image on the slit respectively direct and reversed 180° is sufficient to differentiate or to eliminate the effects (*b*) and (*c*) if any such exist.

The spectrograph employed in the present work is one which we had especially arranged for a study of the radial velocities in the brighter parts of the *Orion* nebula, in continuation of an extended investigation of this object begun in November, 1914. It is in all essential features the original Mills spectrograph,[1] except that the prisms, in a new prism box, have been set at minimum deviation for 4950A. The linear dispersion at the $H\beta$ line, which is central on the plate, is 1 mm. equals 20A. The spectrograph is provided with a constant temperature case and thermostat. As a further precaution against the effects of slight temperature changes, or those due to differential flexure, the comparison spectra of hydrogen and helium were impressed upon the plate eight times at regular intervals during the exposure. All of the spectrograms were taken on Seed's 30 plates. A slit width of 0.075 mm. was employed.

Planetary Nebula N. G. C. 7009

R.A. = 20^h 58^m; Decl. = —11° 48′

The first object studied in this manner was the planetary nebula N. G. C. 7009. A drawing of this nebula, made by Dr. Curtis from a number of Crossley reflector photographs of from 10 seconds to 2 hours exposure times, is reproduced in the accompanying Plate I (*a*).[2] In general outline the image of the nebula is elliptic, with the major axis in position angle 79°. The longer diameter of the ellipse is about 30 seconds of arc. The brighter portion of the nebula, as shown by the lengths of the lines recorded on our spectrograms, has a diameter of about 20 seconds of arc along the major axis and 13 seconds of arc along the minor axis.

It should be said that the relative intensities in different parts of the nebula have been exaggerated in the drawing in order to bring out fainter details. For example, the extensions which terminate in the two condensations, east and west of the main structure, are shown only on the long-exposure photographs and their spectra are much too faint to be recorded with a slit spectrograph with exposures of practicable length.

Four spectrograms of N. G. C. 7009 were secured in November, 1915. The first was obtained with the slit in its usual position angle, i.e., 90°. This plate showed a marked inclination of the nebular lines with reference to those of the comparison spectra. Second and third spectrograms obtained with the slit coincident with the major axis of the nebula, but with the image in positions on the slit respectively direct and reversed 180°, confirmed the results of the first observation. The fourth spectrogram, taken with the slit in position angle 349°, i.e., coincident with the minor axis of the nebula, gave no certain evidence of inclination of the lines.

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[1] *Ap. Jour.*, 8, 123, 1898.

[2] See also the photographs of N. G. C. 7009 by Keeler, *Publ. Lick Obs.*, 8, plate 64, 1908.

The ends of the lines whose source is the western part of the nebula are displaced toward the violet with reference to the ends corresponding to the eastern part. Interpreted as a Doppler-Fizeau effect, the western edge is being carried toward us and the eastern edge away from us by the rotation of the nebula. Maximum rotational speed appears to be attained at about 9 or 10 seconds of arc from the center. The extreme ends of the lines originating in the fainter parts from 10 to 12 seconds from the nucleus do not follow the inclination of the central sections of the lines, but turn back slightly. We interpret this to mean that the outer strata of the nebula are rotating with smaller angular velocities than those of the central regions.

In Table I are collected the data for these spectrograms. The first three columns give respectively the Greenwich Mean Time of the observation, the length of exposure, and the position angle of slit. Columns four, five and six contain the measures, by Messrs. Campbell, Moore, and Green, of the relative velocity in kilometers per second for points in the lines giving maximum displacements, i.e., for points corresponding to regions of the nebula 9 or 10 seconds of arc distant from the center.[3] Measures were made on only the two nebulium lines 5007A and 4959A, since the Hβ line is under-exposed on all of our plates of this nebula. For the three plates obtained with the slit set along or near the major axis of the nebula, the mean of the results by the three observers is, for a point 9 to 10 seconds of arc east of the center with reference to the corresponding point west of the center, a relative velocity of recession of 12.1 km./sec. The agreement in the measures is as good as could be expected, since the selection of the portions of the line giving maximum displacement depends a little upon the judgment of the observer. A determination, made by Mr. Campbell, of the inclination of the lines on the plates of November 14 and 17 by means of measures of the angle between the directions of the comparison lines and of the nebular lines gives for the relative velocity of the two described portions of the nebula, 14.5 km./sec. The results of the two independent methods of measurement are in accord and indicate that the rotational speed of matter in the nebula at a distance of 9 seconds of arc from the nucleus is of the order of 6 kilometers per second. An attempt was made to measure the outer sections of the line which bend backward, but these, while easily seen on the stronger spectrograms, are too faint for accurate measurement. In Plate I (b), is reproduced an enlargement of the spectrogram of November 17, showing the two nebular lines N_1 and N_2. The comparison lines are those of the helium tube and the titanium spark.

TABLE I

	Gr. M. T.	Exposure	Position angle of slit	Relative velocity E–W km./sec.		
				C	M	G
1915	Nov. 14.622	150^m	90°	+11.7	+11.3	+13.7
	17.636	180	259	+10.0	+ 8.7	+14.5
	Nov. 18.634	180	79	+12.0	+14.9	+12.2
				+11.2	+11.6	+13.5
				Mean = +12.1		
				N–S km./sec.		
				M	G	
	Nov. 22.639	120	349	—0.08	—0.19	

The spectrogram of November 22, with slit coincident with the short axis of the nebula, was measured by Messrs. Moore and Green. Their measures of the velocity of a point at the northern edge of the nebula relative to the corresponding point in the southern edge are given in the last line, columns four and five, of Table I. The results appear to show a very small velocity of approach for the northern with respect to the southern edge, but the amount is less than the error of observation and the uncertainty of the correction applied for the curvature of the spectrum lines. We conclude that no evidence of inclination of the lines is shown on this plate.

PLANETARY NEBULA N. G. C. 6543

R.A. = 17^h 58.6^m; Decl. = +66° 38′

The image of the planetary N. G. C. 6543 is in outline that of a rather elongated ellipse, with its major axis in position angle approximately 40°, and the shorter axis in position angle 130°. Five spectrograms of this nebula have been obtained which show it to be an object of unusual interest. Three of the spectrograms were taken with the slit set along the major axis, two of these with the image in one position on the slit and the third with the image reversed 180°. The small stellar nucleus was held in the center of the slit during these exposures. The nebular lines on the three plates have a form which may be roughly described as S shaped, i.e., the line is divided into three sections of approximately equal lengths, the two outer sections having apparently about the same inclinations in one direction, while the central section is inclined in the opposite direction. A plate taken with the slit in the north and south direction shows the same broken form of lines but with less inclination for the central portion than those described above. The fifth spectrogram, secured with slit placed along the minor axis of the nebula, gives practically straight lines, although a slight inclination appears to be indicated by the measures of this plate.

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[3] See drawing of nebular line in the upper part of Plate I (a).

In Table II are given the Greenwich Mean Time, the length of exposure in minutes, and the position angle of the slit for the five observations of N. G. C. 6543.

TABLE II

Gr. M. T.	Exposure time	Position angle of slit
1915 Nov. 29.604	45m	0°
29.675	120	40
Nov. 30.712	220	40
Dec. 6.651	240	130
Dec. 19.667	200	220

All of the plates have been measured independently by Messrs. Moore and Green. The results depend upon the measures of the two nebulium lines N_1 and N_2, except in the case of the spectrogram of November 29.604, on which the 5007 line alone was sufficiently exposed for measurement. Five settings were made on four points of each of the nebular lines and on the comparison lines 5015A and 4922A on both sides of the nebular spectrum, in direct and reverse positions. The four points measured on the nebulium lines were those near the outer ends of the lines and the two in the central part where the direction of inclination changes. To the measures of each point corrections were applied for the inclination of the cross-wire as determined from the comparison lines, and for the curvature of the spectrum lines. The results are expressed as the relative velocity in kilometers per second of approach and recession of the given point with reference to the center of the middle section of the line.

Table III contains the results for the four points a_1, b_1, c_1, d_1 on the nebular lines of the three spectrograms taken with slit set upon the longer diameter of the nebula. If it be assumed that the middle of the line $b_1 c_1$ corresponds to the center of the nebula, then the results exhibit the velocities relative to the center at points along the major axis of the nebula, and at distances northeast of the center for $a_1 = 9''.2$ and $b_1 = 3''.3$, and southwest of the center, $c_1 = 3''.3$ and $d_1 = 10''.2$.

TABLE III

Slit coincident with major axis

Relative velocity in km./sec. with reference to center of $b_1 c_1$

	Image direct				Image reversed				
	November 29		November 30		December 19				
	M	G	M	G	M	G	Mean	Length of line mm.	Corresponding distance in nebulae Seconds of arc
NE a_1	+2.1	+0.9	—1.7	—2.5	+0.8	—0.9	—0.3		
								0.28	5.9
b_1	+4.8	+6.0	+3.9	+4.5	+4.9	+5.1	+4.9		
								0.32	6.7
c_1	—4.8	—6.0	—3.9	—4.5	—4.9	—5.1	—4.9		
								0.33	6.9
SW d_1	+6.4	+5.9	+4.1	+6.6	+3.3	+2.8	+4.8		

The mean of the results by the two observers for the three plates is given in column eight. In the last two columns are the measured distances between the points in millimeters, and the corresponding quantities in seconds of arc for the image of the nebula on the slit, computed from the focal lengths of the camera and collimator and the 36-inch objective. An enlargement of the spectrogram of November 30 is reproduced in Plate I (*c*).

Since the short-exposure spectrogram of November 29.604, taken with slit inclined at an angle of 40° to the major axis of the nebula, is under-exposed, and the measures depend upon a single line, small weight should be assigned to the results for this plate, which are given in the upper half of Table IV. Points a_2 and b_2 are north and c_2 and d_2 south of the center. These results are of interest in that they indicate an inclination of the central portion of the line in harmony with that given by the results for this section in Table III.

TABLE IV

Slit at angle of 40° to major axis

		November 29.604 M	G	Mean	Length of line mm.	Corresponding distance in nebulae Seconds of arc
N	a_2	+3.6	—2.6	+0.5		
					0.26	5.5
	b_2	+2.6	+2.8	+2.7		
					0.31	6.5
	c_2	—2.6	—2.8	—2.7		
					0.30	6.3
S	d_2	+2.2	+6.8	+4.5		
		Slit coincident with minor axis				
		December 6.651				
NW	a_3	+4.2	+4.4	+4.3		
					0.24	5.0
	b_3	+0.9	+0.9	+0.9		
					0.29	6.1
	c_3	—0.9	—0.9	—0.9		
					0.24	5.0
SE	d_3	—1.5	+1.3	—0.1		

The measures of the spectrogram of December 6, taken with the slit coincident with the minor axis of the nebula, are recorded in the second part of Table IV. Our results appear to indicate for the point a_2, some 8 seconds of arc northwest of the center, a velocity of recession of 4.3 km./sec. relative to that of a point at the center, while the inclinations of the other two sections of the line are probably less than the error of measurement.

From the above series of observations of N. G. C. 6543 we seem to have definite evidence that the central portion of the nebula, having a diameter of approximately 6.7 seconds of arc, is rotating about an axis whose position angle is 130°. The direction of rotation is such that a point in the nebula at 3.3 seconds of arc northeast of the center is receding from us and a point at the corresponding distance southwest of the center is approaching us. The observed component of rotational speed of matter at this distance from the center of the nebula is of the order of 5 km./sec.

The inclinations of the two outer sections of the lines which we have denoted by *ab* and *cd* with subscripts, corresponding to regions of the nebulae from 3 to 9 seconds of arc from the center, are such as might be obtained through a rapid decrease in the speed of rotation for the strata lying farther out from the central region. However, an inspection of the results obtained for these outer points, a_1, d_1 of Table III, and a_2, d_2, and a_3 of Table IV will show that the phenomena of the outer strata are more complicated than the simple explanation offered from the observed inclination of the lines. This is not surprising, in view of the very intricate structure of the nebula, as shown by Holden and Schaeberle's drawing[4] and by Keeler's photograph with the Crossley reflector,[5] and confirmed by a long-focus photograph kindly supplied by the Mount Wilson Solar Observatory.

Relative motion within N. G. C. 6543 was suspected by Hartmann[6] from his measures of several plates referring to the center and one plate referring to the north edge of the nebula. His result is, that a point in the north edge of the nebula has a velocity of approach of 9.5 km./sec. with reference to the center of the nebula. This result is not confirmed by our measures given above.

We do not attempt, at this time, a further interpretation of the phenomena of the outer strata, on the basis of our results, as the data are still too meagre for such a study; but we hope to return to this subject and to other interesting features of the planetaries after we shall have been able to secure the required observational data.

Measures of the rotational velocity of a nebula enable us to draw some interesting conclusions concerning its mass. If, for example, we assume that in the case of N. G. C. 7009 the axis of rotation lies in the plane perpendicular to the line of sight and in position angle 169°–349°, then a particle at the distance of 9 seconds of arc from the center and on the major axis of the nebula image has an orbital velocity of 6 km./sec. It is easily shown that for an assumed mass of the nucleus of N. G. C. 7009 equal to that of the Sun, the nebula would be at a distance from us of only 8.9 light years, which must be regarded as an improbably small value. For distances of 100 and 1000 light years, which we have reason to believe are more probable orders of nebular distances, the masses of the nebula would be respectively 11.3 and 113 times that of the Sun, and the corresponding periods of rotation 1371 and 13,710 years. The mean density of the nebula for an assumed distance of 100 light years is of the order of 1×10^{-6} times the density of hydrogen at 0° C. and at a pressure of 1 mm. of mercury; a value which is not out of harmony with the fine bright lines observed in the spectra of planetary nebulae.

In the case of N. G. C. 6543, if we assume the rotational speed of a point 3.3 seconds of arc from the center to be 5 km./sec. about an axis at right angles to the line of sight, then for distances of the nebula of 100 and 1000 light years, the corresponding periods of rotation would be 598 and 5980 years, and the nuclear masses respectively 2.8 and 28 times the solar mass. Its mean density, about five times that of N. G. C. 7009, is still of the order which one would be led to expect from the spectral lines.

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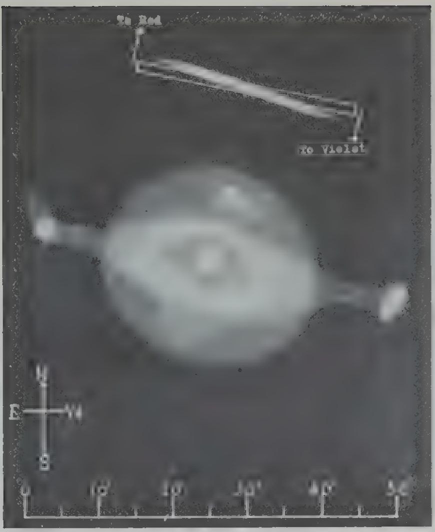
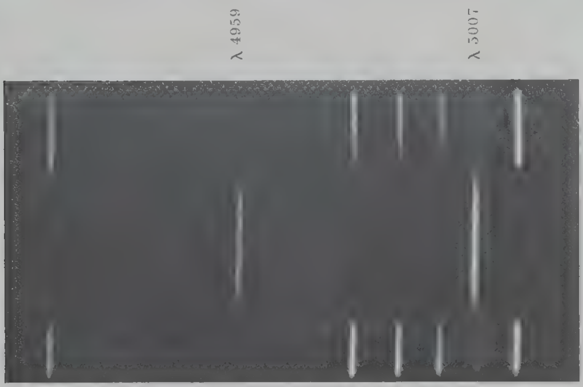
[4] *Mon. Not. R. A. S.*, 48, 388, 1888.

[5] *Publ. Lick Obs.*, 8, plate 57, 1908.

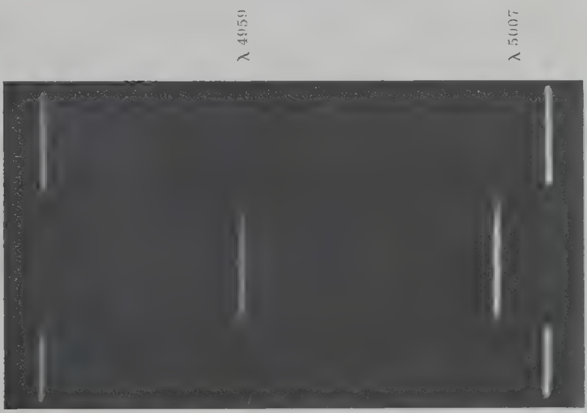
[6] *Ap. Jour.*, 15, 287, 1902.



(*a*) The Planetary Nebula N. G. C. 7009
(Composite drawing, from Curtis's photographs of the nebula made with the Crossley Reflector. The scale is in seconds of arc.)



(*b*) Spectrogram of N. G. C. 7009, November 17, 1915
(Enlargement approximately 20 times.)



(*c*) Spectrogram of N. G. C. 6543, November 30, 1915
(Enlargement approximately 20 times.)

From the above considerations it appears quite probable that the masses of the two planetary nebulae, N. G. C. 7009 and N. G. C. 6543, are several times that of the Sun.

A few speculations concerning N. G. C. 7009 may not be without interest and value.

The faint extensions to the east and to the west of the elliptical figure suggest an encircling ring of matter whose principal plane, passing through the nucleus, passes also near our (the observers') position in space. These extensions terminate in condensations at equal distances from and exactly on opposite sides of the nucleus, which may be largely the effect of the edge-wise projection of the ring, as in the case of the ansae of *Saturn's* rings when the observer is in the plane of the rings. The forms of the two terminating condensations, and especially the wing extending up and out from the eastern condensation, suggest that we are not exactly in the plane of an encircling ring. The form of the main nebula appears to be ellipsoidal and not chiefly elliptical; in other words it seems to be three-dimensional. The space immediately surrounding the central nucleus appears to be relatively vacuous. Aside from the nucleus, the principal mass of nebulosity probably exists in the relatively brilliant ring, roughly elliptical in projection, occupying the regions about midway between the nucleus and the outer boundary of the main nebular structure. The brilliant elliptical ring is probably an ellipsoidal shell. The projection of such a shell upon a plane at right angles to the line of sight would naturally show a relatively dark central area.

If this nebula is in process of development into a solar system, the indications are for a system having certain resemblances to our solar system. Our four outer planets have a combined mass 225 times that of our four inner planets. Similarly, in N. G. C. 7009, we seem to have a paucity of material to form planets near the nucleus and an abundance for planets at greater distances from the nucleus.

In the course of our determinations of nebular velocities in the past three years, we have noted a relatively large number of ring forms among the planetaries. We have observed no extremely-elongated or highly-elliptic gaseous nebulae, and no such nebulae as would result from thin rings seen edgewise. According to the probabilities of the case, if the apparent ring nebulae are really of ring form in space, we should have expected to see a number of very elongated elliptical rings and a certain number of relatively long and narrow nebulae (giving bright-line spectra). We are accordingly led to question whether the ring nebulae are true rings, or whether they are in reality ellipsoidal shells which appear to be rings by virtue of their projection upon the plane of sight.

W. W. CAMPBELL,
J. H. MOORE.

January 24, 1916.

Issued March 18, 1916.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 279

THE WAVE-LENGTHS OF THE GREEN NEBULAR LINES N_1 AND N_2

More than a year ago the original Mills spectrograph was supplied with a new prism box, to receive the three Mills prisms at minimum deviation for 4950A, for use in photographing the green-blue nebular lines under high angular dispersion. Equipped further, with a camera of focal length 126 mm. (4.96 inches), the Mills spectrograph was used extensively in measuring the radial velocities of faint nebulae, including the faint outlying parts of the *Orion* nebula. In October, 1915, the short camera was replaced by the regular Mills camera, of focal length 406 mm. (16.0 inches), in order to study more accurately the radial velocities of the brighter nebulae. As a basis for constructing the reduction curve for the longer camera, we photographed the spark spectrum of titanium and the tube spectra of hydrogen and helium on the same plate. We assumed Kilby's[1] wave-lengths for the titanium lines, W. E. Curtis's[2] wave-length for the hydrogen line $H\beta$ (4861.505A), and Runge and Paschen's[3] wave-lengths for the helium lines (4922.10 and 5015.73A). We endeavored to construct the curve of relationship of micrometer readings and wave-lengths, but found that the helium line at 5015A was far off the curve. Additional photographs of the same elements and photographs of the iron and helium spectra confirmed our conclusion that the assumed wave-length of the helium line, 5015.73, is seriously in error. A wave-length of about 5015.85 was demanded by the curves deduced from the titanium and iron spectra.

Upon examining the literature[4] of the helium spectrum, we noted that the interferometer determinations of the position of the helium line in question, by Rayleigh and by Eversheim, 5015.856 and 5015.859, respectively, are in good agreement with each other and with the indications and requirements of our curves. We accordingly decided to use the results for the helium spectrum obtained by the interferometer method, as the basis for our work involving the green nebular lines. The interferometer position of the other helium line concerned is 4922.11A.

The wave-lengths of the green nebular lines, N_1 (5007A) and N_2 (4959A) had been determined in 1891 by Keeler,[5] in 1902 by Hartmann,[6] and in 1902 by Wright.[7] We had used the weighted mean of Keeler's, Hartmann's and Wright's results as the wave-lengths of the N_1 and N_2 lines in our determinations of the radial velocities of the nebulae in the past three or four years, at Mount Hamilton and at Santiago, Chile. Inasmuch as Wright's measures of the positions of N_1 and N_2 depended strongly upon Runge and Paschen's positions of the helium lines, it seemed to us very desirable that we should use our efficient equipment for a new determination of the wave-lengths of N_1 and N_2.

The apparatus employed was as follows:

(*a*) The 36-inch (91.5 cm.) refractor, focal length 694 inches (17.63 mm.).

(*b*) The original Mills spectrograph,[8] equipped with a new prism box, 4950A in minimum deviation and $H\beta$ central on the plate.

The linear scale of the spectrograms is such that at

5007A, 1 mm. corresponds to $V = 1404$ km./sec.
4959A, 1 mm. corresponds to $V = 1348$ km./sec.
4862A, 1 mm. corresponds to $V = 1236$ km./sec.

During each exposure of a nebular spectrum, the comparison spectra were impressed upon the plate eight times, at appropriate symmetrical intervals; the hydrogen spectrum twice on each side of the nebular spectrum, and the helium spectrum twice on each side. On the last three spectrograms of the *Orion* nebula the titanium spark spectrum also was photographed twice on each side, and the titanium lines at 4981 and 4991A were used, along with the hydrogen and helium lines, in fixing the positions of the nebular lines.

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[1] Kilby, *Ap. Jour.*, **30**, 247–255, 1909.

[2] Curtis, *Proc. Roy. Soc. Lon.*, **90**, 613, 1914.

[3] Runge and Paschen, *Ap. Jour.*, **3**, 10, 1896.

[4] Kayser, *Handbuch der Spectroscopie*, **5**, 518–520, 1910.

[5] Keeler, *Publ. Lick Obs.*, **3**, 227, 1894.

[6] Hartmann, *Ap. Jour.*, **15**, 291, 1902.

[7] Wright, *L. O. Bull.*, **1**, 156, 1902.

[8] For description, see *Ap. Jour.*, **8**, 123, 1898.

Spectrograms were secured for the wave-length determinations, in October and November, 1915, as follows:

	R.A. 1900.0	Decl. 1900.0	No. of plates
N. G. C. 6572	$18^{h}\ 7^{m}2$	+ 6° 50′	4
N. G. C. 7027	21 3.3	+41 50	5
Orion nebula	5 30.	— 5 27	10

The N_1, N_2 and $H\beta$ nebular lines are recorded on all of the plates. Each plate was measured by Mr. Campbell, Mr. Moore and Miss Hobe, the three observers using different measuring microscopes, except that Campbell and Moore used the same microscope on three plates. All of the reductions were made by Miss Hobe after the measures were completed.

It was assumed that the normal wave-length of the $H\beta$ nebular line is identical with that of the $H\beta$ comparison line, and that the observed displacements of the $H\beta$ nebular lines were due to the relative radial velocities of the nebulae and the observer. The observed positions of the N_1 and N_2 nebular lines were assumed to differ from their normal positions by virtue of radial velocities equal to those indicated by the $H\beta$ nebular lines on the same spectrograms. The measured positions were corrected accordingly, and the resulting normal wave-lengths of N_1 and N_2 were obtained as shown in the accompanying table. The results set opposite the initials C, M, and H are those obtained from the measures by Campbell, Moore, and Miss Hobe, respectively. The weights were assigned to the observations, on the basis of a critical comparison of the spectrograms and of the observing records, before the reductions began. The weights for the spectrograms of N. G. C. 7027 are systematically smaller than for those of the other two nebulae, because the $H\beta$ line is weak relatively to N_1 and N_2 in N. G. C. 7027. The spectrograms of the *Orion* nebula are really entitled to greater relative weight than we have given them, because for the brighter regions in that object the $H\beta$ line is comparable in photographic intensity with the N_1 and N_2 lines, the intensity of $H\beta$ usually falling between the intensities of N_1 and N_2. For N. G. C. 6572 and especially for N. G. C. 7027 the N_1 and N_2 lines must be somewhat over-exposed in order to record the $H\beta$ line with the intensity required for accurate measurement of its position. For certain

Plate No.		N_1 A	N_2 A	Plate Wt.
		N. G. C. 6572		
N 287	C	5007.014	4959.037	
	M	6.959	8.983	1
	H	6.938	8.963	
N 289	C	7.009	9.095	
	M	7.021	9.109	1
	H	6.994	9.110	
N 293	C	7.054	9.117	
	M	7.067	9.133	1
	H	7.028	9.077	
N 300	C	7.040	9.102	
	M	7.024	9.068	1
	H	7.006	9.041	
Means	C	5007.029	4959.088	
	M	7.018	9.073	4
	H	6.992	9.048	
Mean		5007.013	4959.070	4
		N. G. C. 7027		
N 288	C	5007.019	4959.058	
	M	7.011	9.100	½
	H	6.972	9.063	
N 290	C	7.044	9.118	
	M	7.034	9.099	½
	H	7.012	9.091	
N 291	C	7.055	9.085	
	M	7.122	9.152	¼
	H	7.040	9.068	
N 294	C	7.040	9.083	
	M	7.064	9.090	½
	H	7.041	9.156	
N 301	C	7.052	9.067	
	M	7.028	9.055	¼
	H	7.049	9.068	
Means	C	5007.039	4959.084	
	M	7.046	9.098	2
	H	7.017	0.094	
Mean		5007.034	4959.092	2
		Orion NEBULA		
N 292	C	5007.081	4959.095	
	M	7.076	9.108	1
	H	7.073	9.112	
N 295	C	7.005	9.042	
	M	7.014	9.075	1
	H	7.005	9.082	
N 296	C	7.036	9.094	
	M	7.080	9.109	1
	H	7.016	9.099	
N 299	C	7.053	9.132	
	M	7.013	9.117	1
	H	7.025	9.076	
N 302	C	7.002	9.046	
	M	6.989	9.030	½
	H	6.932	9.020	
N 303	C	7.029	9.093	
	M	7.069	9.129	1
	H	7.018	9.091	

Plate No.		N_1 A	N_2 A	Plate Wt.
N 305	C	5007.019	4959.037	
	M	7.005	9.067	1
	H	6.979	9.091	
N 309	C	6.958	9.028	
	M	7.029	9.008	1
	H	6.991	9.009	
N 311	C	7.036	9.138	
	M	7.005	9.025	1
	H	7.018	9.136	
N 312	C	7.018	9.078	
	M	7.056	9.107	2
	H	7.014	9.095	
Means	C	5007.024	4959.080	
	M	7.038	9.083	10½
	H	7.011	9.085	
Mean		5007.024	4959.083	10½

FOR THE THREE NEBULAE

		N_1 A	N_2 A	Plate Wt.
Means	C	5007.027	4959.082	
	M	7.034	9.082	16½
	H	7.007	9.077	
Mean		5007.023	4959.080	16½

plates of the latter objects we reduced the effective exposure times for N_1 and N_2 to one-quarter and one-half the exposure times for $H\beta$ by means of a diaphragm immediately in front of the photographic plate. The advantage in using this device to make the densities of the three lines nearly equal is probably balanced by the evident disadvantages. Plate No. 312 is an especially good one, and it was given double weight.

If weight 1 be assigned to each of the nineteen spectrograms, we obtain for the mean of the nineteen results:

	N_1	N_2
C	5007.030 ± .0082	4959.081 ± .0102
M	.035 ± .0117	.082 ± .0139
H	.008 ± .0108	.071 ± .0149
Mean	5007.024 ± .006	4959.078 ± .008

As these values differ inappreciably from the weighted means, it is apparent that the system of weighting employed is an unimportant matter.

The probable errors assigned to the means, ±.006 and ±.008, are a little larger than the values computed from the probable errors set opposite the results obtained by the individual observers, on the assumption that these results are independent of each other. This assumption is not strictly true, as the three sets of measures are based upon the same nineteen plates and are affected by the same "plate errors." These errors are thought to be small: the slit was always filled with light from objects whose images are much larger than the slit; the comparison spectra were impressed upon each plate from eight to twelve times; and a reasonably constant temperature was maintained throughout the exposure. It is possible that assigned probable errors ±.007 and ±.009 would be better measures of the accuracy attained.

The wave-lengths resulting from this investigation are:

N_1	5007.023	(Rowland system)
	5006.847	(International system)
N_2	4959.080	(Rowland system)
	4958.902	(International system)

In the following table we collect the results obtained by modern observers.

WAVE-LENGTHS OF THE NEBULAR LINES N_1 AND N_2

	N_1	Wt.	N_2	Wt.
Keeler	5007.05±.03	1	4959.17±.04	¼
Hartmann	7.04	¼	9.17	¼
Wright	6.993±.012	2	9.095±.013	2
Campbell & Moore	7.023±.006	3	9.080±.008	3
Mean	5007.018		4959.094	(Rowland system)

Keeler's numerous results for N_1 were obtained by visual methods, and it is probably fair to give them weight 1 on the scale adopted. The value which Keeler gave for N_2 was 4959.02A, but Hartmann's observations of the relative intensities of the components of the double iron line at 4957.480 and 4957.785A, which Keeler used as a reference line in determining the position of N_2, make it probable that Keeler really observed only the component at 4957.785A, instead of a blend of the two components. Hartmann's opinion that Keeler's value for N_2 should be increased on this account is undoubtedly correct in principle. Whether the increase should be $\frac{1}{2}$ (.785 — .480) A = 0.15A, as Hartmann proposed, seems doubtful, in view of the unknown and unknowable constants of the electric spark which Keeler used. Duplicating Keeler's conditions as well as he could, Hartmann estimated that the less refrangible component of the iron doublet at 4957A has five times the intensity of the more refrangible component, and therefore that the wave-length of the comparison line as used by Keeler was really 4957.785A, instead of 4957.634A. Our experiments with photographs of this iron doublet may be worth recording.

(*a*) The position of the recorded line in the arc spectrum of iron is about 4957.63: the two components seem to be equally effective, as Hartmann and others have found.

(*b*) The position of the line in the spark spectrum of iron, when the line is not over-exposed, is about

4957.79: the violet component is ineffective, in agreement with Hartmann's visual observation.

(*c*) In both the arc and the spark spectra of iron the line at 4957, as photographed on ordinary plates, is by far the most intense iron line in the whole range of spectrum to the red of the close doublet at 4920A. To record the other iron lines needed for the comparison spectrum in the region embracing N_1 and N_2 and not to over-expose the 4957 line is difficult. The center of this line in the spark spectrum, when over-exposed, seems to us to be slightly to the violet of 4957.79: the fainter component seems to become effective.

(*d*) When self-induction has been used in obtaining the spark spectrum of iron, the wave-length of the iron line in question is appreciably less than 4957.79.

We are using $4959.02 + 0.15 = 4959.17$ as Keeler's wave-length of N_2, but are assigning it weight ¼, in view of some uncertainty affecting the iron comparison line, but especially in view of the small number of visual observations involved.

Hartmann's positions of N_1 and N_2 depend upon only three spectrograms: one obtained with 1-prism dispersion, and two with 3-prism dispersion. In reality, the first of the 3-prism plates, according to Hartmann's description of the Hβ line (“a very weak trace of Hβ”), should be discarded, inasmuch as the wave-length determinations of N_1 and N_2 depend absolutely upon the measured position of Hβ. His final wave-lengths depend very largely upon the measured displacement of Hβ (“Hβ rather weak”) on the third plate alone. To two of our own plates having Hβ slightly under-exposed, each measured by three observers, we have assigned weights ¼, and they enter into the final results with still smaller weights.

Wright's observations, with 3-prism dispersion in 1901 and with 3-prism and grating dispersion in 1912–13, are described, or referenced, in the paper following this one. To his wave-lengths of N_1 and N_2 we have given weight 2.

To our own determination we have ventured to assign weight 3.

We recommend that the above weighted mean values be rounded off to

5007.02	4959.09	(Rowland system)
5006.84	4958.91	(International system)

W. W. Campbell,
J. H. Moore.

December 8, 1915.

THE WAVE-LENGTHS OF THE NEBULAR LINES N_1 AND N_2

Values of the wave-lengths of the nebular lines N_1 and N_2 were published about fourteen years ago in one of these Bulletins.[1] The measurements on which those values depend were made almost entirely with reference to the hydrogen line Hβ and the helium lines 4922A and 5015A as comparisons. The assumed wave-lengths of these reference lines, based on the best data then available, were 4861.50, 4922.10 and 5015.73A. In presenting the results attention was called to the fact that the single iron comparison line measured on these plates gave a very considerable residual with reference to this system. This fact did not at the time, however, appear to warrant the suspicion of inaccuracy in the assumed wave-lengths of the comparison lines.

As has been pointed out in the foregoing article by Messrs. Campbell and Moore, the helium wave-lengths have been re-determined by both Eversheim and Rayleigh through the use of interference methods, and the assumed wave-lengths of the third reference line mentioned above is shown to be greatly in error. This error was naturally transferred, in part, to the nebular wave-lengths, with the result that my published values of N_1 and N_2 are too small. A new reduction of the measures, using the corrected values of the wave-lengths of the comparison lines has been made. These wave-lengths, as at present assumed, are:

λ (Rowland)	Element	Authority
4861.50	Hydrogen	W. E. Curtis
4922.11	Helium	Rayleigh and Eversheim
5015.86	Helium	Rayleigh and Eversheim

The corresponding new values of N_1 and N_2, as derived from the observations described in the earlier Bulletin, are:[2]

Orion nebula (2 obsns.)	N. G. C. 7027 (1 obsn. of wt. ½)
4959.07	4959.00
5007.00	5006.91

Orion nebula	N. G. C. 7027
4959.07	4958.95
5006.91	5006.80

Since the measures above described were published I have re-observed these lines occasionally in connection with a more extensive investigation of the spectra of the nebulae. The instrument used was the same 3-prism spectrograph with which the earlier observations were made, except in two cases where the comparatively high dispersion of a grating was used. The reductions of the more recent observations are based almost entirely on the iron and titanium spectra, as published by Burns and Kilby respectively. In the following list are all of my observations of these lines, including the older ones referred to above. The symbol (*) indicates that a grating was used.

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[1] *L. O. Bull.*, 1, 153, 1902.

[2] The wave-lengths, as originally given are:

Date	Object	(Rowland)	(Rowland)	
1901 Dec. 16	Orion	4959.09	5007.04	
1901 Dec. 17	Orion	59.04	06.95	
1912 Jan. 12	Orion	59.14*	06.96*	
1912 Jan. 31	Orion	---------	07.08* wt. ½	
1912 Aug. 21	N.G.C. 6572	59.12	07.03	
1912 Aug. 30	N.G.C. 6572	59.19	07.04	
1901 Sept. 15	N.G.C. 7027	59.00 wt. ½	06.91 wt. ½	
1912 Aug. 16	N.G.C. 7027	59.02	06.91	
1913 Aug. 21	N.G.C. 7027	59.09	06.99	
1912 Aug. 30	N.G.C. 7027	59.11	07.02	
	Means	4959.095±.013	5006.993±.012	(R.)
		4958.92	5006.82	(I.A.)

The correction for the elimination of radial velocity, etc., was in each case determined from the $H\beta$ line on the plate under consideration. The measured displacement of this line was multiplied by a factor representing the wave-length ratio, and applied as a correction to the measured wave-length of the nebular line. This procedure assumes that the small displacement due to curvature of the lines bears the same proportionality to wave-length that the velocity correction does. While this is not strictly true the error introduced by the assumption is inappreciable.

My observations are not so numerous as those of Messrs. Campbell and Moore, and it may be that the conditions under which mine were made were not so favorable. My apparatus was made of wood, and theirs of metal, which would give them the advantage of greater stability; still, with the method of reduction used, the determinations are almost wholly differential, and most errors would naturally be eliminated.[3] The results are probably of value as having been secured with a separate set of prisms, in a different mounting, and in two cases with a grating. The discrepancy between our respective values is of the order of twice the probable error, which may be considered a not unsatisfactory agreement between independent determinations.

W. H. Wright.

January, 1916.

Issued March 18, 1916.

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[3] The differential character is perhaps lacking, in some respects, with regard to the measurement of the spectrum of N. G. C. 7027. In this object the lines N_1 and N_2 are so bright, as compared with the $H\beta$ nebular line, whose displacement is used to standardize them, that they are over-exposed on those plates which exhibit $H\beta$ at its proper density. This is especially true in the case of N_1, which is much the stronger of the two. The fact has been referred to by Messrs. Campbell and Moore, who in some of their observations attempted to mitigate the evil by partial exposures. In general, such a condition is liable, if there should be certain defects in the optical system of the spectrograph, to introduce an error corresponding to the *magnitude equation* encountered in other branches of astronomical work. The effect in one spectrograph or another would depend on the character of any instrumental imperfection. My values of N_1 as measured in this nebula are .03A smaller than the adopted mean, and their omission would result in a value of 5007.01 for N_1. The results of Messrs. Campbell and Moore appear to be, on the other hand, systematically too large in this nebula. Their value from this object exceeds their general mean by .01A. Freed from the observation of this nebula our respective results for N_1 are:

5007.011 (W)
5007.021 (C & M)

This brings the two sets of determinations into better agreement. It is considered preferable, however, to keep all of the observations in striking the mean for the final result.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 280

ELEMENTS AND EPHEMERIS OF COMET a 1916 (NEUJMIN)

The results given below were obtained from the following observations:

1916 Gr.M. T.	Comet's Apparent		Observer
	α	δ	
Feb. 29.6120	8^{h} 58^{m} $46^{s}.28$	$+13^\circ$ 35′ $18''.2$	Van Biesbroeck, Yerkes
Mar. 3.4877	8 59 22.9	12 04 05	Hartwig, Bamberg
Mar. 7.6585	9 01 03.5	+ 9 53 20	Aitken

ELEMENTS

$T =$ March 11.2195 Gr. M. T.

$$\left.\begin{aligned}\omega &= 193^\circ\ 44'.1\\ \Omega &= 327\ \ \ 38.8\\ i &= \ \ 10\ \ \ 29.6\end{aligned}\right\}1916.0$$

$e = 0.55465$

$\log q = 0.12528$

$\log a = 0.47658$

$\mu = 684''.14$

$P = 5.186$ years

CONSTANTS FOR THE EQUATOR 1916.0

$$x = r\,[9.99793]\sin(251^\circ\ 49'.1 + v)$$
$$y = r\,[9.92761]\sin(158\ \ 17.2 + v)$$
$$z = r\,[9.73342]\sin(170\ \ 34.0 + v)$$

EPHEMERIS FOR GREENWICH MEAN MIDNIGHT

1916	True α	True δ	Log Δ	Br.
Mar. 16.5	9^{h} 07^{m} 46^{s}	$+$ 5° 26′.4	9.6112	1.03
18.5	09 46	4 29.7		
20.5	11 56	3 34.3		
22.5	14 17	2 40.4	9.6312	0.93
24.5	16 48	1 48.0		
26.5	19 29	0 57.1		
28.5	22 19	$+$ 0 07.8		
Mar. 30.5	25 19	— 0 40.0	9.6625	0.79
Apr. 1.5	28 27	1 26.3		
3.5	31 44	2 11.2		
5.5	35 08	2 54.6		
7.5	38 39	3 36.6	9.6977	0.66
9.5	42 17	4 17.2		
Apr. 11.5	9 46 02	— 4 56.5		
Apr. 13.5	9^{h} 49^{m} 52^{s}	— 5° 34′.5		
15.5	53 48	6 11.3	9.7356	0.53
17.5	9 57 49	6 46.8		
19.5	10 01 54	7 21.2		
21.5	06 03	7 54.5		
23.5	10 16	8 26.7	9.7750	0.43
25.5	14 32	8 57.8		
27.5	18 52	9 28.0		
Apr. 29.5	23 15	9 57.2		
May 1.5	10 27 40	—10 25.6	9.8156	0.34

Brightness March 3 = 1.00

The last observation available is one by Van Biesbroeck at the Yerkes Observatory. This was sent by mail by Director Frost. The observation and its representation are:

1916 Gr.M.T.	Comet's Apparent		(O — C)	
	α	δ	$\cos\delta\Delta\alpha$	$\Delta\delta$
Mar. 8.5666	9^{h} 01^{m} $32^{s}.26$	$+9^\circ$ 25′ $08''.2$	$+2''$	$-2''$

The comet is receding from both the Sun and the Earth. The nearest approach to the Earth occurred near the time of discovery, February 24. It was then estimated to be of magnitude 10.5.

Observations for the computation of an orbit from a shorter arc were not available. A parabolic orbit computed from the observations given above gave a residual of 4′ in the first declination, which could not be removed by a differential correction.

JESSICA M. YOUNG,
HAMILTON M. JEFFERS,
TRACY A. PIERCE.

BERKELEY ASTRONOMICAL DEPARTMENT,
March 14, 1916.

Issued March 18, 1916.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 281

ELEMENTS AND EPHEMERIS OF COMET *e* 1915 (TAYLOR)

The elements given below were derived from three normal places, formed from observations over as long an arc as was available. The adopted normal places corrected for parallax are:

Gr.M.T.	α (1916.0)	δ (1916.0)
1915 Dec. 8.5000	$80^\circ\ 25'\ 40''.7$	$+\ 0^\circ\ 21'\ 48''.6$
1916 Jan. 8.5000	76 40 33.4	+10 36 28.4
1916 Apr. 5.6669	113 54 38.7	+33 03 48.4

The dates of the normal places have been corrected for aberration. The resulting elements are:

$$\text{Epoch} = 1916 \text{ Jan. } 8.5 \text{ Gr. M. T.}$$

$$M_0 = 356^\circ\ 31'\ 33''.0$$

$$\left.\begin{array}{l}\omega = 354^\circ\ 49'\ 00''.1\\ \Omega = 113\ \ 53\ \ 16.3\\ i = \ 15\ \ 31\ \ 43.7\end{array}\right\}1915.0 \qquad \left.\begin{array}{l}\omega = 354^\circ\ 49'\ 01''.6\\ \Omega = 113\ \ 54\ \ 05.1\\ i = \ 15\ \ 31\ \ 43.5\end{array}\right\}1916.0$$

$$e = 0.546458$$

$$\log a = 0.535922$$

$$\mu = 557''.3450$$

$$T = 1916 \text{ Jan. } 30.9403 \text{ Gr. M. T.}$$

$$P = 6.3662 \text{ years}$$

CONSTANTS FOR THE EQUATOR, 1915.0

$$x = r\,[9.986582] \sin(199^\circ\ 30'\ 11''.9 + v)$$

$$y = r\,[9.981743] \sin(113\ \ 47\ \ 29.1 + v)$$

$$z = r\,[9.573930] \sin(\ 70\ \ 52\ \ 22.8 + v)$$

CONSTANTS FOR THE EQUATOR, 1916.0

$$x = r\,[9.986585] \sin(199^\circ\ 31'\ 03''.4 + v)$$

$$y = r\,[9.981752] \sin(113\ \ 48\ \ 15.3 + v)$$

$$z = r\,[9.573864] \sin(\ 70\ \ 53\ \ 04.2 + v)$$

In the formation of the normal places all observations at hand which were near the adopted dates were investigated, and the more consistent of them were utilized. The observations used, together with their representations by the final elements, are:

Gr.M.T.	Comet's Apparent α	Comet's Apparent δ	O — C cosδΔα	Δδ	Observer, place
1915 Dec. 5.3908	$5^h\ 23^m\ 24^s.12$	$-\ 0^\circ\ 11'\ 43''.7$	$+5''.2$	$+5''.3$	Millosevich, Rome
6.6957	5 22 42.9	+ 0 01 36	+2	+3	Burton, Washington
7.7719	5 22 07.5	+ 0 13 24	—2	+4	Van Biesbroeck, Yerkes
8.5337	5 21 41.90	+ 0 22 03.6	—2.6	0.0	Thiele, Hamburg
8.7004	5 21 36.8	+ 0 24 05	+1	+1	Burton
9.6935	5 21 02.60	+ 0 36 04.8	—3.5	—1.1	Van Biesbroeck
9.7877	5 20 59.6	+ 0 37 16	0	—1	Aitken
10.4364	5 20 36.93	+ 0 45 30.5	+1.7	+2.2	Palisa, Vienna
1916 Jan. 7.6431	5 06 49.73	+10 14 31.7	—2.3	—1.5	Dawson, Ann Arbor
7.6736	5 06 49.28	+10 15 19.4	—2.9	—0.4	Dawson
8.4104	5 06 43.81	+10 34 02.9	+2.0	—1.5	Palisa, Vienna
8.6645		+10 40 34.1		—0.9	Dawson
8.6751	5 06 42.35		+5.5		Dawson
9.4956	5 06 37.30	+11 01 48.7	—1.8	—1.3	Hartwig, Bamberg
11.2661	5 06 34.86	+11 47 29.1	+0.7	+3.0	Strömgren, Copenhagen

The observations used to form the third normal place are photographic. They were made with the Crossley reflector, and are point images. The positions are (uncorrected for parallax):

	Gr.M.T.	α (1916.0)	δ (1916.0)	O — C cos δΔα	Δδ	Observer
1916 Apr.	5.6663	7^{h} 35^{m} $36^{s}.96$	+33° 03′ 47″.3	—2″.2	—0″.8	Green
	5.6825	7 35 39.88	33 03 48.0	+2.9	—0.1	Green

The residuals used in forming the normal places were derived by a comparison of the foregoing observations with the elements given in A. N. 4827.

The comet is quite well situated for further observation, and, since more observations are desirable, an ephemeris has been computed. It is now a rather difficult object for visual observation, being only of about the fifteenth magnitude, and probably it will be impossible to observe the comet when the Moon is between the first and last quarter. However, for the sake of continuity, the ephemeris has been continued over these intervals. The estimates made of the brightness at the date for which the brightness is taken as unity varied between the tenth and twelfth magnitudes.

Ephemeris for Greenwich Mean Midnight

1916	True α	True δ	log Δ	Br.
May 26.5	9^{h} 46^{m} 40^{s}	+27° 19′.1	0.2864	0.09
28.5	51 19	26 56.4		
May 30.5	9 55 55	26 33.4		
June 1.5	10 00 29	26 10.1		
3.5	05 01	25 46.4		
5.5	09 30	25 22.4		
7.5	13 57	24 58.1	0.3225	0.07
9.5	18 22	24 33.5		
11.5	22 44	24 08.7		
13.5	27 03	23 43.7		
15.5	31 20	23 18.5		
17.5	35 35	22 53.1		
19.5	39 48	22 27.4	0.3565	0.06
21.5	43 59	22 01.7		
23.5	48 08	21 35.8		
25.5	52 14	21 09.8		
27.5	10 56 19	20 43.7		
June 29.5	11 00 21	20 17.5		
July 1.5	04 22	19 51.2	0.3883	0.05
3.5	08 20	19 24.8		
July 5.5	11 12 17	+18 58.4		
July 7.5	11^{h} 16^{m} 12^{s}	+18° 32′.0		
9.5	20 06	18 05.6		
11.5	23 57	17 39.2		
13.5	27 47	17 12.8	0.4180	0.04
15.5	31 36	16 46.4		
17.5	35 23	16 20.1		
19.5	39 08	15 53.8		
21.5	42 52	15 27.6		
23.5	46 35	15 01.4		
25.5	50 16	14 35.2	0.4455	0.03
27.5	53 55	14 09.1		
29.5	11 57 34	13 43.1		
July 31.5	12 01 11	13 17.3		
Aug. 2.5	04 47	12 51.5		
4.5	08 22	12 25.9		
Aug. 6.5	12 11 56	+12 00.4	0.4707	0.03

Brightness December 8 = 1.00

While the proofs of this Bulletin were being read a letter was received from Professor E. E. Barnard, giving details as to the physical appearance of the comet. The comet was observed to be double on February 9, the nuclei being 10″ apart. By March 24 the southern comet, which was originally brighter by two magnitudes, had faded until it was no longer visible, and the companion of the original comet was all there was to be seen. These observations would seem to indicate a possibility of the third normal place being in error by a few seconds of arc, owing to the fact that the center of illumination may not have coincided with the center of mass.

It is interesting to note that, with the exception of the longitude of the node, the elements of this comet are in general agreement with those of Comet *e* 1909 (Daniel).

HAMILTON M. JEFFERS,
FERD. J. NEUBAUER.

BERKELEY ASTRONOMICAL DEPARTMENT,
May 14, 1916.

NOTE ON THE RETURN OF COMET *e* 1909 (DANIEL)

New elements have been computed for this comet, based upon normal places formed from observations extending from December 7, 1909, to April 11, 1910. The change in the nature of the orbit is so small that the elements published in LICK OBSERVATORY BULLETIN, No. 179, may be considered satisfactory.

The undisturbed time of perihelion is 1916 May 23.422 Gr. M. T. The following tabulation shows that the comet will not be favorably situated for observation:

Date, Gr.M.T.	Right Ascension Comet	Right Ascension Sun	Declination Comet	Declination Sun	Log Δ
1916 May 23.5	4h 30m	4h 01m	+22°5	+20°7	0.377
July 2.5	6 54	6 46	+29.1	+23.0	0.391
Aug. 11.5	9 11	9 25	+27.0	+15.1	0.418
Sept. 20.5	10 59	11 51	+20.3	+ 0.9	0.443
Oct. 30.5	12 23	14 19	+13.6	—13.9	0.452

These positions may be considerably in error due to the fairly close approach to *Jupiter*. During most of December, 1911, and January, February, and March, 1912, the comet was within seven-tenths of an astronomical unit of *Jupiter*. From September, 1911, to August, 1912, it was within one astronomical unit of the planet.

S. EINARSSON,
MARGARET HARWOOD.

BERKELEY ASTRONOMICAL DEPARTMENT,
May 14, 1916.

Issued May 22, 1916.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 282

ELEMENTS AND EPHEMERIS OF COMET *b* 1916 (WOLF)

The geocentric distance of this comet at the time of discovery (1916 April 27) was greater than that of any other comet at discovery. It was approximately 4.1 astronomical units, according to the elements given below. On account of this great distance and consequent slow motion there is a large range of possible solutions for an orbit based upon short intervals.

After the announcement of the discovery Professor Barnard of the Yerkes Observatory found that he had an image of the object on plates of April 24. This is the earliest observation available. Using this and observations of May 4 by Van Biesbroeck and May 10 by Barnard an orbit was computed. These observations and also one on May 6 by Barnard were satisfactorily represented by a parabola with perihelion date in 1914 and an inclination of 95° with only one parabolic solution possible. Just as this orbit was completed Van Biesbroeck's elliptic elements were received. These were so radically different that an erroneous observation was feared and the elements were withheld pending further investigation. In the meantime an observation of May 23 by Van Biesbroeck was kindly sent by Director Frost of the Yerkes Observatory. Comparison of this observation with our parabola showed a residual of +2′ 42″ in α and —51″ in δ. On the basis of these residuals the parabola was corrected differentially so as to give another parabola representing the right ascensions and declinations of May 10 and May 23, and the right ascension of April 24, leaving a residual of only 1″ in the declination of April 24. These three observations, the elements and ephemeris based upon them are:

	Comet's Apparent		
1916 Gr.M.T.	α	δ	Observer
April 24.6365	$12^h\ 41^m\ 42^s.4$	+2° 10′ 23″	Barnard, Yerkes.
May 10.6331	12 34 36.0	+3 24 13	Barnard.
May 23.7135	12 30 37.2	+4 08 13	Van Biesbroeck, Yerkes.

ELEMENTS

$$T = 1917 \text{ June } 16.4806 \text{ Gr. M.T.}$$

$$\left.\begin{aligned} \omega &= 120^\circ\ 37'\ 07''.9 \\ \Omega &= 183\ \ 16\ \ 58.8 \\ i &= 25\ \ 40\ \ 06.4 \end{aligned}\right\} 1916.0$$

$$\log q = 0.226855$$

(O–C)	I	III
$\cos\delta\Delta\alpha$	0″	0″
$\Delta\delta$	+1	0

CONSTANTS FOR THE EQUATOR 1916.0

$$x = r\,[9.999866] \sin(\ 33^\circ\ 34'\ 41''.8 + v)$$
$$y = r\,[9.999682] \sin(303\ \ 37\ \ 57.6 + v)$$
$$z = r\,[8.657307] \sin(\ 90\ \ 30\ \ 25.9 + v)$$

EPHEMERIS FOR GREENWICH MEAN MIDNIGHT

1916 Gr. M.T.	True α	True δ	$\log\Delta$	Br.
June 2.5	$12^h\ 28^m\ 59^s.5$	+4° 30′ 12″	0.6201	1.04
4.5	28 48.8	33 29		
6.5	28 41.4	36 22		
8.5	28 37.2	38 51		
10.5	28 36.2	40 56	0.6249	1.05
12.5	28 38.5	42 38		
14.5	28 44.0	43 56		
16.5	28 52.7	44 51		
18.5	29 04.7	45 23	0.6300	1.07
20.5	29 19.9	45 33		
22.5	29 38.2	45 20		
24.5	29 59.8	44 46		
26.5	30 24.5	43 50	0.6352	1.07
28.5	30 52.4	42 33		
June 30.5	12 31 23.5	+4 40 55		

Ephemeris for Greenwich Mean Midnight—(*Concluded*)

1916 Gr. M.T.		True α			True δ			log Δ	Br.
July	2.5	12^{h}	31^{m}	57^{s}7	+4°	38′	56″		
	4.5		32	35.0		36	37	0.6403	1.08
	6.5		33	15.3		33	58		
	8.5		33	58.7		31	01		
	10.5		34	45.0		27	45		
	12.5		35	34.3		24	11	0.6451	1.09
	14.5		36	26.5		20	19		
	16.5		37	21.5		16	10		
	18.5		38	19.4		11	44		
	20.5		39	20.0		07	01	0.6495	1.11
	22.5		40	23.4	4	02	02		
	24.5		41	29.6	3	56	48		
	26.5		42	38.5		51	18		
	28.5		43	50.0		45	33	0.6532	1.13
	30.5		45	04.2		39	34		
Aug.	1.5		46	21.0		33	21		
	3.5		47	40.3		26	54		
	5.5		49	02.2		20	15	0.6562	1.16
	7.5		50	26.5		13	23		
	9.5		51	53.3	3	06	19		
	11.5		53	22.5	2	59	04		
	13.5		54	54.0		51	37	0.6584	1.19
	15.5		56	27.9		44	00		
	17.5		58	04.1		36	12		
	19.5	12	59	42.6		28	14		
	21.5	13	01	23.3		20	07	0.6598	1.23
	23.5		03	06.2		11	50		
	25.5		04	51.4	2	03	24		
	27.5		06	38.8	1	54	50		
Aug.	29.5	13	08	28.4	+1	46	08	0.6602	1.27

Brightness May 10 = 1.00.

With the exception of the element ω this orbit resembles that of Wolf's periodic comet 1884 III, which has a period of approximately seven years. To test the possibility of identity an ellipse of a seven-year period was forced through the same five coordinates used for the parabola leaving all of the residual in the first declination. This residual amounted to +31″.

The accuracy of the observations was tested by representing all of the available observations not used in the computation. As the maximum residual of these is 3″ it is seen that all of the observations are of a high degree of accuracy so that the residual of +31″ resulting from the seven-year ellipse is too large to be accounted for by accumulated error of observation.

To test the range of period for an ellipse to represent these observations an orbit with a fifty-year period was computed. This leaves a residual of +7″ in the first declination. As 7″ or 8″ is about the limit of accumulated error of observation it is safe to say that, if later computations based upon longer arcs show the orbit to be an ellipse, the period cannot be much less than fifty years and probably will be greater. It may be of interest to add that the three observations used in this computation are completely represented by an orbit that is slightly hyperbolic. Therefore the complete range of solution from the available observations extends from an ellipse with a period of about fifty years to an hyperbola, with a lower limit of eccentricity of about 0.86.

This investigation shows, therefore, that this comet is not Wolf's periodic comet of 1884 III, since the perturbations computed by Kamensky (*cf. A. N.* **4460**) cannot account for a large change in the period.

For the sake of comparison the elements of Wolf's periodic comet and those of the seven-year and fifty-year ellipses mentioned above are here tabulated:

1884 III	Ellipse Seven-year Period	Ellipse Fifty-year Period
ω = 172° 49′	170° 54′	126° 10′
☊ = 206 38	187 22	184 07
i = 25 16	40 34	27 49
log q = 0.2022	0.1824	0.2290
e = 0.557	0.5842	0.8752
	$\Delta\delta_1$ = +31″	+7″

The observations available and not used in this computation, together with their representation by our parabolic orbit are:

1916 Gr.M.T.		Comet's Apparent α			Comet's Apparent δ			(O–C) cos δΔα	(O–C) Δδ	Observer
May	4.8565	12^{h}	36^{m}	55^{s}6	+2°	59′	51″	+0^{s}1	0″	Van Biesbroeck, Yerkes.
	6.6559	12	36	10.3	3	07	46	+0.2	+1	Barnard, Yerkes.
	6.6846	12	36	09.5	3	07	53	+0.1	+2	Burton, U. S. Naval Observatory.
	8.6889	12	35	20.9	3	16	21	0.0	+1	Hubble, Yerkes.
	18.6680	12	31	55.7	3	53	09	+0.1	—1	Van Biesbroeck.
	22.6721	12	30	51.9	4	05	19	0.0	0	Barnard.
	27.6893	12	29	48.6	4	18	17	—0.1	0	Barnard.
	27.7933	12	29	47.4	4	18	30	—0.1	—2	Aitken.
	28.6536	12	29	38.8	4	20	28	+0.1	—2	Van Biesbroeck.
May	30.7009	12	29	19.9	+4	24	22	+0.1	(—30)	Van Biesbroeck.

There is probably an error of 30″ in the transmission of the declination of the observation of May 30. The last four of these observations were received after the orbit was completed. With the exception of the declination of the last one it is seen that they are represented with great accuracy.

Berkeley Astronomical Department,
June 2, 1916.
Issued June 5, 1916.

R. T. Crawford,
Dinsmore Alter.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 283

MODERN POSITIONS OF ASTRONOMISCHE GESELLSCHAFT ZONE STARS

The star places of the A. G. zones are in general use at the present time for purposes where only a moderate degree of precision is required. For more accurate places modern observations must be secured, partly owing to the uncertainty of possible proper motion in the interval of thirty years or more since the epoch of those zone observations. The A. G. zones were planned to secure observations of every star of the Bonner Durchmusterung of the ninth magnitude or brighter. Earlier zone work and Gould's zones at Cordoba aimed at a nearly uniform distribution of stars, without being inclusive to any definite limiting magnitude.

Modern observations are also useful as a criterion to indicate the accuracy of the earlier star places, for the discussion of their systematic errors, and for the detection of proper motions. Some of the proper motions which were suspected at the epoch of the A. G. zone reductions, from comparison with the older zone catalogues, are not confirmed by recent observations, and the discordance must be attributed to the effect of errors in the various determinations.

In 1914 some six hundred A. G. stars were observed with the meridian circle of the Lick Observatory, and the observations can be used for the purposes stated above. The list of stars was selected by Dr. Frank Schlesinger to be used for standard stars in the reductions of the photographs of the zone between the limits +2° and —2° 10′ declination. The photographic plates secured under his direction at the Allegheny Observatory cover an area of slightly over four degrees in declination by six degrees in right ascension, or about six times the area of the plates used in the International Chart of the sky. There are thus about ten of the standard stars on each plate. The observed places of the standard stars will serve as the basis for the reduction of about 7100 stars in the zone, a somewhat smaller number than that of the corresponding area in the A. G. catalogues.

Of the stars selected as standards, 400 are catalogued in the Nicolaief A. G. zone, 202 are in the Albany A. G. zone only, and 88 of the stars are contained in the overlapping region of the two zones. One other star of the Albany catalogue was also observed here, and the comparisons thus include 603 stars. The star places of the A. G. zones are strictly differential, having been based upon the places of the bright stars of the fundamental system of Dr. Auwers, in the neighborhood of each zone. The same fundamental star system has been used in the reduction of the Lick Observatory observations, but it had been subjected to a thorough revision by Dr. Auwers, in the interval following the A. G. epoch. At least two observations of each star were made in the A. G. zones, and the requirements were that the probable error of a star position should not exceed $\pm 0''.6$ in either co-ordinate. The greater part of the work exceeded this requirement in precision.

The 1914 Lick Observatory series includes four observations of each star, two in each position of the instrument, fixed circle east and west. The probable errors have been derived from the discussion of all the observations, and are, $\pm 0^s.013$ in right ascension, and $\pm 0''.19$ in declination, for the mean of four observations. The effect of division error is included in the probable error of declination, and the two co-ordinates thus have equal precision.

In bringing up the A. G. zone places to the present epoch, proper motions have usually been applied when these had been determined. But when the Lick Observatory positions do not confirm the suspected proper motions, these have been rejected. Over twenty new proper motions have also been adopted on the evidence of these comparisons, but only when supported by the comparison with the older zone observations, preceding the A. G. epoch. A list of new and old proper motions, with some few corrections to the A. G. catalogues will be given below. In general, suspected proper motions of less than 0″1 per year have not been adopted on the evidence of these comparisons, as differences of $0^{s}2$ and of 3″ might be due to the accidental errors of the old observations.

A rigorous comparison with all old observations would undoubtedly bring to light many small proper motions, but the old catalogues often require systematic corrections, which should first be thoroughly investigated. The mean differences between the Lick Observatory positions and those of the two A. G. catalogues are as follows:

L. O. — N., $+0^{s}009$, and +0″15, for 400 stars, at average declination —0°8

L. O. — A., $-0^{s}037$, and —0″17, for 203 stars, at average declination +1°6.

The first comparison, with Nicolaief, includes the 88 stars common to both A. G. catalogues. If these be included in the comparison with Albany there results: L. O. — A., $-0^{s}039$, and —0″14 for 291 stars; which differs but slightly from the above comparison of the stars as originally selected by Dr. Schlesinger. The correction for magnitude equation, $-0^{s}008$ per unit of magnitude, has been applied to my observed values of the right ascensions of the faint stars in this series. The mean of the magnitudes of the fundamental stars is 3.9, and the average of the corrections for the faint stars is $-0^{s}036$, corresponding to an average magnitude of 8.4.

It is evident that the application of the magnitude equation has brought the right ascensions of Nicolaief nearly into conformity with my recent observations. No magnitude equation was applied in the Nicolaief zone reductions. Without the application of my magnitude equation the Albany right ascensions would be in precise agreement with mine, which is an indication that the magnitude equation of Professor Lewis Boss, who observed all the transits at Albany, was similar to my own. He determined his correction at the time of the zone observations, obtaining an average of $0^{s}014$ per unit, with some evidence of variation at different parts of the scale of magnitudes, but the correction was not applied to the right ascensions of the Albany catalogue.

If the Lick Observatory series of star places be considered as a homogeneous standard, with which the places of the A. G. catalogues can be compared, the two catalogues have evident systematic differences. For the means of the two zones these would amount to approximately $0^{s}05$, and 0″3, for the two co-ordinates, respectively. The comparison of the stars common to both catalogues gives the relative differences: N. — A., $-0^{s}51$, and —0″76. The discordant values of the differences in declination indicate a systematic variation, which deserves special investigation. All of the stars in the overlapping region of 20′ of declination were made the subject of comparison in the Introduction to the Nicolaief zone catalogue, and were tabulated in detail in the notes to that catalogue. The differences there presented were: N. — A., $-0^{s}065$, and —0″74, for 656 stars. And in the *Astronomische Nachrichten,* volume 161, page 35, are published the results of the comparison of these two catalogues, made by Dr. Auwers, as part of the exhaustive comparison of all the overlapping regions in the various A. G. catalogues that had been printed at that time, 1903. He derived the mean differences: N. — A., $-0^{s}061$, and —0″70, for 673 stars. The difference between the results of the last two comparisons is due to the rejection of individual cases of suspected proper motion.

The difference in the right ascensions of the two catalogues can be accounted for by the existence of different magnitude equations for the various observers, of the usual form. It is not so easy to account for the comparatively large difference in the declinations. Dr. Auwers discussed the differences with respect to magnitude for both co-ordinates, and derived a correction for each, depending upon the magnitudes of the stars. The range of brightness is small in the stars selected for this list, the greater part of them being between eighth and ninth magnitudes, and we may properly look for the origin of the systematic variations in the combination with respect to declination. The differences from the Lick Observatory places are arranged below, in Tables I and II, in groups of half a degree of declination. The tables give the number of stars in each group, and the mean declinations of the groups. In the comparison with the Nicolaief catalogue no systematic variation with respect to declination is evident in $\Delta\alpha$; but the variation in $\Delta\delta$ is notable, and the residuals are tabulated accordingly for each group. These residuals give a rate of +0″25 for one degree, and in the last column are tabulated the differences, O.–C., between the observed values of $\Delta\delta$ and the values computed with this rate. In place of a mean residual of ±0″32, the mean of the last column is ±0″20.

In Table II the similar groups for the Albany catalogue, treated in a similar manner, show no systematic variation in $\Delta\delta$; but the variation in $\Delta\alpha$ is plainly evident. It amounts to $+0^{s}044$ for one degree, and by taking account of this variation in $\Delta\alpha$ the mean group residual, $\pm 0^{s}016$, is replaced by the mean difference O.–C., $\pm 0^{s}005$. In the last two groups of Table I are included the stars observed at Albany, besides a few which are now outside the limits of the Albany zone, owing to precession in the interval. The first two groups of Table II include these same stars, all of the first group and nearly one-half the second group being found in the Nicolaief catalogue. But few stars were selected in this list within one degree of the equator.

Systematic variations with respect to the declination of the stars thus appear in $\Delta\alpha$ at Albany, and in $\Delta\delta$ at Nicolaief. These variations could not have been detected in the small overlapping area of the two zones, but become evident in this method of more extended comparison with modern observations.

TABLE I

L. O.—Nic. 1914.0

No.	Decl.	$\Delta\alpha$	$\Delta\delta$	v	O.–C.
26	$-2^{\circ}.1$	$-0^{s}.008$	$-0''.12$	$-0''.42$	$-0''.02$
101	-1.7	$+\ .008$	$+\ .14$	$-\ .16$	$+\ .14$
112	-1.3	$+\ .018$	$-\ .18$	$-\ .48$	$-\ .28$
53	-0.8	$+\ .016$	$+\ .07$	$-\ .23$	$-\ .15$
8	-0.3	$+\ .032$	$+\ .74$	$+\ .44$	$+\ .39$
6	$+0.3$	$-\ .035$	$+\ .47$	$+\ .17$	$-\ .03$
38	$+0.8$	$-\ .015$	$+\ .90$	$+\ .60$	$+\ .28$
56	$+1.2$	$+\ .015$	$+\ .40$	$+\ .10$	$-\ .32$
Group Means		$+0.004$	$+0.30$	± 0.32	± 0.20

Rate of $\Delta\delta = +0''.25$ to one degree.

TABLE II

L. O.—Alb. 1914.0

No.	Decl.	$\Delta\alpha$	v	O.–C.	$\Delta\delta$
33	$+0^{\circ}.9$	$-0^{s}.067$	$-0^{s}.028$	$-0^{s}.002$	$-0''.09$
119	$+1.3$	$-\ .043$	$-\ .004$	$+\ .005$	$-\ .12$
107	$+1.7$	$-\ .033$	$+\ .006$	$-\ .003$	$-\ .22$
32	$+2.1$	$-\ .013$	$+\ .026$	$.000$	$+\ .02$
Group Means		-0.039	± 0.016	± 0.005	-0.10

Rate in $\Delta\alpha = +0^{s}.044$ to one degree.

The differences are arranged in Table III, for the investigation of systematic variation with respect to right ascension. The table gives the number of stars, and the mean differences, for each hour of right ascension, in each catalogue, and the differences, N.—A., which correspond to the means of the declinations of the zones. A periodic term has been derived for $\Delta\alpha$, of similar character for both catalogues, and has been tabulated in the columns adjoining the observed $\Delta\alpha$. The table includes the average residual, v, for the hourly groups, and the average difference, O.–C., between the hourly observed values of $\Delta\alpha$ and the quantity derived by the use of the periodic variation. The individual values of $\Delta\alpha$ were tested with the application of this periodic variation, and only a very slight decrease in the average of the residuals resulted. The conclusion would be that the individual differences are mainly due to the accidental errors of observation, and that these quite overwhelm the periodic variations.

The fact that the comparisons with both the A. G. catalogues show similar periodic terms in $\Delta\alpha$ would indicate that the Lick Observatory right ascensions may include a periodic term of this character. But it is quite probable that the revision of the fundamental system, to which reference was made above, may have introduced a slight change of periodic nature in the standard right ascensions. The respective columns of $\Delta\delta$ do not give any clear indication of periodic variation of simple form. The accidental errors are undoubtedly large enough to mask the effect of small periodic terms. The fact that the average hourly residuals of the differences between the A. G. catalogues are of the same order as the residuals from the comparison with the modern observations is an indication that the differences are mainly due to accidental errors.

Without including the effect of systematic variations, and allowing for the differences due to the errors of the Lick Observatory determinations, the probable errors of the A. G. catalogue star positions brought up to the present epoch have been derived from the residuals of all the stars of this list.

They are: Nicolaief, $\pm 0^{s}.040$, and $\pm 0''.81$;
and, Albany, ± 0.044, and ± 0.58.

The right ascensions are of about the same degree of precision, while the Albany declinations have double the weight of those of Nicolaief. These probable errors are in nearly the same proportion as those derived at the epoch of observation by the respective observers, allowing for an appreciable increase due to the undetermined small proper motions. The known proper motions had been applied before summing up the residuals. It is plain that good modern determinations would have approximately a weight of ten, in comparison with the places of these A. G. catalogues brought up to this epoch. The greater part of the circle readings for the Albany A. G. zone were made by the writer.

If the increase in the probable errors of the old star positions may be ascribed mainly to the effect of proper motion, we have the means of deriving the average proper motion for stars of this class. One cannot safely rely upon the comparison of probable errors for exact deductions, since it is well known that probable errors derived by the comparison of different measures in a series are likely to be smaller than those derived from two separate sets of measurements. This should be taken into consideration in the estimation of the probable errors above of the A. G. catalogues brought up to the present time. The average number of observations in the two A. G. catalogues has been found to be 2.5 per star. Taking the probable errors of a single observation published in the Albany cata-

TABLE III

L. O.—A. G. 1914.0

		NICOLAIEF				ALBANY			N.-A.	
h	No.	Δα	Per.	Δδ	No.	Δα	Per.	Δδ	Δα	Δδ
0	18	+0ˢ036	+0ˢ04	—0″67	14	—0ˢ004	—0ˢ01	—0″44	—0ˢ040	+0″23
1	17	+ .045	+ .03	+ .07	13	— .014	— .02	— .18	— .059	— .25
2	17	+ .016	+ .03	+ .40	12	— .052	— .02	+ .09	— .068	— .31
3	20	+ .046	+ .02	+ .62	11	+ .001	— .03	+ .25	— .045	— .37
4	16	+ .056	+ .02	+ .18	12	+ .008	— .03	— .14	— .048	— .32
5	16	+ .035	+ .01	— .41	12	— .041	— .04	+ .17	— .076	+ .58
6	19	.000	+ .01	+ .50	15	— .059	— .04	+ .37	— .059	— .13
7	19	+ .027	.00	+ .60	11	— .082	— .05	+ .54	— .109	— .06
8	16	— .028	.00	+ .88	10	— .042	— .05	+ .17	— .014	— .71
9	14	— .029	— .01	+ .10	12	— .046	— .06	— .17	— .017	— .27
10	18	— .009	— .01	— .16	10	— .087	— .06	— .45	— .078	— .29
11	16	— .028	— .02	+ .05	11	— .051	— .07	— .30	— .023	— .35
12	18	— .003	— .02	— .52	9	— .073	— .07	— .01	— .070	+ .51
13	17	— .009	— .01	+ .37	12	— .072	— .06	— .63	— .063	—1.00
14	11	— .044	— .01	— .28	13	— .055	— .06	— .87	— .011	— .59
15	17	+ .010	.00	+ .37	12	— .087	— .05	— .15	— .097	— .52
16	15	— .004	.00	+ .07	14	— .075	— .05	— .42	— .071	— .49
17	18	— .007	+ .01	— .03	12	— .044	— .04	— .49	— .037	— .46
18	14	.000	+ .01	— .25	14	— .049	— .04	+ .02	— .049	+ .27
19	18	+ .014	+ .02	— .58	12	— .008	— .03	— .32	— .022	+ .26
20	18	— .007	+ .02	+ .10	13	— .023	— .03	+ .13	— .016	+ .03
21	15	+ .027	+ .03	+ .11	13	+ .007	— .02	— .35	— .020	— .46
22	18	+ .019	+ .03	+ .43	12	— .044	— .02	— .03	— .063	— .46
23	15	+ .039	+ .04	+ .13	12	+ .010	— .01	+ .08	— .029	— .05
Sum	400				291					
Mean	17	+0.009	+0.01	+0.09		—0.040	—0.04	—0.13	—0.049	—0.22
v		±0.023		±0.30		±0.025		±0.27	±0.021	±0.30
O.-C.			±0.015				±0.018			

logue introduction, for stars up to the ninth magnitude, we have for the probable errors of a star position, ±0ˢ022, and ±0″33. From the mean differences between two single observations, for stars of this class, published in the introduction to the Nicolaief catalogue, I have derived the probable errors of a star position, ±0ˢ030 and ±0″54. Adopting these probable errors, at the respective epochs of observation, and the probable errors derived above for the present epoch, the increase would be, ±0ˢ038, and ±0″48, in the Albany positions, and ±0ˢ026, and ±0″60, in the Nicolaief positions. With an average difference of epochs of thirty-four years in the former, and of twenty-nine years in the latter, the annual proper motions would be:

Albany,	±0ˢ0011, and	±0″014;
Nicolaief,	±0.0009, and	±0.021.

The means of the two are, ±0ˢ0010, and ±0″018, giving an annual average proper motion of ±0″023, on the arc of a great circle. The larger proper motions had been eliminated in the comparisons from which these results are derived. There are about fifty such proper motions listed below, or one for every twelve stars on the average. Even with the uncertainty involved in this means of deriving the average change of position, the general conclusion has some fair amount of probability. Over six hundred residuals are included in the available material. Including the computed proper motions, the annual average would be ±0″03 for all the stars.

In Table IV are listed the proper motions used in the reduction of the A. G. star places, with the indications of possible proper motions which have not yet been confirmed. Seven of the old zone catalogues have been used for comparisons, and some results from modern authorities have been added. When the respective A. G. catalogues contain notes of proper motion, these have been adopted without change; unless the Lick Observatory place does not confirm the movement, when the suspected proper motion has been rejected. About one-third of the proper motions published in the A. G. catalogues were thus rejected. A few errors are noted in the original catalogues. It would have been appropriate to recompute the proper motions, in many cases referred to in the original catalogue notes, but, for the present purpose it has seemed best to leave the results in this form, as the Δα and Δδ of the modern observations show clearly the evidence of confirmation, or variation from the original values.

The table refers to the Albany, and the Nicolaief catalogues, by their respective star numbers. The observed $\Delta\alpha$ and $\Delta\delta$ give the corrections to the A. G. catalogues, derived from the Lick Observatory positions, after the proper motions noted at the right hand have been applied. The abbreviations of the list refer to the following authorities, for confirmation of the proper motion.

A. Argelander zones.
B. Bessel zones.
G. Göttingen zones.
L. Lalande zones.
M. Munich zones.
P. Paris zones.
S. Schjellerup zones.
Bausch. Bauschinger, Catalogue of Proper Motions.
Cin. 18. Cincinnati, vol. 18, six hours only.
and
Al. Notes to Albany A. G. Catalogue.
Nic. Notes to Nicolaief A. G. Catalogue.

TABLE IV

Proper Motions, A. G. Catalogues, Nicolaief and Albany

Star	$\Delta\alpha$	$\Delta\delta$	Proper motion		
N 99	+0ˢ28	—1″5	Not confirmed.		B. M. G.
N 210	+0.27	—5.3	No old observations.		
N 317	—0.60	+0.8	Error 0ˢ5 (?)		B. M.
A 550	+0.14	+1.6		—0″1	Al.
N 430	+0.05	+2.9	—0ˢ0192	—0.367	Cin. 18.
N 589	+0.06	—0.2	+0.0067	—0.048	Cin. 18.
A 1196	+0.08	—0.2		—0.1	L. B. S.
N 1060	+0.08	—0.5	+0.0058	—0.058	Cin. 18.
A 1879	+0.01	+0.5		—0.1	L. B. S.
N 1505	+0.04	—1.9		—0.1	M.G.
A 1979	—0.07	+0.3	P.M. Al. not confirmed.		
A 2299	—0.05	+0.9	P.M. Al. not confirmed.		
A 3322	+0.04	+1.6	P.M. Al. not confirmed.		
N 2630	—0.18	+2.0	P.M. Nic. not confirmed.		
N 2678	+0.04	+1.7	P.M. Nic. not confirmed.		
N 2767	—0.04	—1.7	—0ˢ007, (in place of —0ˢ07).		
N 3110	—0.02	—0.1		—0″206	Nic.
A 4169	—0.02	+1.5	P.M. Al. not confirmed.		
N 3266	—0.02	—0.5	—0ˢ007		L. B. M.
N 3322	+0.05	—0.6	+0.0085	—0″285	Nic.
N 3356	—0.02	+2.1	—0.006		L. B. M.
N 3391	—0.03	+1.6	—0.006		L. B. M.
A 4472	+0.09	—1.1	+0.001	—0.19	Al.
N 3418	+0.06	+1.5	—0.0079	—0.184	Bausch.
N 3436	+0.02	—0.4	+0.0094	—0.119	Bausch.
N 3438	+0ˢ02	+1″1	—0ˢ005		B. M.
A 4540	+0.06	—0.3	—0.009		L. M.
N 3455	—0.04	—1.3	+0.006	—0″06	Nic.
A 4638	—0.08	+2.0	—0.0024	—0.092	Al.
N 3543	+0.04	+1.4	—0.010		L. M.
N 3562	+0.03	—0.1	—0.006		B. M.
A 4715	—0.02	—3.7	Not confirmed.		B.
N 3626	0.00	+1.1	—0ˢ006		L. B. M.
A 4796	—0.09	—3.5	No old catalogues.		
N 3691	—0.01	0.0	—0ˢ005		L. B. M.
A 4921	—0.05	—0.7	+0.002	+0″08	Al.
N 3716	+0.01	+0.8	—0.007		L. B. M.
N 3757	—0.06	—1.5	P.M. Nic. not confirmed.		
N 3816	—0.01	—1.9	P.M. Nic. not confirmed.		
N 3916	+0.02	+1.4	+0ˢ007	—0″24	Nic.
N 3923	0.00	+2.5	—0.004		L. B. M.
N 3942	—0.01	—0.6	—0.006		B. M.
N 4012	—0.01	+1.5	—0.008		B. M. S. G.
N 4262	+0.04	—1.1	+0.004		B. M.
N 4351	—0.03	+0.5	Error 10″ Nic.		A 5801.
A 5861	+0.02	—2.0	P.M. Al. not confirmed.		
N 4725	+0.05	—0.3		—0″10	Nic.
N 5326	—0.07	(+9.4)	Error 10″ Nic.		L. B. M. P.
N 5386	+0.04	+0.6	+0ˢ004		L. M.
N 5409	—0.03	+0.7		—0″1	L. M.
N 5414	+0.05	+0.6	+0.010		Nic.
N 5501	+0.10	+0.7	+0.006		L. B. M.
A 7631	+0.09	—0.8	—0.009	—0.18	Al.
A 7709	—0.13	+0.3		—0.1	L. B.
N 5638	—0.01	+1.6	+0.006		L. M.
A 7813	—0.07	—1.0	+0.009	+0.09	Al.
A 7888	—0.06	+1.0	P.M. Al. not confirmed.		
N 5770	+0.14	—2.2	—0ˢ010	—0″11	Nic.
N 5784	—0.09	—1.7	+0.008		L. B. M.
A 7990	+0.05	+1.1	+0.0055	—0.054	Al.
A 8009	+0.14	+1.7		—0.173	Al.
N 5810	0.00	—3.2	+0.013	—0.19	Nic.
N 5831	+0.05	+1.5	—0.011	—0.15	M. A. G.
A 8090	+0.09	+1.4	+0.003	—0.07	Al.
A 8164	+0.03	—1.4	+0.0639	—0.934	Al.
N 5904	+0.35	+2.1	No old catalogues.		
N 5911	+0.13	+0.6	+0ˢ010	—0″14	M. G.

A portion of this discussion was presented in a note at the meeting of the *American Association for the Advancement of Science*, last August, at Berkeley.

R. H. Tucker.

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ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 284

ON THE RADIAL VELOCITY OF β URSAE MAJORIS

In February, 1908, Ludendorff announced[1] that the radial velocity of *β Ursae Majoris* (R. A. = 10 h. 56 m.) is variable, on the basis of sixteen Potsdam observations. The series of observations was greatly extended in March–June, 1908, and the seventy-one velocities measured between May, 1901, and June, 1908, were used by Ludendorff to determine[2] the spectroscopic orbit. He obtained a revolution period of 27.16 mean solar days, a total (double) amplitude of 15.5 km./sec. and an eccentricity of 0.79. Ludendorff's individual observations are plotted in figure 1 as open (light) circles and his computed velocity curve as the broken line.

Seven Mount Hamilton observations secured with the Mills spectrographs from March, 1901, to November, 1907, had given no indications of variable velocity, and they did not support Ludendorff's conclusions. After the appearance of his paper we obtained twenty-nine additional spectrograms, from October, 1910, to May, 1914, and these likewise failed to confirm. The total range in our observed velocities was but 5.5 km./sec., whereas Ludendorff's range was 20 km./sec. The differences in our velocities were no greater than the character of the spectrum (Class A) would lead us to expect, and they showed no relationship to Ludendorff's curve.

Guthnick and Prager observed the brightness of *β Ursae Majoris* in 1915 at Berlin–Babelsberg with a photo-electric photometer and found it to vary through a total range of 0.02 magnitude, in a period of 0.3122 mean solar day.[3] They then plotted Ludendorff's radial velocity observations with this period and found that they were represented by a spectroscopic orbit whose eccentricity is 0.23 and whose maximum and minimum radial velocities are —11.5 and —17.4 km./sec. respectively—a range of 5.9 km. We have plotted Ludendorff's sixty velocities obtained in 1908, with period 0.3122 day, in figure 2, and from the means of overlapping groups of single observations we have fixed the position of the velocity curve (dotted line) which best represents the observations. The range of the curve is from —12.5 to —17.2 km./sec., or 4.7 km.

The thirty-six Mount Hamilton observations were compared with the Kuthnick and Prager orbit, and seemed not to confirm it. However, this result was not conclusive, because of possible limitations to the accuracy of the period 0.3122 day. Guthnick and Prager assign the value 0.31306 sidereal day = 0.3122 mean solar day, but do not otherwise publish their estimate of its degree of accuracy. Their observations seem to have extended at least from March 21 to May 4, 1915, and they applied this period to Ludendorff's observations extending over more than five months in 1908. Our twenty-eight observations made in 1912–13–14, or the fifteen in 1914, are perhaps not opposed to a period of 0.3122 day and a double amplitude of 2 km./sec. or less, but the observations are not uniformly distributed with reference to the phases of the cycle, as they should be for a good test, and we think that the variations in our results may be accidental.

In dealing with a binary of such short period of revolution and small range of radial velocities—in this case seven and a half hours and 5± km. respectively—there is a great advantage in making a continuous series of observations extending over at least one whole period, and in measuring the spectrograms in as nearly continuous succession as possible: the exposure times for stellar spectrum and comparison spectrum can be made of the right length and essentially alike for all the plates of the series; systematic errors arising from instrumental conditions or from the personal equations of the plate measurers will affect all observations of the series about equally; and the differential character of measures looking for small variations of velocity is better preserved. Three such series of spectrograms of β *Ursae Majoris*—thirty-six plates in all—were obtained with the three-prism Mills spectrograph on the nights of February 1, 13 and 14, 1916, by Messrs. J. H. Moore, G. F. Paddock and W. K. Green. The photographs, on Seed 23 plates, are of high quality, essentially perfect in all respects. They were measured, with but little interruption for other duty, by Miss Hobe and Dr. Paddock, using different microscopes.

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[1] *Astr. Nach.*, 177, 235, 1908.

[2] *Astr. Nach.*, 180, 271, 1909.

[3] *Sonderabdruck aus Arbeiten aus den Gebieten der Physik, Math. u. Chemie*, 655–7, 1915.

The eleven velocities observed on February 1 seemed to be periodic and in agreement with Guthnick and Prager's value, 0.3122 day, but with a double amplitude not exceeding 2 km./sec.; but the results from twelve plates on February 13 and from thirteen plates on February 14 did not conform so fully as we had expected to those of February 1. The thirty-six observations obtained in February, 1916, are plotted in figure 3 on the basis of a period equal to 0.3122 day, and continued through a second cycle with the interval 0.312 day. We are unable to say that the results indicate a periodic variation through an appreciable or dependable range of velocities. The total observed range is but 4.8 km., and a smaller range could hardly be expected for a constant-velocity star whose spectrum is of the type concerned. It should be said that the thirty-six results plotted in figure 3 depend upon the same eighteen absorption lines on thirty of the spectrograms, but for five spectrograms one observer omitted one of the lines and for one spectrogram two of the lines. The magnesium line, 4481A, which is by far the best line on the plates, was measured on all of the thirty-six spectrograms. The velocities observed on February 1, 1916, are systematically smaller than the others.

Ludendorff's results plotted in figures 1 and 2 depend upon from 1 to 16 weak lines respectively in the spectrum of the star, and not at all upon the line 4481A, which he excluded. His first paper on the subject (1907) describes the line at 4481A as "gut meszbar," whereas the other lines are "wegen ihrer groszen Schwäche meist nur ungenaue Einstellungen gestatten"; and his second paper states that the faint lines which he used were "zum gröszten Teil an der Grenze der Sichtbarkeit," and that upon the different plates very different lines were measured. The line 4481A gave velocities differing from those afforded by the very faint and difficult lines, and the character of the 4481A line seemed to vary, apparently as an effect of the second component of the binary, and the results for that line were accordingly excluded from his published velocities. Ludendorff had also the impression that changes in the intensities of the lines occurred.

The Mills plates do not indicate that the 4481A line gives velocities appreciably different from those coming from the other lines. Dr. Paddock has compared many of the Mount Hamilton plates with one another on the Hartmann comparator and has found no evidences of variable breadth or density of the absorption lines.

OBSERVATIONS MADE AT LICK OBSERVATORY

Date, G. M. T.	J. D. 241	V n lines	n	V_1 4481A	V_2 $(n-1)$ lines	Obs.	Phase P 27.16d	 P 0.3122d	Remarks
1901 Mar. 12	5456.911	—14.	1	—14	......	W.	9.95		Poor plate; wt. ½; not worth two measures
1903 Jan. 14	6129.088	—12.4	2	— 9	—16	C.	3.13		
May 12	6247.685	—12.5				C.	13.08		Overexposed; wt. ½; measure lost
1904 Jan. 13	6493.078	—12.5	4	— 7	—14.4	M.	14.04		
		— 9.8	4	— 7	—10.7	W.			
		—11.7	4	— 9	—12.7	H.			
1907 Mar. 14	7649.835	—12.2	*			H.W.	2.92		
		— 9.6	5–4	—11	— 9.4	H.			
May 9	7705.749	—12.8	4–6	—15	—12.3	H.	4.51		
		—11.1	*			H.W.			
Nov. 25	7905.078	—10.1	3	—14	— 8.0	H.	13.72		Poor plate; wt. ½
		— 8.7	4	—16	— 6.4	H.			
1910 Oct. 30	8975.008	—12.2	12	—15	—12.0	Y.	24.41		Plate defective; should have been rejected before measuring; wt. ¼
		—12.4	*			H.W.			
		— 6.6	9–9	—11	— 6.0	H.			
1912 Jan. 4	9406.963	—10.8	10	—10	—10.9	W.	21.80		
		—11.8	10	—12	—11.8	H.			
Apr. 15	9508.923	—11.3	9	—11	—11.3	W.	15.12		
		—10.4	9	—13	—10.1	H.			
May 5	9528.771	—14.4	7–6	—14	—14.4	W.	7.81		
		—15.4	10–13	—14	—16.1	H.			
May 9	9532.782	—13.3	6	—13	—13.4	W.	11.82		
		—14.2	6	—12	—14.5	H.			

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* Measured on the Hartmann spectro-comparator.

Observations Made at Lick Observatory—(*Continued*)

Date, G. M. T.	J. D. 241	V n lines	n	V₁ 4481A	V₂ (n — 1) lines	Obs.	Phase P 27.16d	 P 0.3122d	Remarks
May 27	9550.809	—11.8	6	—12	—11.7	W.	2.69		
		—11.2	11	— 8	—11.6	H.			
June 5	9559.708	—14.4	4–10	—14	—14.6	W.	11.59		
		— 8.6	7–6	—11	— 8.2	H.			
June 16	9570.707	— 9.2	11–2	— 9	— 9.5	W.	22.59		
		— 9.7	7	—11	— 9.5	H.			
1913 Feb. 27	9826.967	—10.9	6	—10	—11.1	W.	7.25		
		—10.1	12	— 8	—10.3	H.			
Mar. 2	9829.821	—11.6	5	—10	—12.1	W.	10.10		
		—10.2	11	—12	—10.0	H.			
Mar. 2	9829.889	—13.5	6	—12	—13.8	W.	10.17		
		— 9.9	10	—10	— 9.8	H.			
Mar. 3	9830.877	—10.8	8	—11	—10.8	W.	11.16		
		—11.4	9	— 9	—11.7	H.			
Mar. 3	9830.928	—12.3	9	—12	—12.3	W.	11.21		
		—10.5	13	— 9	—10.6	H.			
Mar. 30	9857.800	— 9.8	5	— 7	—10.4	W.	10.92		
		—12.3	2	—11	—13.6	H.			
	242								
1914 Feb. 26	0190.898	—12.7	14	—14	—12.6	H.	18.10		
		— 9.9	14	—13	— 9.7	P.			
Feb. 26	0190.940	—11.4	14	—13	—11.2	H.	18.14		
		—12.5	13	—14	—12.4	P.			
Mar. 3	0195.919	—12.4	17	— 9	—12.6	H.	23.12		
		—13.7	14	—12	—13.9	P.			
Mar. 3	0195.953	—11.5	16	— 9	—11.7	H.	23.15		
		— 8.5	14–14	— 8	— 8.6	P.			
Mar. 9	0201.910	—11.3	16	—14	—11.1	H.	1.95		
		—12.8	14	—11	—13.0	P.			
Mar. 9	0201.956	—14.1	17	—15	—14.1	H.	2.00		
		—13.9	14	—13	—14.0	P.			
Mar. 13	0205.907	—11.8	18	— 9	—12.0	H.	5.95		
		—11.6	14	—10	—11.7	P.			
Mar. 14	0206.935	—10.5	17–18	—12	—10.4	H.	6.98		
		—12.2	14	—10	—12.3	P.			
Mar. 17	0209.942	—13.8	17	—17	—13.6	H.	9.98		
		—13.0	14	—15	—12.9	P.			
Mar. 17	0209.991	—14.2	18	—13	—14.3	H.	10.03		
		—12.4	14	—14	—12.3	P.			
Mar. 18	0210.837	—11.7	17	—14	—11.6	H.	10.88		Plate sent to Europe after one measure
May 8	0261.682	—15.2	16–18	—10	—15.5	H.	7.40		
		—13.3	14	—12	—13.4	P.			
May 8	0261.718	—13.1	16	—14	—13.0	H.	7.44		
		—14.7	14–14	—15	—14.7	P.			
May 10	0263.690	—14.3	15	—11	—14.5	H.	9.41		
		—12.4	14	—14	—12.4	P.			
May 10	0263.726	—12.4	14	—16	—12.1	H.	9.45		
		—14.5	14	—14	—14.5	P.			
1916 Feb. 1	0895.686	— 9.1	17	— 9	— 9.2	H.	16.73	0.55	
		— 9.2	18	— 9	— 9.2	P.			
Feb. 1	0895.725	— 9.6	17	—10	— 9.6	H.	16.76	0.58	
		—11.6	18	— 8	—11.9	P.			
Feb. 1	0895.763	— 9.9	18	— 7	—10.1	H.	16.80	0.63	
		—10.6	18	— 6	—10.9	P.			
Feb. 1	0895.798	—10.8	16	—10	—10.8	H.	16.84	0.66	
		—11.5	18	—11	—11.5	P.			
Feb. 1	0895.833	— 8.3	17	— 5	— 8.5	H.	16.87	0.69	
		— 9.2	18	—11	— 9.1	P.			
Feb. 1	0895.869	— 9.5	18	— 7	— 9.6	H.	16.91	0.73	
		— 9.7	18	— 9	— 9.8	P.			
Feb. 1	0895.905	— 7.8	18	—10	— 7.7	H.	16.94	0.77	
		— 8.4	18	— 8	— 8.4	P.			
Feb. 1	0895.947	— 9.4	18	—10	— 9.4	H.	16.99	0.81	
		— 9.3	18	— 7	— 9.4	P.			
Feb. 1	0895.978	— 9.4	18	—10	— 9.4	H.	17.02	0.84	
		—10.5	18	—10	—10.5	P.			
Feb. 2	0896.009	—11.3	17	— 7	—11.6	H.	17.05	0.56	
		—11.7	18	— 9	—11.8	P.			
Feb. 2	0896.041	— 8.6	18	— 5	— 8.8	H.	17.08	0.59	
		—10.1	18	— 7	—10.3	P.			
Feb. 13	0907.658	—11.7	18	—10	—11.8	H.	1.54	0.66	
		—11.4	18	—12	—11.4	P.			
Feb. 13	0907.734	— 9.6	18	—12	— 9.5	H.	1.61	0.73	
		— 9.9	18	—10	— 9.8	P.			
Feb. 13	0907.782	— 9.6	18	—11	— 9.6	H.	1.66	0.78	
		—12.0	18	—11	—12.1	P.			
Feb. 13	0907.817	— 9.2	18	—10	— 9.2	H.	1.70	0.82	
		—10.8	18	—11	—10.8	P.			

Observations Made at Lick Observatory—(*Continued*)

Date, G. M. T.	J. D. 241	V *n* lines	*n*	V_1 4481A	V_2 $(n-1)$ lines	Obs.	Phase P 27.16d	 P 0.3122d	Remarks
Feb. 13	0907.851	— 9.3	18	—11	— 9.2	H.	1.73	0.85	
		—10.4	18	—13	—10.3	P.			
Feb. 13	0907.881	— 9.8	18	—11	— 9.8	H.	1.76	0.57	
		—12.5	18	—10	—12.6	P.			
Feb. 13	0907.911	—10.6	18	—11	—10.6	H.	1.79	0.60	
		—10.7	18	—11	—10.7	P.			
Feb. 13	0907.936	—10.8	18	—11	—10.8	H.	1.82	0.63	
		—12.0	18	—11	—12.1	P.			
Feb. 13	0907.962	—11.0	18	—12	—11.0	H.	1.84	0.65	
		—11.9	18	—11	—11.9	P.			
Feb. 13	0907.989	—10.9	18	— 9	—11.0	H.	1.87	0.68	
		—10.5	18	—11	—10.5	P.			
Feb. 14	0908.017	—12.3	18	— 9	—12.4	H.	1.90	0.71	
		—12.8	18	—11	—12.9	P.			
Feb. 14	0908.049	—11.3	18	—10	—11.4	H.	1.93	0.74	
		—12.0	18	—12	—11.9	P.			
Feb. 14	0908.660	— 9.8	18	—11	— 9.8	H.	2.54	0.72	
		—12.4	18	—10	—12.5	P.			
Feb. 14	0908.696	—10.2	18	— 8	—10.3	H.	2.58	0.76	
		—12.1	18	—12	—12.1	P.			
Feb. 14	0908.728	—11.0	17	— 9	—11.1	H.	2.61	0.79	
		—10.8	18	— 9	—10.9	P.			
Feb. 14	0908.758	—11.0	18	— 8	—11.1	H.	2.64	0.82	
		—11.7	18	—12	—11.7	P.			
Feb. 14	0908.786	—11.2	18	—14	—11.0	H.	2.67	0.85	
		—13.0	18	—11	—13.2	P.			
Feb. 14	0908.816	— 9.6	18	— 8	— 9.7	H.	2.70	0.57	
		—11.3	18	— 8	—11.4	P.			
Feb. 14	0908.848	— 9.8	18	—10	— 9.8	H.	2.73	0.60	
		—11.8	18	— 8	—12.0	P.			
Feb. 14	0908.879	—11.8	18	— 8	—12.1	H.	2.76	0.63	
		—13.3	18	—10	—13.5	P.			
Feb. 14	0908.920	—11.5	18	—10	—11.6	H.	2.80	0.67	
		—11.3	18	—12	—11.3	P.			
Feb. 14	0908.949	—11.8	18	—11	—11.8	H.	2.83	0.70	
		—12.8	18	—10	—12.9	P.			
Feb. 14	0908.980	—11.6	18	— 8	—11.8	H.	2.86	0.73	
		—11.0	18	—10	—11.0	P.			
Feb. 15	0909.012	— 9.5	18	—12	— 9.4	H.	2.89	0.76	
		—11.9	18	—13	—11.8	P.			
1916 Feb. 15	0909.046	—10.4	18	— 8	—10.5	H.	2.93	0.80	
		—10.2	18	—11	—10.2	P.			

Initial following V is that of measurer—W. = R. E. Wilson, C. = H. D. Curtis, M. = J. H. Moore, H. = Miss Hobe, H.W. = H. C. Wilson, Y. = R. K. Young, P. = G. F. Paddock.

Epoch for phases of long period, J. D. 2418000.

Epoch for phases of short period, J. D. 2420907.

The table contains the Mount Hamilton observational data. The third column, V, is the observed velocity with reference to the Sun, depending upon all of the *n* lines measured. V_1 is the velocity given by the 4481A line alone and V_2 is the velocity resulting from the other *n*-1 lines. The phases for the two periods, 27.16 days and 0.3122 day, are computed from Julian days 2418000 and 2420907 respectively. The phases quoted for the shorter period may be reduced to values less than 0.3122 by subtracting one period (0.3122) or two periods (0.6244) from them. The seventy-two Mount Hamilton observations have been plotted on figure 1, as filled (dark) circles. They show no apparent relation to the 27.16-day period and curve. There are only forty-five plotted points for the seventy-two observations, because there was not room to plot the thirty-six points for the long series of observations obtained on February 1, 13 and 14, 1916. The results for each of these three nights were combined into three mean results before plotting; that is, the thirty-six measures were plotted as nine observations. Only the thirty-six 1916 observations are used in figure 3, because we are not justified in extending the period 0.3122 day to the 1901–14 observations.

It is not impossible that the Mount Hamilton observations will be found to support a periodic variation of very small amplitude—it could scarcely exceed 1.8 km./sec.—and we hope that somebody with less observing opportunity and more computing opportunity will make the effort. Evidences of revolution of the line of apsides should perhaps be looked for at the same time. Our plates are available for loan to any experienced measurer of spectrograms who may have a plan for remeasuring them.

It may not be amiss to caution some readers of this article against making a mistake to which the inexperienced are liable. Every series of observations contains unavoidable errors. Even if the function maintains a constant value, a plot of the observed values, on the basis of any assumed period, is liable to show a slight maximum at some one phase and a minimum at another phase. If the plotted points are carried forward through a second cycle, a third cycle, etc., the accidental features are carried forward also. The casual observer of the diagram sees the maxima and the minima repeating themselves faithfully, and concludes that a real variation is involved, whereas the apparent periodicity is due to the mechanical repetition of accidental peculiarities in the one plotted cycle.

W. W. CAMPBELL,
ADELAIDE M. HOBE.

April 5, 1916.

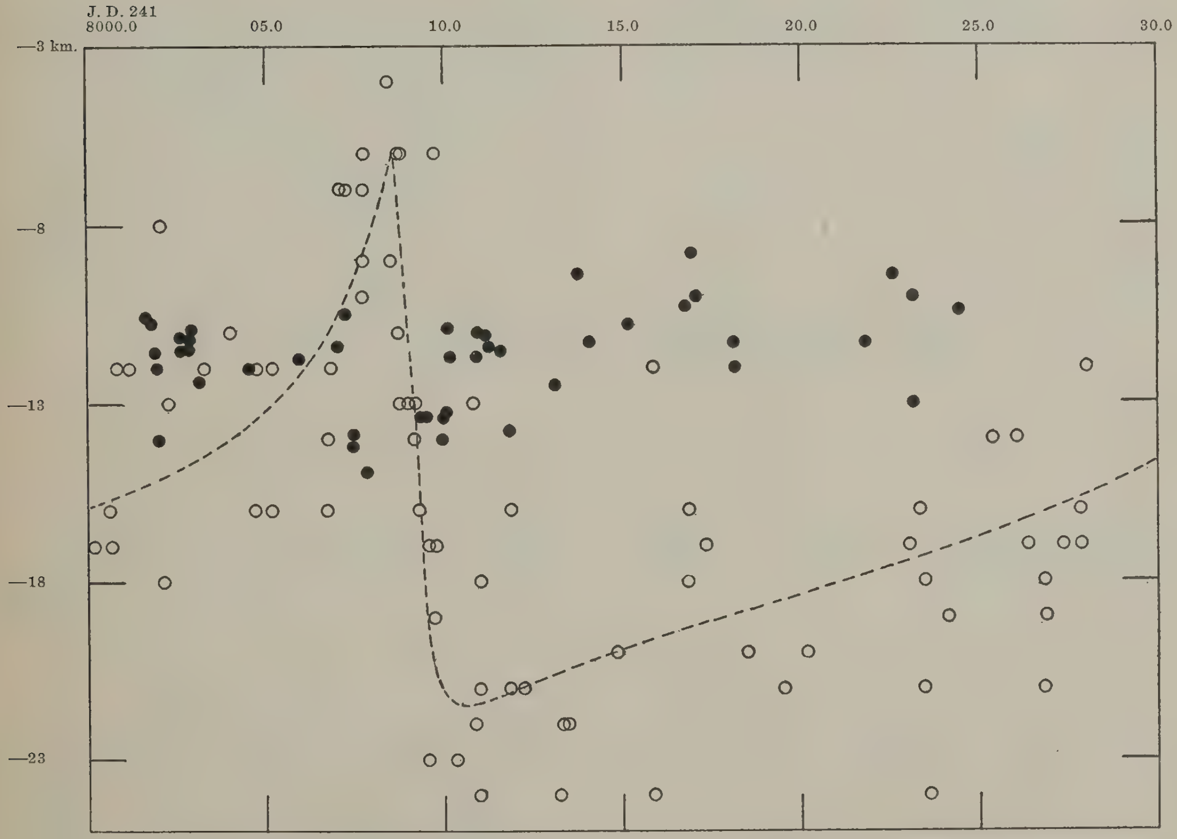

Fig. 1.—Open circles are Potsdam observations. Period = 27.16 mean solar days. Filled circles are Mount Hamilton observations, 1901–16. Broken line is velocity curve determined by Ludendorff.

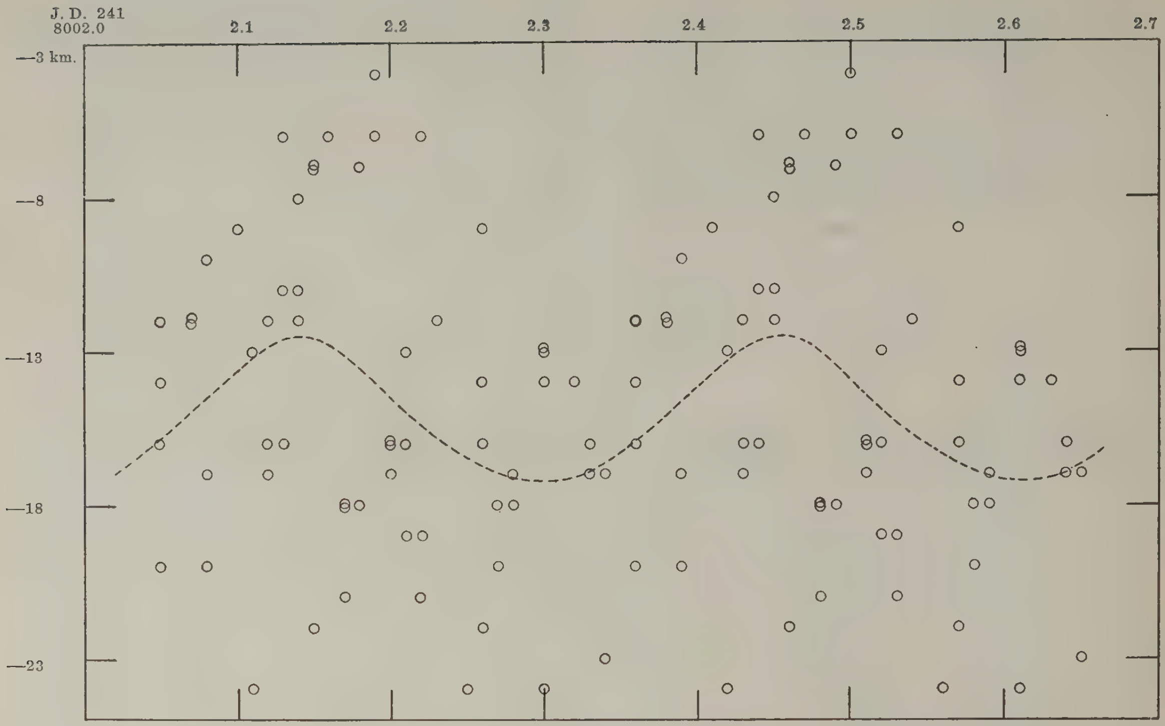

Fig. 2.—Potsdam observations made in 1908. Period = 0.3122 mean solar day.

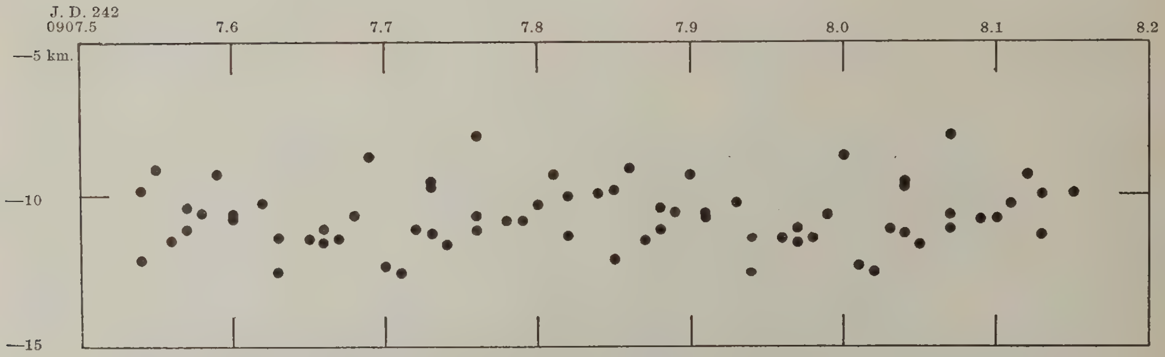

Fig. 3.—Mount Hamilton observations made in February, 1916. Plotted on Period 0.3122 mean solar day.

ON SYSTEMATIC ERRORS IN THE MEASURED INTERVAL BETWEEN CLOSE LINES

In the year 1897, after a considerable number of Mills three-prism spectrograms had been measured, I noticed an interesting form of systematic error of measurement which led at once to the taking of precautions to eliminate its effects. The components of each of the two pairs of iron comparison lines at 4250A and 4271A are about two-thirds of an angstrom unit apart and clearly separated on the spectrograms, but the measures made the interval between the components always smaller than the wave-lengths determined by Rowland required.[1] This was true in both systems of measurement: (*a*) in which the micrometer wire was made to bisect each line, and (*b*) in which the wire was set against each side of the line and the mean of the readings in the two positions taken as the reading on the center of the line; but the discrepancy (in my measures at least) was the greater by method (*a*). It seemed clear that the measure of one or both of the lines was affected by the presence of the close companion, in the sense that the interval between the centers of the lines was measured too small. It was sought to eliminate the effect upon the velocities of stars resulting from the individual absorption lines in the spectral regions concerned by using the mean of the readings on the two lines of each pair as the zero reading for the mean of the wave-lengths of the lines of the pair.

The next three observers had personal equations of the same sign as mine, and larger, but Dr. Burns's measures showed a small discrepancy of the opposite sign.

At my request Miss Hobe has copied from the records the measures of the two pairs of lines on some of the spectrograms secured from 1896 to 1902 with the original Mills spectrograph and has combined and tabulated them as below. There is no reason to expect that the measures not used in making the tabulation would give different results. The value of one revolution of the micrometer screw is $r = 0.5$ mm.

	4250.287—.945A			4271.325—.934A		
Measurer	No. of measures	Measured interval		No. of measures	Measured interval	
W. W. Campbell	23	0.1193±0.0006		50	0.1031±0.0004	
W. H. Wright	63	.1171	2	7	.0971	8
H. M. Reese	115	.1178	2	1	.097	
H. D. Curtis	50	.1163	3	24	.0983	7
Keivin Burns	143	.1220	2	247	.1080	2
Means for first four observers	251	.1175		82	.1011	

The computed intervals for Rowland's wave-lengths and for Burns's[2] wave-lengths, from the reduction curve which fits a long region of the Mills spectra, are:

	4250A	4271A
Rowland	0.1209	0.1082
Burns	0.1206	0.1054

Expressed in angstrom units, the intervals between the pairs of lines in Rowland's and Burns's tables are as in columns two and three below, and the intervals as measured by the first four observers and by Keivin Burns are as in columns four and five.

	Laboratory		Mills spectrograph	
Pairs of lines	Δ Rowland	Δ Burns	4 observers	K. Burns
4250.287–.945	0.658A	0.657A	0.639A	0.664A
4271.325–.934	0.609	0.593	0.569	0.608

We encountered several interesting examples of the phenomenon in October, 1915, when we attempted to construct a reduction curve for the region 4713A to 5016A, from the laboratory spectra of iron, titanium, hydrogen and helium, to be used in determining the wave-lengths of the green nebular lines (*Lick Obs. Bull.*, **9**, 6, 1916). The high dispersion or interferometer positions of the laboratory lines that did not have companion lines nearer than two or three angstroms fell upon the curve, within the usual unavoidable errors; but we could not get close though clearly separated lines to fall on the curve. When the helium and iron spectra were photographed upon the same plate, for example, the helium line at 4922.1A always fell above the curve of the iron lines by an amount three or four times the possible accidental error of observation. This discrepancy repeated itself on several plates. It was attributed to the influence of the strong iron line at 4920.8A upon the micrometer measures. We then placed a diaphragm immediately in front of the photographic plate which cut out the iron lines that would have fallen within a few angstroms of the helium lines at 4922A and 5015A. The iron spectrum was photographed with the diaphragm in place and the helium spectrum with the diaphragm removed. The helium lines were then found to lie upon the iron dispersion curve, and in the same manner upon the titanium curve.

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[1] The wave-lengths which I used for these pairs of lines were those assigned by Rowland to the corresponding absorption lines in the solar spectrum; at that time we seemed not to know the wave-lengths in the laboratory spectrum of iron so accurately as we knew the wave-lengths of the iron lines in the solar spectrum. The discrepancies do not modify the conclusions of this paper appreciably.

[2] *Lick Obs. Bull.*, **8**, 37, 1913.

It seems clear, from these results, that observers may readily over-estimate the accuracy or reliability of measured intervals between pairs of bright lines that are close but well separated, and that different observers may have quite different systematic errors.

When I was measuring stellar spectrograms I received the impression that the same kind of error may occur in measuring the interval between close absorption lines, and that the observer's judgment in bisecting such lines is affected by the degree of darkening of the continuous spectrum. The mean of the measured positions of two such lines seems to be reliable, but the results for the individual lines may have quite small weight. If one line of such a pair is unsuitable for measurement, on account of unsymmetrical distribution of the silver grains or for other reasons, both lines of the pair should usually be rejected. The close pairs of stellar absorption lines chiefly concerned in forming the impression were those at 4250A and 4271A; but as that region of the spectrum on the three-prism plates frequently suffered from different degrees of under-exposure, I did not pursue the subject to a definite conclusion.

There seems to be no reason why laboratory determinations of the wave-lengths of very close lines may not be affected with errors of the same kind.

April 5, 1916.

W. W. CAMPBELL.

WOLF-RAYET BANDS IN THE NUCLEI OF THREE NEBULAE: N. G. C. 6826, N. G. C. 418_2, N. G. C. 40

Among the plates taken in connection with Dr. Campbell's programme for the determination of the radial velocities of the planetary nebulae a few have shown the spectrum of the nucleus. Since the nuclei of several planetary nebulae have recently been found to have the Wolf-Rayet character of spectrum,[1] a description of the nuclear spectra shown on the above-mentioned plates will be of interest. With Dr. Campbell's permission the data furnished by the plates are here presented.

A bright band at about 4686A in the spectrum of the nucleus of the planetary nebula N. G. C. 6826 was noticed by Dr. P. W. Merrill while measuring a one-prism plate taken August 9, 1913. This was recognized as the well-known Wolf-Rayet radiation at 4686A common to both the Wolf-Rayet stars and the nebulae. Further examination discloses another less conspicuous bright band at about 4657A which is not given in Dr. Campbell's list[2] of Wolf-Rayet bands. Keeler[3] observed N. G. C. 6826 and glimpsed bright points in the continuous spectrum, but could not measure their positions. Measurement of the plate gives the following wave-lengths and approximate widths:

N. G. C. 6826; R. A. $= 19^h\,42^m.2$; Decl. $= +50^\circ\,16'.0$

Measured wave-lengths	Width	Intensity
4686A	4A	Fair
4657A	4A	Faint

A one-prism plate of the nebula N. G. C. 418_2 taken October 16, 1913, shows two bright bands in the continuous spectrum of the nucleus. Measurement of this plate gives the following details:

N. G. C. 418_2 = D. M. —12° 1172; R. A. $= 5^h\,22^m.9$; Decl. = —12° 46′

Measured wave-lengths	Width	Intensity
4686A	3A	Faint
4650A	8A	Fair

A conspicuous bright band was seen in the spectroscope upon observing the nebula N. G. C. 40, on October 6, 1913. The exposure then made was not sufficiently long to secure any of the nebular lines, and the plate shows the bright band alone. This band was at once measured and found to correspond to a wave-length of 4652A,[4] which is that of a well-known Wolf-Rayet band. This was, so far as known, the first instance of its observation in connection with a nebula. Another plate of N. G. C. 40 secured on September 26, 1914, with three prisms and 5-inch camera, was given longer exposure and careful guiding of the nucleus upon the mid-point of the slit. The slit was lengthened to admit the full diameter of the nebular image. As this plate contains a great deal of detail it has been thoroughly measured and will be described in connection with a brief description of the nebula itself. The results of the measurement are:

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[1] W. H. Wright, *Ap. Jour.*, 40, 466, 1914; *Proc. N. A. S.*, 1, 266, 1915; paper presented to the A. A. A. S., Palo Alto, 1915.

[2] *A. and Ap.*, 13, 468, 470, 1894.

[3] *Publ. Lick Obs.*, 3, 212, 1894.

[4] The wave-length of this band is given as 4688A in the Harvard List of Gaseous Nebulae, *Annals H. C. O.*, 56, 175, 1912, and in the classification of the Spectra of Gaseous Nebulae, *Annals H. C. O.*, 76, 21, 1915. This must have been a misidentification.

N. G. C. 40; R. A. $= 0^{h}\,7^{m}.6$; Decl. $= +71^{\circ}\,32'$

Object	Measured wave-length of point of maximum intensity	Width of band	Intensity	Notes
Line Hβ	4861.6A	......	Strong	Narrow; extending throughout nebula
Band Hβ	4862.2A	14A	Faint	Localized in the central nucleus
Band	4788.3A	13A	Faint	Localized in the central nucleus
Band	4687.4A	15A	Fairly strong	Localized in the central nucleus
Band	4652.1A	22A	Very strong	Localized in the central nucleus

A direct photograph of N. G. C. 40 taken with the Crossley reflector shows a ring of irregular distribution and intensity broken on the north and south sides. The brightest parts are on the east and west, the west being the most intense. The interior is filled with faint network of nebulosity. In the middle is a central star or nucleus of about the 11th magnitude.

The spectrographic plate shows monochromatic lines emitted by the nebula and a faint and narrow continuous spectrum emitted by the nucleus. The latter extends from about 4600A to 5000A and exhibits four bright bands. The strongest is that at 4652A. Its point of maximum intensity is nearer its violet edge and it seems to shade off in intensity on its red edge. The next strongest band is at 4686A. The other two are faint. That at 4862A corresponds to Hβ radiation, being essentially symmetrically placed with respect to the Hβ line which crosses it at right angles, the two giving the appearance of a plus sign. This was, so far as known at the time of observation, the first observed instance of a broad emission band of hydrogen in the nucleus of a nebula. These four bands are well known in Wolf-Rayet spectra.

The monochromatic radiations of this nebula are represented on this plate principally by the Hβ line and faintly by Hγ, while traces of helium are suspected particularly at 4922A at that point which corresponds to the bright western rim of the nebula. The nebular line at 5007A also is very faintly traced across the spectrum, indicating its existence throughout the nebula. The Hβ line shows considerable structure. It is most intense at the rims and the nucleus, the intensity diminishing in the order, western rim, nucleus, eastern rim. A narrow thread of light extends between these portions of the line, its source being the interior nebulosity.

G. F. Paddock.

November, 1915.

AN ERROR IN RADIAL VELOCITY OBSERVATIONS ARISING FROM NON-UNIFORM SLIT ILLUMINATION*

Some observations of the planet *Venus* secured with the three-prism Mills spectrograph attached to the 36-inch refractor give strong evidence that an imperfect or unsymmetrical illumination of the slit, by the light source, may introduce error in the resulting radial velocities. These observations were secured when the terminator of *Venus* was nearly a straight line. The slit was made parallel to the terminator and was placed either just inside the terminator or just inside the section of the limb immediately opposite the center of the terminator, as judged from the image of the planet on the slit plates. Ten observations of the radial velocity of the planet were obtained with the slit just inside the terminator and ten with the slit just inside the limb, with the prism box and camera directed to the west. The spectrograph was then rotated 180° about the axis of the telescope until the prism box and camera pointed east, with the slit again parallel to the terminator line. In this position of the spectrograph nine observations were made with the slit just inside the terminator and nine with the slit just inside the limb.

In the first position of the spectrograph, that is, with camera west, the ten observed radial velocities of the limb minus the ten of the terminator had an average value of —0.51 ±0.08 km. per second.

In the second position of the instrument, that is, with camera east, the nine observed radial velocities of the limb minus the nine of the terminator had the average value of +0.75 ±0.12 km. per second.

The difference of the two mean results, 1.26 km., is, in my opinion, attributable to the fact that the slit was not uniformly illuminated by the planets' image. Guiding was based upon a visual image of the planet, as formed by the 36-inch objective and by a correcting lens one meter in front of the slit. This correcting lens is figured for the blue and violet rays. Guiding was chiefly by virtue of the yellow rays. The yellow image was undoubtedly larger than the blue-violet image to which the radial velocity measures were intended to relate. The slit was probably not so far inside the boundary lines of the blue-violet image as the visual guiding image indicated. It is probable that the blue-violet image was so much smaller than the guiding image that the former did not always extend entirely across the slit. The systematic errors are in the direction explainable by the assumption that in half the observations the image was stronger on the east edge of the slit than on the west edge, and vice versa for the other half of the observations. The slit width used was 0.0014 inch (0.036 mm.).

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* Reprinted from *Publ. Astr. Soc. Pac.*, **27**, 129–30, June, 1915.

Errors of the same nature probably enter into radial velocity results for stars, provided the observer in guiding upon the stellar image has a systematic tendency which leads him to keep the center of the image nearer one edge of the slit than the other, or in case the image is not symmetrical, owing to faulty collimation, atmospheric refraction, etc., or if the adjustment of the telescope is such that with the progress of time the stellar image drifts systematically off the slit in one direction.

The high degree of accuracy calls for care in the symmetry of the stellar image and symmetry of the image with reference to the central line of the slit of the spectrograph.

It is possible that atmospheric dispersion distorted the image of *Venus* sufficiently to introduce guiding errors in the radial velocities of the planet described above.

The measures referred to in this paper were all made by Miss A. M. Hobe.

W. W. Campbell.

Issued September 30, 1916.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 285

ON THE ORBIT OF AETHRA*

The first part of a study of the orbit of the planet *Aethra* was published in *L. O. Bulletin*, **275**. The aim of the study was to find the cause of the uncertainty in the orbit, to find the range of solution, and, if possible, to determine such an orbit that the planet could be rediscovered. A considerable portion of the work was devoted to a discussion of a minor planet observation made at the Lowell Observatory with the forty-inch reflector, 1913 June 10. The conclusion was that there was a strong possibility of identity of this minor planet with *Aethra* and a search ephemeris was published, on this assumption, for the opposition of November 1915. At this opposition the planet was far north of the equator but was near aphelion and therefore comparatively faint. A great amount of cloudy weather rendered a search impossible at the Lick Observatory and no information has been received of an attempt having been made at any other place. The study of the orbit has been continued and makes identity seem more probable, as will be shown in this paper.

The value of μ adopted in the first paper to represent the observation made at the Lowell Observatory in 1913 most satisfactorily is capable of minor improvement because of a slight error of computation. The value of μ adopted, viz., $\mu = 883''.425$, is, therefore, not the best obtainable. The residuals of the 1913 observation as published for this value are $\Delta\alpha = +2'.4$ and $\Delta\delta = +72'.5$. These should have been $\Delta\alpha = +31'.9$ and $\Delta\delta = +62'.5$. A new value of μ has now been interpolated to satisfy the Lowell observation in α. This value is $883''.4657$ and gives $\Delta\alpha = \pm 0'.0$ and $\Delta\delta = +60'.0$. Since the δ residual is considerably smaller than previously given and, as explained in *L. O. Bulletin*, **275**, is a measure of the probability of identity, it now seems even more probable that the object was *Aethra*. The new elements are given with their residuals in Table VII. (The numbering of the tables is continued from *L. O. Bulletin*, **275**.)

Another feature which seems to show rather strong probability of identity is the fact that with the value of μ necessary to represent the 1913 observation, the planet is always far south of the equator when opposition comes near perihelion time, and is near aphelion when the opposition is in good position for northern observatories. On account of the very large eccentricity of the orbit of *Aethra* there is a great difference in its brightness at different oppositions. The difference between $\mu = 883''.4657$ for *Aethra* and one-fourth the mean daily motion of the Earth is less than four seconds. The period of *Aethra* is, therefore, almost exactly four years. As a result of this near commensurability the oppositions, through long periods of time, will be similar to those which occurred either four years or a multiple of four years previously. The accompanying diagram shows the relative positions which can occur at oppositions of *Aethra* and the Earth. This diagram shows clearly the great variations in the heliocentric and geocentric distances at times of opposition. It explains in large measure the puzzling fact that *Aethra* can become as bright as it does at times and yet has escaped observation through forty years. The oppositions of 1873 and 1913 were similar oppositions, at which the planet was bright and far south of the equator. The next opposition will be similar to these. On December first the planet will have a declination of —24° and a magnitude of 10.6. February first it will be forty-one degrees south and will have a magnitude of 10.4. During March and the early part of April it will be about forty-five degrees south and will have a magnitude of about 10.0. It then begins its return north, reaching closest approach to the Earth about May 8, with a magnitude of 9.9, and opposition about May 19. These magnitudes are based on the 1873 observations, but according to the Lowell photographic observation should be somewhat brighter. Next June it will be again approximately in the position observed in 1873 and 1913. An ephemeris has been computed at half-month intervals from the first of December to the first of August. This ephemeris is published as Table VIII.

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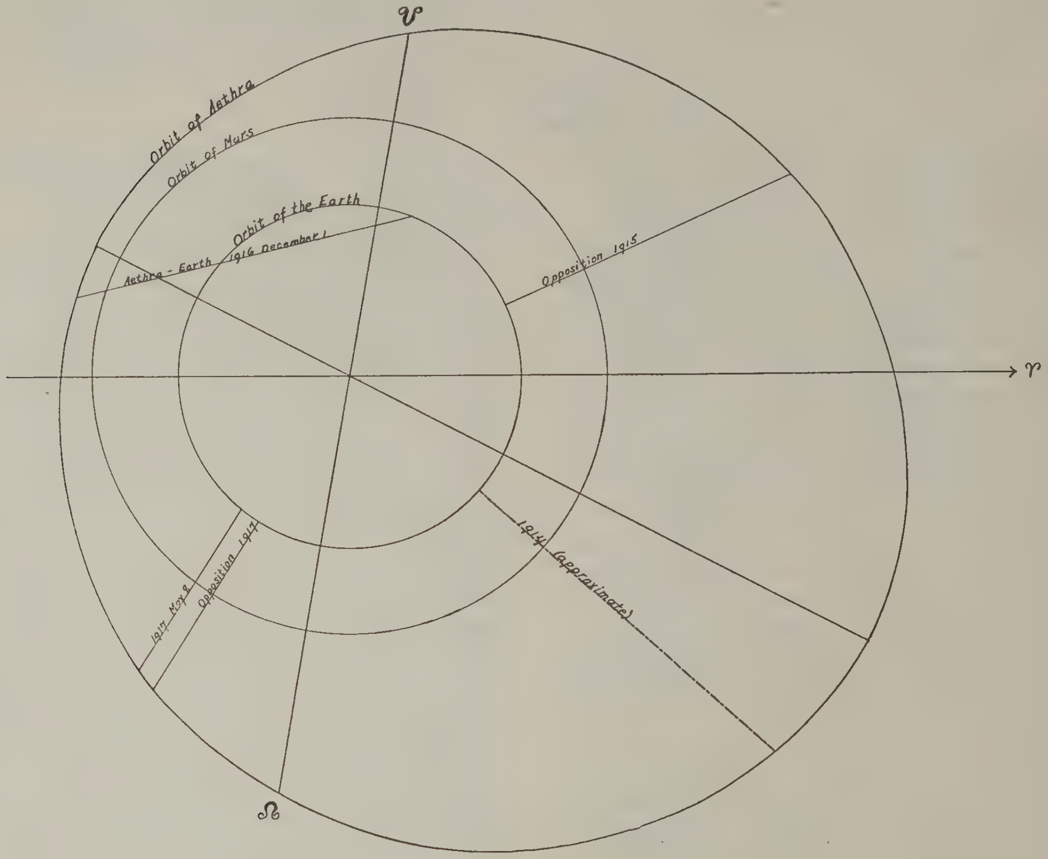
* Continued from *L. O. Bulletin*, 275.

The apparent close approach of the orbits of *Mars* and *Aethra* in the diagram is due to the projection of these orbits on the plane of the ecliptic, the bodies themselves being widely separated in declination.



In Table V of the former bulletin are given parabolic elements which lie just at the edge of the range of possible solutions. It was felt worth while to extend this work in the other direction. Accordingly a circular solution was made. The sum of the residuals is 39″4 which, according to the criterion adopted in the last paper, is just over the borderline of possible solutions. In Leuschner's method a circular solution is obtained by first computing the conditioned ellipse $r = a$, and then obtaining the circular elements, if the criteria show a circular solution to fall within the range of possible solutions (*Publications of the Lick Observatory,* Volume VII, pages 305–306). In the present case the fact that in a general solution r is also very nearly equal to a, will cause the conditioned solution to lead to practically the same very eccentric orbit, so that forcing it into a circle leads to difficulties. The circular elements are given in Table IX.

Since the interpolation to represent the α of the Lowell observation was based only on the 1873 observations it did not remove the δ residual. It was planned to correct the orbit differentially and thus represent the δ as accurately as the α. This was attempted by means of the closed expressions for f and g (*Publications of the Lick Observatory,* Volume VII, pages 262–266). The differential correction was based on one of the initial orbits approximately representing the 1913 observation. It was found that the equations of condition which had to be solved would not lead to a determinate solution of the unknowns. In view of the already satisfactory representation of the orbit finally adopted in Table VII no further attempt at improvement was made.

It has been shown that the range of possible values of μ is very large unless *Aethra* be identical with the asteroid observed in 1913. When a definite value of μ is assumed the other elements become determinate. However, they change very much as μ is varied. This may be seen by an inspection of the elements published. It is, therefore, advisable to publish elements with their variations as μ changes. This has been carried out using two constants, one a coefficient of $\Delta\mu$ and the other of $(\Delta\mu)^2$. The values of the elements from which the solutions for these constants have been made are those of Table I corresponding to $\mu = 800''.243$, $861''.394$, and $896''.930$. These are given in Table X. With these approximate general elements it should be possible to settle very quickly the question of identity between *Aethra* and any asteroid that may be found.

TABLE VII

ADOPTED ELEMENTS

$$\begin{array}{l} M = 12^\circ\ 53'\ 39''.5 \\ \left.\begin{array}{l} \Omega = 260\quad 18\quad 46.9 \\ \omega = 252\quad 05\quad 15.5 \\ i = 24\quad 09\quad 33.4 \end{array}\right\} 1917.0 \\ \phi = 20\quad 19\quad 23.8 \\ \mu = 883''.4657 \\ \log a = 0.4025447 \end{array}$$

Epoch 1917 January 1.0 Gr. M. T.

RESIDUALS (O–C)

1873 Normal Places

	$\Delta\alpha$	$\Delta\delta$
I	—0″.1	—2″.0
III	±0.0	—1.4

1913 Lowell Observation

	$\Delta\alpha$	$\Delta\delta$
	±0″.0	+60″.0

TABLE VIII

EPHEMERIS 1916–1917

Gr. M. T.	True α	True δ	$\log\rho$	Magnitude
1916 Dec. 1.0	12h 23m	—24° 08′	0.29830	10.6
1916 Dec. 16.0	13 00	29 14	0.28168	10.5
1917 Jan. 1.0	13 40	34 03	0.26293	10.5
Jan. 16.0	14 17	37 50	0.24365	10.4
Feb. 1.0	14 54	41 01	0.22029	10.4
Feb. 15.0	15 23	43 06	0.19701	10.3
March 1.0	15 47	44 32	0.17090	10.2
March 16.0	16 05	45 21	0.14047	10.1
April 1.0	16 12	45 21	0.10737	10.0
April 16.0	16 08	44 15	0.08017	9.9
May 1.0	15 54	41 47	0.06330	9.9
May 16.0	15 36	37 55	0.06393	10.0
June 1.0	15 19	32 53	0.08767	10.2
June 16.0	15 11	28 20	0.12851	10.4
July 1.0	15 10	24 39	0.18011	10.7
July 16.0	15 16	21 55	0.23551	11.1
1917 Aug. 1.0	15 27	—20 10	0.29377	11.4

TABLE IX

CIRCULAR ELEMENTS

$T = 1873$ June 25.5 Berlin M. T.

$$\begin{array}{l} \left.\begin{array}{l} \Omega = 259^\circ\ 04' \\ u_0 = 6\quad 07 \\ i = 15\quad 16 \end{array}\right\} 1873.0 \\ \mu = 1106''.062 \\ \log a = 0.337485 \end{array}$$

TABLE X

GENERAL ELEMENTS

Epoch Berlin M. T. 1873 June 25.5

$$\begin{array}{l} M = 69^\circ\ 32' + [2.6248]\ \Delta\mu'' + [9.4530]\ (\Delta\mu'')^2 \\ \left.\begin{array}{l} \Omega = 259\quad 43 - [0.6360]\ \Delta\mu'' - [7.8723]\ (\Delta\mu'')^2 \\ \omega = 252\quad 13 - [0.0668]\ \Delta\mu'' + [8.6770]\ (\Delta\mu'')^2 \\ i = 24\quad 42 - [1.9553]\ \Delta\mu'' - [8.9008]\ (\Delta\mu'')^2 \end{array}\right\} 1873.0 \\ \phi = 21\quad 35 - [2.2955]\ \Delta\mu'' - [9.1276]\ (\Delta\mu'')^2 \\ \mu = 861''.394 \\ \log a = 0.409870 \end{array}$$

DINSMORE ALTER.

BERKELEY ASTRONOMICAL DEPARTMENT,
September 6, 1916.

Issued September 30, 1916.

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ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 286

EPHEMERIS OF COMET *b* 1916 (WOLF)

After having passed conjunction this comet will again be in favorable position for observation in December of this year. An ephemeris is therefore here given extending from 1916 Nov. 29.5 to 1917 March 5.5. The elements upon which this ephemeris is based are those published in *Bulletin No.* 282. Comparison of an observation by Barnard of July 5 (the last available) with these elements shows (O–C), $\Delta\alpha = +0^s3$; $\Delta\delta = -1''$.

Ephemeris for Greenwich Mean Midnight

1916 Gr.M.T.	True α			True δ			log Δ	Br.
Nov. 29.5	15^h	06^m	47^s9	—5°	07′	13″	0.5862	2.96
Nov. 30.5		08	25.9		10	18		
Dec. 1.5		10	04.4		13	19	0.5829	3.04
2.5		11	43.3		16	16		
3.5		13	22.6		19	08	0.5794	3.13
4.5		15	02.4		21	56		
5.5		16	42.6		24	40	0.5760	3.22
6.5		18	23.2		27	20		
7.5		20	04.3		29	55	0.5724	3.32
8.5		21	45.8		32	25		
9.5		23	27.8		34	51	0.5687	3.42
10.5		25	10.2		37	11		
11.5		26	53.1		39	27	0.5650	3.52
12.5		28	36.4		41	38		
13.5		30	20.2		43	43	0.5612	3.63
14.5		32	04.4		45	43		
15.5		33	49.0		47	38	0.5574	3.74
16.5		35	34.1		49	27		
17.5		37	19.7		51	11	0.5534	3.86
18.5		39	05.7		52	49		
19.5		40	52.2		54	22	0.5494	3.98
20.5		42	39.1		55	49		
21.5		44	26.5		57	10	0.5453	4.11
22.5		46	14.3		58	25		
23.5		48	02.5	—5	59	33	0.5411	4.25
24.5		49	51.2	—6	00	35		
25.5		51	40.3		01	31	0.5369	4.39
26.5		53	29.9		02	21		
27.5		55	19.9		03	04	0.5326	4.54
28.5		57	10.3		03	40		
29.5	15	59	01.2		04	10	0.5282	4.70
30.5	16	00	52.5		04	33		
Dec. 31.5	16	02	44.3	—6	04	49	0.5237	4.86
1917 Gr.M.T.	True α			True δ			log Δ	Br.
Jan. 1.5	16^h	04^m	36^s5	—6°	04′	58″		
2.5		06	29.1		05	00	0.5192	5.03
3.5		08	22.2		04	55		
4.5		10	15.7		04	43	0.5146	5.20
5.5		12	09.7		04	23		
6.5		14	04.1		03	56	0.5099	5.39
7.5		15	58.9		03	21		
8.5		17	54.2		02	39	0.5052	5.58
9.5		19	49.9		01	49		
10.5		21	46.1	—6	00	51	0.5004	5.79
11.5		23	42.7	—5	59	45		
12.5		25	39.8		58	31	0.4955	6.00
13.5		27	37.3		57	09		
14.5		29	35.3		55	39	0.4906	6.23
15.5		31	33.7		54	00		
16.5		33	32.5		52	13	0.4856	6.46
17.5		35	31.8		50	17		
18.5		37	31.5		48	13	0.4806	6.71
19.5		39	31.6		46	00		
20.5		41	32.2		43	37	0.4754	6.96
21.5		43	33.2		41	06		
22.5		45	34.6		38	25	0.4702	7.23
23.5		47	36.4		35	36		
24.5		49	38.7		32	37	0.4650	7.51
25.5		51	41.4		29	29		
26.5		53	44.5		26	12	0.4597	7.81
27.5		55	48.0		22	45		
28.5		57	51.9		19	08	0.4543	8.12
29.5	16	59	56.2		15	22		
30.5	17	02	01.0		11	26	0.4489	8.44
Jan. 31.5		04	06.2		07	20		
Feb. 1.5		06	11.8	—5	03	04	0.4434	8.78
2.5		08	17.8	—4	58	38		
3.5		10	24.2		54	02	0.4379	9.14
4.5		12	31.0		49	16		
5.5		14	38.3		44	20	0.4323	9.51
6.5		16	46.0		39	13		
7.5		18	54.0		33	56	0.4267	9.90
8.5		21	02.4		28	28		
9.5		23	11.3		22	50	0.4210	10.31
10.5		25	20.6		17	01		
11.5		27	30.3		11	02	0.4153	10.73
12.5		29	40.4	—4	04	51		
13.5		31	50.9	—3	58	30	0.4096	11.18
14.5		34	01.8		51	57		
Feb. 15.5	17	36	13.1	—3	45	14	0.4038	11.65

1917 Gr.M.T.		True α			True δ			log Δ	Br.
Feb.	16.5	17^{h}	38^{m}	$24^{s}.7$	—3°	38′	20″		
	17.5		40	36.7		31	14	0.3979	12.14
	18.5		42	49.1		23	57		
	19.5		45	01.9		16	29	0.3921	12.65
	20.5		47	15.1		08	50		
	21.5		49	28.6	—3	01	00	0.3862	13.19
	22.5		51	42.5	—2	52	58		
	23.5		53	56.7		44	45	0.3802	13.75
	24.5		56	11.3		36	21		
	25.5	17	58	26.3		27	45	0.3743	14.34
	26.5	18	00	41.6		18	58		
	27.5		02	57.3		09	59	0.3683	14.96
Feb.	28.5	18	05	13.3	—2	00	49		
Mar.	1.5	18^{h}	07^{m}	$29^{s}.6$	—1°	51′	28″	0.3623	15.60
	2.5		09	46.3		41	55		
	3.5		12	03.3		32	11	0.3563	16.27
	4.5		14	20.6		22	16		
Mar.	5.5	18	16	38.3	—1	12	09	0.3502	16.97

Brightness May 10, 1916 = 1.00.

R. T. CRAWFORD.

BERKELEY ASTRONOMICAL DEPARTMENT,
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Issued October 4, 1916.

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ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 287

SIX STARS WHOSE RADIAL VELOCITIES VARY

Following is a list of six stars which were found to have variable velocities, from observations secured by the D. O. Mills Expedition to the Southern Hemisphere, prior to August 1, 1913. Their announcement has been held awaiting the confirmation of further observations. Except where otherwise noted the plates were secured with a spectrograph of two-prism dispersion. The last column gives the number of stellar lines measured, or when this number is followed by the letter "s" the number of sections measured with the Hartmann spectro-comparator.

E Puppis ($\alpha = 7^h\ 8^m.9$; $\delta = -40^\circ\ 19'$; Mag. 5.4; Class A5)

Gr. M. T.	Vel. km.	Observer	Measured by	
1912 Feb. 10.667	—45.3	Moore	Mrs. Moore	7 s
	—43.9		Moore	15 s
1913 Feb. 25.610	+23.7	Moore	Mrs. Moore	4 s
	+20.5		Moore	17 s
Oct. 15.867	+20.5		Sanford	26
	+25.7		Moore	11 s
1916 Feb. 17.631	+16	Scott	Wilson	8, 1 prism
	+28		Scott	7

A. G. C. 15640 ($\alpha = 11^h\ 21^m.4$; $\delta = -63^\circ\ 25'$; Mag. 5.3; Class F2)

Gr. M. T.	Vel. km.	Observer	Measured by	
1912 Feb. 15.866	+ 2.6	Sanford	Mrs. Moore	11 s
	— 1.4		Sanford	15 s
	— 0.5		Moore	15 s
1913 Jan. 21.856	—15.8	Moore	Mrs. Moore	12 s
	—18.9		Mrs. Moore	20 s
	—21.7		Moore	17 s, stronger?
	+30.0			14 s
Feb. 8.815	+24.2	Moore	Mrs. Moore	13 s
	+22.1		Mrs. Moore	15 s
	+23.6		Moore	14 s, stronger?
	—24.1			14 s

On the spectrograms of January 21 and February 8, 1913, the lines are double. The two spectra are of nearly equal intensity, although a study of the lines which do not appear to be affected by the blending of one component with the other component of a neighboring line seems to show that the stronger component is the one indicated above. For many of the sections coincidences of this component with the lines of the standard plate (*α Carinae*) were more readily obtained than in the case of the other component. For this reason it was the only one measured by one of the observers.

A. G. C. 16939 ($\alpha = 12^h\ 21^m.0$; $\delta = -62^\circ\ 34'$; Mag. 5.1; Class B5)

Gr. M. T.	Vel. km.	Observer	Measured by	
1911 May 31.595	+26	Moore	Mrs. Moore	5
	+15		Miss Hobe	6, 8, mean of 2 measures
	+28		Moore	6
1913 Feb. 17.854	+52	Sanford	Mrs. Moore	4, mean of 2 measures
	+51		Miss Hobe	9
	+57		Moore	3
Mar. 10.801	+30	Sanford	Mrs. Moore	1
	+35		Miss Hobe	7
Apr. 24.683	+23	Sanford	Mrs. Moore	4
	+30		Miss Hobe	8
1916 Feb. 17.855	+43		Wilson	11, 1 prism
	+42		Scott	10

The spectrum consists of fairly broad lines of hydrogen, helium, carbon and magnesium. The appearance of the lines on the spectrogram of February 17, 1913, seems to be different from that of the lines on the other three plates obtained with the two-prism spectrograph. Some of this may arise from differences of exposure, but part of it appears to be a real change in the character of the lines. A comparison of the two-prism spectrograms on the Hartmann spectro-comparator verifies the variation in velocity indicated above.

ε Normae ($\alpha = 16^{h}\,19^{m}.8$; $\delta = -47^\circ\,20'$; Mag. 4.7; Class B4)

Gr. M. T.	Vel. km.	Observer	Measured by	
1908 July 23.565	+ 2	Paddock	Sanford	
	+ 7		Olivier	
	+ 4		Miss Hobe	7
1911 May 16.780	— 14	Paddock	Mrs. Moore	
	— 10		Miss Hobe	8
May 29.728	— 13	Moore	Mrs. Moore	2
	— 11		Miss Hobe	5
June 12.710	—144⎫		Miss Hobe	8 double lines
	+ 80⎭			10
1912 May 25.754	— 20	Sanford	Mrs. Moore	3, Hγ double?
	+ 4		Miss Hobe	
1915 June 6.736	+ 19	Scott	Wilson	4, 1 prism
	+ 14		Scott	3
Aug. 17.566	+ 85⎫		Scott	8, double,
	—119⎭			1 prism
	+ 84⎫		Wilson	2, double
	— 87⎭			

The spectrum contains broad lines of hydrogen and helium. On the spectrograms of June 12, 1911, and August 17, 1915, both spectra appear; while on those of May 29, 1911, May 25, 1912, and June 6, 1915, the lines are hazy, with the appearance of being double. The separation, however, is not sufficient to permit of approximate measurement.

κ Scorpii ($\alpha = 17^{h}\,35^{m}.6$; $\delta = -38^\circ\,59'$; Mag. 2.5; Class B2)

Gr. M. T.	Vel. km.	Observer	Measured by	
1904 May 31.704	—21	Palmer	Miss Hobe	3, III prisms
1907 July 30.529	—25	Paddock	Curtis	3, III prisms
	—36		Miss Hobe	3
Sept. 7.503	+ 2	Paddock	Curtis	3, III prisms
	+18		Miss Hobe	3, mean of 2 measures
1911 May 18.828	— 9	Moore	Mrs. Moore	4
	— 9		Miss Hobe	3
	— 5		Moore	3
July 31.659	+45	Paddock	Sanford	3
	+45		Moore	3
	+36		Miss Hobe	3, mean of 2 measures
1912 June 10.769	—21	Moore	Moore	3
	—17		Miss Hobe	3
1915 June 20.767	—35	Wilson	Wilson	10, 1 prism
	—27		Scott	12
June 22.788	—24	Scott	Wilson	10, 1 prism
	—12		Scott	9

The spectrum contains rather broad lines of hydrogen and helium. A comparison of the spectrograms of May 18 and July 31, 1911, on the Hartmann engine checks the difference in velocity indicated above for these two observations. The lines on the three plates obtained with three-prism dispersion are very broad, and the results from these spectrograms are therefore entitled to small weight.

A. G. C. 29075 ($\alpha = 21^{h}\,6^{m}.6$; $\delta = -39^\circ\,50'$; Mag. 5.3; Class F5)

Gr. M. T.	Vel. km.	Observer	Measured by	
1912 June 30.858	—40.5	Sanford	Mrs. Moore	
	—40.6		Moore	15 s
	—41.9		Moore	14 s
1913 June 7.927	+ 0.7	Sanford	Mrs. Moore	9 s
	— 4.3⎫		Moore	Stronger
	—85.1⎭			11 s
Sept. 26.603	—50.8		Sanford	9 s
	—45.4		Moore	11 s
Oct. 17.583	—46.7		Sanford	10 s
	—42.1		Moore	7 s

The spectra of both components are visible on the plate of June 7, 1913. That of the stronger component only was measured by one of the observers. The last three plates are underexposed, and the last one is so faint that the results from it are entitled to smaller weight than those from the other measures.

J. H. Moore.

Mount Hamilton,
June 1, 1916.

FOUR STARS WHOSE RADIAL VELOCITIES ARE VARIABLE

Of the stars whose radial velocities are being determined by the D. O. Mills Expedition to the Southern Hemisphere a number observed during the past year give indications of having variable velocities. For several of these the observed variations are such as to make a confirmation by further observations desirable. The range observed for four stars is sufficient, however, to remove all doubt as to the reality of this variation, and for these the necessary data are given below. Except where noted the spectrograms were obtained with a one-prism spectrograph. The number of lines from which the radial velocity was determined follows the measurer's name.

A. G. C. 8815 ($\alpha = 6^{h}\,57^{m}.0$; $\delta = -25^\circ\,4'$; Mag. 5.8; Class B3)

Gr. M. T.	Vel. km.	Observer	Measured by	
1915 Mar. 14.559	+12	Wilson	Sanford	10
	+ 6		Scott	8
	+ 9		Wilson	8
Oct. 19.861	—11	Scott	Wilson	7, II prisms
	—13		Scott	5
Oct. 25.853	—11	Wilson	Wilson	9
	— 7		Scott	6
Nov. 10.783	+53	Wilson	Wilson	11
	+58		Scott	12

The spectrum contains several good lines for either one- or two-prism dispersion, the principal ones being of hydrogen, helium, calcium and magnesium.

A. G. C. 10211 ($\alpha = 7^h\ 44^m5$; $\delta = -46° 22'$; Mag. 5.3; Class B2)

Gr. M. T.	Vel. km.	Observer	Measured by	
1912 Feb. 11.679	+19	Paddock	Sanford	3, II prisms
	+17		Mrs. Moore	1
Dec. 6.778	+17	Sanford	Mrs. Moore	3, II prisms
	+17		Sanford	3
1913 Dec. 26.722	+87		Sanford	3, II prisms
	+90		Wilson	
1916 Feb. 21.560	+ 4		Wilson	8
	+13		Scott	8
Feb. 24.660	+41		Wilson	12
	+52		Scott	10

The lines of hydrogen and helium upon which the measures depend are rather broad for the dispersion of two prisms. The character of the lines appears to change.

K Velorum ($\alpha = 9^h\ 14^m7$; $\delta = -50° 38'$; Mag. 5.3; Class A)

Gr. M. T.	Vel. km.	Observer	Measured by	
1913 Feb. 11.744	+ 86	Sanford	Mrs. Moore	2, II prisms
	+ 89		Moore	2
1914 Mar. 19.660	+105±		Wilson	1, II prisms
	+110±		Campbell	1
	+124±		Scott	1
Apr. 16.601	+ 44		Wilson	2, II prisms
	+ 45		Sanford	2
	+ 24		Scott	2, mean of 2 measures
1916 Feb. 16.760	+ 17		Wilson	5
	+ 20		Scott	4
Feb. 23.596	+ 80		Wilson	7
	+ 98		Scott	6

With two-prism dispersion the lines of hydrogen and magnesium are rather difficult, as the $H\gamma$ is broad and the 4481 line is faint. On the spectrograms obtained with the one-prism spectrograph the hydrogen lines are fairly sharp, and their measures are entitled to more weight than those of the higher dispersion spectrograms.

χ^2 Hydrae ($\alpha = 11^h\ 1^m1$; $\delta = -26° 45'$; Mag. 5.6; Class B8)

Gr. M. T.	Vel. km.	Observer	Measured by	
1915 June 24.506	— 57	Scott	Wilson	2
	— 56		Scott	2, 4, mean of two
June 25.483	+134	Scott	Wilson	3, 5, mean of two
	+156		Scott	5, 5, mean of two
July 6.497	+ 95	Scott	Wilson	2, 2, mean of two
	+ 68		Scott	2, 4, mean of two

The spectrum contains broad lines of hydrogen, fair magnesium and very weak helium lines. The helium lines are not in general measurable.

Ralph E. Wilson.

Observatory of the D. O. Mills Expedition,
Santiago, Chile,
March, 1916.

RS SAGITTARII, ALGOL VARIABLE AND SPECTROSCOPIC BINARY

RS *Sagittarii* ($18^h\ 11^m0$, $-34° 8'$), a variable star of the *Algol* type according to the *Second Catalogue of Variable Stars* (*Annals Harvard College Observatory,* **55**, 22, 1907), was observed along with a group of other variables in 1909 at Santiago, Chile, for the purpose of finding variable radial velocity. A single prism and 16-inch camera were used, these giving the lowest dispersion then available in the equipment of the D. O. Mills Expedition. Nine plates of RS *Sagittarii* were obtained. Preliminary measures of the wide lines of the Class A spectrum shown by this star give a range of 160 km., that is, from +80 km. to —80 km. This result indicates that the star is a spectroscopic binary, one body eclipsing the other, in view of the nature of the light curve. Definitive measures of velocity will be published later. Further observations and measures of this star in 1915 with one-prism dispersion, by Dr. R. E. Wilson at Santiago, corroborate this variation of velocity, giving a range of 140 km.

G. F. Paddock.

June, 1916.

NOTES ON THE SPECTRUM OF γ *ARGUS*

On examining three three-prism plates of γ *Argus* taken by the D. O. Mills Expedition at Santiago, Chile, with reference to the character and place of the Wolf-Rayet band at 4652A, which is represented by or at least included in the broad bright band extending from about 4640A to 4670A, I noticed a faint indication of a break or division in the band at about 4655A. This appearance, however, could as easily be interpreted as a faint, narrow absorption line, particularly in view of the existence of a line at 4654.736A in the iron spectrum. But narrow iron lines have not thus far been observed in Class O spectra. It occurred to me that this broad band might be composed of two overlapping bands corresponding to the effective wave-lengths 4652A and 4658A, which are now known to be represented in Wolf-Rayet spectra. One of the three plates was therefore measured and the following results obtained for the broad band and the neighboring band at 4688A.

γ ARGUS ($8^h\ 6^m5$, —47° 2′; Plate 1089 IV, Jan. 22, 1907)

Bright bands	Measured wave-lengths	Middle of band	Width	
Violet side	4641A	4648A		14A+
Division	4655		30A±	
Red side	4671	4663		16A+
Violet side	4679	4688	19A+	
Red side	4698			

The widths given in the above table must not be taken as definite or exact. If the 30-angstrom band is composed of two overlapping bands, the widths of the latter must be greater than the widths of the divisions as noted in the table.

These bands have no defined edges, but shade off over intervals of several angstroms, so that it is difficult to designate beginning, ending, or mid-point. The values in the third column are the means of those in the second. The mean values 4648A and 4663A do not correspond to any Wolf-Rayet bands thus far observed. But if this broad band of γ *Argus* is composed of two overlapping bands, then their mid-points would be closer together than indicated by the values 4648 and 4663, and the effective wave-length of the violet component would be greater than 4648A and that of the red component less than 4663A. Thus it would not be impossible for this broad band to consist of the two Walf-Rayet bands known as 4652A and 4658A.

An extension of the idea of complex or merging bands would be that this is the nature or explanation of others in other Wolf-Rayet stars, especially those in this same region, some of which are one hundred or more angstroms wide, extending from about 4600A to 4700A; for example, that in the star SDM —21° 4864 ($18^h\ 2^m5$). In this region of spectrum are known at least six Wolf-Rayet bands, namely, 4615, 4626, 4633, 4652, 4658, 4688. The investigation of these extended bands with various dispersions and varied exposures and otherwise would probably lead to interesting results.

Upon referring to Dr. Campbell's observations of γ *Argus,* made with single-prism dispersion in 1893, it is found that he distinguished two maxima in the broad band extending from 4640A to 4670A, which caused him to say in his published description of the spectrum of γ *Argus:*[1] "The broad band 4651 is strongly suspected to be double, with components near 4643 and 4659." He has not noted the double appearance of this band in the spectrum of any other Wolf-Rayet star.[2] Dr. Campbell's values 4643, 4651, 4659, for the mid-points of the whole band and of its divisions, are thus systematically four angstroms less than those in the table above. The detection of the apparent division in this band on both one- and three-prism plates gives it a certain amount of probabilty of existence, whatever its cause or explanation may be.

The bright band at 4688A is described in the *Harvard Annals*[3] as follows: "A narrow dark line is superposed on this bright band"; "the bright bands 4682 to 4700, . . are reversed." Neither of these effects is discernible on our three-prism plates. In fact, these plates show no features except the two bright bands noted in the table above. The continuous spectrum is almost uniform except for an extremely slight and extended absorption in the regions of Hγ and Hδ. Campbell's observations[4] and those at Harvard Observatory[3] indicate narrow dark lines across the bright Hβ band and others.

There has recently appeared in *Monthly Notices*[5] an article by Mr. W. M. Worssell of the Union Observatory, Johannesburg, South Africa, upon the spectrum of γ *Argus,* containing a partial description of his work with the Franklin-Adams prismatic camera, and the results of a study by Professor Fowler of South Kensington, England, of one of Mr. Worssell's plates of γ *Argus.* The plates were exposed sufficiently to obtain the less refrangible part of the spectrum, thus overexposing the blue region. A table is given of twenty-one lines or bands found in the region 6700A to 4862A. A few identifications are made, namely, for hydrogen, helium and possibly enhanced carbon; but the particular point of interest here is the finding of ten or eleven instances of reversal, four strong or definite and six or seven faint or suspected. The reversals of helium D_3 and hydrogen Hβ are the most conspicuous. No mention is made of reversal being seen in the two bright bands at 4686A and 4652A which are conspicuous in the overexposed region of the spectrum. However, in view of the evidence of reversal found by Worssell and Fowler, it is very possible that the division seen on our Southern Mills plates in the band 4640A to 4670A may be a reversal of the band 4652A if this be the effective wave-length of the whole band.

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[1] *Astr. and Astroph.,* **12,** 555, 1893.

[2] *Astr. and Astroph.,* **13,** 455 to 467, 1894.

[3] *Annals H. C. O.,* **28,** 246, 247, 1901.

[4] *Astr. and Astroph.,* **13,** 457, 1894.

[5] *Mon. Not. R. A. S.,* **76,** 418, 1916.

Recent one-prism plates of γ *Argus* taken at Santiago, Chile, by Wilson and Scott give indication that the absorption lines at Hδ, Hγ and He 4471 are bordered by slight brightening of the continuous spectrum. This accords with the description given by Harvard Observatory,[6] and may place these lines in the class of reversed lines or bands. Mr. Worssell, however, did not detect any brightening on his plates, saying that the hydrogen lines to violet of Hβ appear to be wholly dark. There is also indication that the reversal increases in strength toward the violet, which property would associate the spectrum of γ *Argus* with the spectra of the bright-line stars of Class B.[7] On the Santiago one-prism plates bright helium 4922A and 4713A are not perceptible; also on the bright helium 5015A the reversal, ''feeble'' according to Mr. Worssell, is not discernible. The bright band 4442A is strong on these plates. Hydrogen and helium absorption bands are conspicuous in the violet, namely, at 3875A (unknown), 3888A (He), 3889A (Hζ), 3970A (Hε), and 4026A (He). In fact, the whole stretch of spectrum presented on these plates shows a wavy intensity.

Further observation and study are quite likely to bring out critical data or facts concerning this peculiar spectrum.

G. F. Paddock.

June, 1916.

Issued October 21, 1916.

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[6] *Annals H. C. O.*, **28**, 247, 1901.

[7] Campbell, *Ap. Jour.*, **2**, 177, 1895; Merrill, *Lick Obs. Bull.*, **7**, 175, 1913.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

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THE SPECTROSCOPIC BINARY h 4 CENTAURI[1]

The southern star h *Centauri,* for which the *Harvard Rerived Photometry* gives

> α (1900) $= 13^{h}\ 47^{m}.4$, δ (1900) $= -31°\ 26'$,
> Visual magnitude 4.76, spectral class B5A,

was observed by the D. O. Mills Expedition at Santiago, Chile, in the years 1908 to 1911, and announced as a spectroscopic binary in *Lick Observatory Bulletin,* **6**, 56, 1910. In 1910 a series of plates was taken to obtain the period and thence the other orbit elements. From May 4, 1908, to August 6, 1911, 37 plates were taken, one early plate with three prisms and 36 plates with two prisms giving a dispersion of about 24A per mm. Iron arc comparison and Seed 30 plates were used. Except on the first two dates, when one plate was experimental and the other of higher dispersion, exposures ranged from 76 to 130 minutes, as demanded by the atmospheric conditions. The slit widths were .046, .047, and .048 mm. The adjoined table of observations gives the usual data, namely, the number of the plate, the Greenwich date, the Julian Day, the phase with respect to the time of periastron passage, the observed velocities, the residuals from the final ephemeris, the quality of the plate, the initial of the observers Moore and Paddock, and, lastly, notes.

Spectral Class, List and Character of Lines.—h *Centauri* belongs to the spectral class B5, which is included in the group called *Orion* or *helium* stars. Lines of helium are conspicuous and more numerous than lines of other elements. In scanning the extent of spectrum caught upon the plates, namely, from 3900A to nearly 5000A, although ill-focused toward these limits, twenty absorption lines are easily seen, while numerous traces of lines are detectable. The twenty lines are as follows:

Line	λ	Appearance
K	3933	Narrow, sharp
Hϵ	3970	Broad
He	4009	Faint, ill defined
He	4026	Strong
Hδ	4102	Strong
He	4121	Fair
Si	4128	Fair
Si	4131	Fair
He	4143	Fair, ill defined
He	4169	Faint, ill defined
?	4233.4	Faint, ill defined
Hγ	4340	Strong
He	4388	Fair
He	4437	Faint, ill defined
?	4469.3	Faint, ill defined
He	4471	Good
Mg	4481	Good
He	4713	Faint
Hβ	4861	Strong
He	4922	Fair

The identification of these lines is well established, with the exception of those at 4233.4A and 4469.3A. The former is seen in spectra of many early class stars and the latter is often seen, although confused in its form or appearance. Close approximations are found in Rowland's *Preliminary Table of Solar Spectrum Wavelengths,*[2] designated iron and titanium. In *Kayser's Handbuch VI* are also found close values. These two lines are not given in Pickering's[3] list of lines in Class B stars, nor mentioned by Frost[4] in his study of *Orion* stars. Lockyer and Baxendall,[5] however, have found lines at 4233.3A, 4468.7A, and 4469.5A

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[1] No. 3586 in Boss's *Preliminary General Catalogue.*

[2] *Ap. Jour.,* **1**, 1895.

[3] *Annals H. C. O.,* 28, 226, 1901.

[4] *Publ. Yerkes Obs.,* 2, 156, 1903.

[5] *Publ. Solar Physics Committee, Catalogue Brighter Stars, Spectra,* 1902; *Origin of Lines, Stellar Spectra,* 1910.

Observations and Measures of h Centauri

No.	Plate	Date	J. D. = 241	Phase	Vel.	O – C	Qual.	Obs.	Notes
1	1928 II	1908 May 4	8066.673	5.342	+18.4	+0.6	g	P	+21.2, +15.7
2	2695 II	1909 Aug. 8	8527.518	2.078	+ 4.4	+0.5	f	P	3 prisms
3	2985 II	1910 Jan. 23	8695.835	4.147	+27.0	+4.8	g	P	
4	3004 II	Jan. 31	8703.806	5.191	+18.3	—1.0	g	P	
5	3036 I	Feb. 17	8720.819	1.423	— 8.2	—1.2	g	P	
6	3063 III	Feb. 24	8727.872	1.549	— 6.6	—1.8	g	P	
7	3065 II	Feb. 25	8728.788	2.465	+12.2	+2.9	g	P	
8	3085 I	Mar. 3	8734.854	1.604	+ 1.0	+4.8	g	P	
9	3089 III	4	8735.849	2.599	+10.7	—0.3	g	P	
10	3095 IV	5	8736.863	3.613	+20.9	+0.9	g	P	
11	3103 II	8	8739.860	6.610	—12.1	—2.3	g	P	
12	3109 II	9	8740.877	7.627	—17.0	+1.5	g	P	
13	3113 III	10	8741.840	1.663	+ 1.2	+4.0	g	P	
14	3116 I	11	8742.826	2.649	+12.3	+0.7	g	P	
15	3121 II	12	8743.837	3.660	+19.8	—0.5	g	P	
16	3127 III	14	8745.881	5.704	+10.4	—2.2	g—	M	
17	3132 IV	15	8746.867	6.690	—13.8	—2.0	g—	M	
18	3138 I	20	8751.862	4.758	+17.8	—4.1	g	P	
19	3144 I	21	8752.871	5.767	+ 5.9	—5.5	g	M	
20	3149 III	22	8753.799	6.695	— 3.6	+8.3	g	P	
21	3311 I	May 10	8802.753	7.160	—22.5	—2.8	f	P	—19.9, —25.1
22	3313 I	11	8803.571	1.051	—17.6	—4.1	f	M&P	
23	3318 IV	21	8813.683	4.236	+27.4	+5.1	f—	M	+26.2, +28.6
24	3322 I	23	8815.719	6.272	+ 3.7	+4.2	f	M	
25	3325 IV	24	8816.653	7.206	—21.5	—1.5	g	P	
26	3337 II	27	8819.628	3.254	+12.1	—5.4	g	P	+12.2, +12.0
27	3340 II	29	8821.624	5.250	+17.8	—1.0	g	P	
28	3346 IV	30	8822.646	6.272	— 6.8	—6.3	g	M	
29	3452 IV	Aug. 19	8903.505	4.007	+23.8	+2.0	g	P	
30	3453 I	21	8905.501	6.003	+12.1	+5.7	g—	P	
31	3456 IV	22	8906.499	....	------	----	p	P	Too poor for use
32	4073 I	1911 July 23	9241.553	2.632	+ 8.5	—2.9	g	P	
33	4082 II	25	9243.510	4.589	+25.5	+3.2	g	P	
34	4085 IV	31	9249.519	3.671	+16.8	—3.5	g	P	+19.9, +13.8
35	4090 IV	Aug. 4	9253.544	0.769	—18.5	—0.8	g	P	
36	4095 II	5	9254.523	1.748	+ 0.4	+1.7	g	P	
37	4101 II	6	9255.512	2.737	+11.4	—1.2	g	P	+13.7, +9.1

in their spectra of *Sirian* type (Class A) stars and designate their origin as iron and chromium, titanium, and iron, respectively. Lockyer[6] found an enhanced iron line at 4233.32A, an enhanced chromium line at 4233.50A, and an enhanced titanium line at 4468.66A. Neovius[7] and others found oxygen lines close to the places of these two lines. This correspondence might be significant in view of the numerous identifications of lines in certain *Orion* stars with lines of oxygen. But Gill[8] in his comparison of oxygen lines with lines of β *Crucis* (Class B1) does not mention the two under discussion here, and Frost[9] in his suggested division of Class B stars does not put any star of Class B5 (except 67 *Ophiuchi*, B5p) in the group showing oxygen lines. Identification with oxygen may then be doubtful and that with iron or titanium most probable, particularly in view of the correspondence of these lines with *Sirian*[10] (Class A) lines and the appearance of metallic (mostly enhanced) lines, even though faint, and few in classes A and B as found by Miss Waterman.[11]

Measures.—The plates were measured in the course of the interval of observation of the series, using from 4 to 10 lines. Velocities were finally reduced from four lines, which alone could be considered sufficiently satisfactory on all the plates. The Hartmann-Cornu formula and Rowland wavelengths were used for the tabular differences representing zero velocity with respect to the observer.

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[6] *Publ. Solar Physics Committee, Tables of Enhanced Lines,* 1906.

[7] *Kayser,* 6, 213, 215, 1912.

[8] *Ap. Jour.,* **10**, 280, 1899.

[9] *Publ. Yerkes Obs.,* 2, 249, 1903.

[10] *Publ. Solar Physics Committee, Catalogue Brighter Stars, Spectra,* 58, 60, 1902; *Annals H. C. O.,* **28**, 58, fourth column, 1897.

[11] *Lick Obs. Bull.,* **8**, 1, 1913.

Wavelengths.—The published determinations of the wavelengths of the four lines selected for measurement are:

Line	International System	Rowland System	Observer	Reference
Hγ	....	4340.634	Rowland	*Ap. Jour.*, **1**, 300, 1895.
Hγ	4340.467	.634	W. E. Curtis	*Proc. Roy. Soc.*, 90, 613, 1914.
He	....	4388.100	Runge u. Paschen	*Kayser's Handbuch*, V, 1910.
He	....	4471.676	Runge u. Paschen	*Kayser's Handbuch*, V, 1910, weighted mean of double line
He	4471.482	4471.652	Rayleigh	*Kayser's Handbuch*, V, 1910
He	4471.493	4471.663	Eversheim	*Kayser's Handbuch*, V, 1910
He	....	4471.693	Albrecht	*Publ. Lick Obs.*, **9**, 78, 1907, derived from stellar spectra
Mg	....	4481.400	Adams	*Ap. Jour.*, **15**, 216, 1902
Mg	....	etc.	Various	*Lick Obs. Bull.*, **8**, 168, 1915

The wavelength of Hγ is evidently well established. There is but one value available for He 4388A. Eversheim's value for He 4471A is probably the most reliable of the several available, it being obtained with the interferometer on the International System. A provisional value for Mg 4481A must be adopted here for the reasons given in the discussion of its wavelength in connection with the spectroscopic binary ν_n *Eridani*.[12] With respect to one iron comparison line there has been some question, that at 4482A. Under high dispersion two close lines are found at this point and are given by Rowland and by Burns as follows:

Observer	Fe λ	System	Intensity	Weighted mean	System	Reference
Rowland	4482.338	Row.	5	4482.376	Row.	*Ap. Jour.*, **1**, 381, 1895
	4482.438	Row.	3			
Burns	4482.176	I. A.	3	4482.395	Row.	*Lick Obs. Bull.*, **8**, 37, 1913
	4482.262	I. A.	4			
Hartmann	4482.397	Row.	....			*Phys. Zeit.*, **4**, 429, 1902

Hartmann's determination was made with the three-prism spectrograph and iron arc, with which apparatus the close lines are blended into a single line. Since this value and the weighted mean of Burns's components agree, the latter has been adopted for the present use. Wavelengths for the other iron comparison lines agree very closely in Rowland's and Burns's tables. It is noticed that none of the lines here employed for comparison are among the "sensitive lines" subject to the "pole effect" recently investigated by St. John and Babcock.[13]

The values selected for the four star lines under consideration and those adopted for the iron comparison lines, which in each case are those nearest to the star lines measured, are as follows:

Line	Wavelengths	Weights
Fe	4337.216	
Hγ	4340.634	1
Fe	4383.720	
He	4388.100	2
Fe	4466.727	
He	4471.663	4
Fe	4482.395	
Mg	4481.400	3

Lines Weighted.—Weights for the four star lines were adopted as shown in the table because of the different characters of the lines and unequal consistency in their measures. The lines 4471A and 4481A are better than 4388A and 4340A. The latter, Hγ, is broad and uncertain. The first, 4471A, is the best and gives the most consistent measures. Six plates, those numbered 1928, 3311, 3318, 3337, 4085, 4101, at first giving outstanding or critical values of velocity, were remeasured and the mean of the two measures used. The separate measures are placed in the notes to the table of observations.

Period.—Plots of the observations show the period to be a little less than seven days. Superposition of plots of the observations within the three intervals 1910 March 3 to 22, 1910 May 21 to 30, 1911 July 23 to August 6, give values differing one and two hundredths and a mean of 6.928 days. After adjusting the first and last observations the value 6.927 days has been adopted.

Elements.—By the Lehmann-Filhés[14] method for spectroscopic binary orbits and use of the planimeter the values

$$A = +18.5 \qquad z_1 = 47$$
$$B = +25.5 \qquad z_2 = -57$$

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[12] *Lick Obs. Bull.*, **8**, 168, 1915.

[13] *Ap. Jour.*, **42**, 231, 1915.

[14] *Astr. Nach.*, **136**, 17, 1894.

were obtained, and thence preliminary elements. A preliminary ephemeris was then derived from the following values:

$$\left.\begin{array}{r@{\;=\;}l} P & 6.927 \text{ days} \\ \mu & 51^{\circ}\!.97 \\ \gamma & +5 \text{ km.} \\ K & 21 \text{ km.} \\ e & .15 \\ \omega & 149^{\circ} \\ T & \text{J. D. } 2418733.3 \end{array}\right\} \text{Preliminary}$$

Solution for the definitive elements was performed by least squares on the basis of Schlesinger's[15] equation of condition, the normal equations being

$$\begin{array}{rrrrrl} 36x + & 4.607y - & 3.261z - & 14.552u - & 0.151w & = +12.3 \\ & +16.807 - & 0.535 - & 0.063 - & 0.228 & = +0.61 \\ & & +19.193 + & 1.227 + & 14.196 & = -11.08 \\ & & & +9.630 + & 0.003 & = -19.20 \\ & & & & +10.857 & = -7.96 \end{array}$$

which give

$$x = -1.26,\ y = +0.37,\ z = +0.66,\ u = -3.98,\ w = -1.61$$

The corrections to the provisional elements are then

$$\begin{array}{r@{\;=\;}l} \delta\gamma & +0.21 \text{ km.} \\ \delta K & +0.37 \text{ km.} \\ \delta e & +0.08 \\ \delta\omega & -1^{\circ}\!.77 \\ \delta T & -0.05 \text{ days} \end{array}$$

Definitive elements consequently are, together with their probable errors:

$$\left.\begin{array}{r@{\;=\;}ll} P & 6.927 \text{ days} & \pm <.001 \text{ day (estimated)} \\ \gamma & +5.2 \text{ km.} & \pm 1.1 \text{ km.} \\ K & +21.4 \text{ km.} & \pm 0.65 \text{ km.} \\ e & 0.23 & \pm 0.028 \\ \omega & 147^{\circ}\!.23 & \pm 11^{\circ}\!.13 \\ T & \text{J. D. } 2418733.25 & \pm 0.17 \text{ days} \\ a \sin i & 1{,}984{,}000 & \pm 61{,}400 \text{ km.} \\ \dfrac{m_s^3 \sin^3 i}{(m+m_s)^2} & 0.0065 & \end{array}\right\} \text{Definitive}$$

The probable error of a single plate is ±2.5 km. The sum of squares of the residuals, 487 from the preliminary ephemeris, reduces to 424 for the corrected ephemeris.

A true value of the period, that which is unaffected by the relative motion of the Sun and the binary system, is only one ten-thousandth less than the apparent period as observed. The greatest effect on the observed velocities producible by the Earth's motion in its orbit is .07 km., while the maximum possible effect due to the primary's orbital motion is less than .001 km. Corrections to the period and to the phases of the observed velocities are therefore negligible.

Upon the definitive elements is based the velocity curve shown in the accompanying diagram. The circles, representing the 36 observed velocities, have a diameter of 2.5 km. to indicate the probable error of observation. A cross toward the right hand indicates the earliest observation and one toward the left the last. A bar marks the place of periastron.

The Mass Ratio and Relative Magnitudes.—The expression and value given last in the list of definitive elements is difficult of interpretation because of the impossibility of determining the inclination of the orbit plane. A point or two, however, may be drawn from it. Campbell[16] and Schlesinger[17] have derived a theoretical mean value of $\sin^3 i = 0.6$ for uniformly distributed orbit inclinations. Data of binaries showing both spectra indicate an average mass several times the Sun's mass. Assuming, then, $\sin^3 i = 0.6$ and values of the total mass from 0.5 to 5⊙, the ratio of the masses ranges from 0.4 to 0.1. The greater the total mass and the greater the inclination the less the ratio. For a lower limit of total mass of ½⊙ and an upper limit of ratio of masses of ½, the inclination is 43° 45′. It may consequently be said that for a total mass between ½ and 5⊙ and an inclination greater than 44° the ratio of masses is less than ½. The secondary mass of h *Centauri* is therefore very probably small, and probably very small. Other properties being equal or similar, the relative luminosity is proportional to the $M^{2/3}$, and therefore if the mass-ratio is between ½ and $1/10$⊙ the secondary is between ½ and 1½ magnitudes fainter than the primary. It is found[18] in general that a difference of brightness between the components of a binary greater than one magnitude renders it scarcely possible to detect the secondary spectrum or any appreciable disturbance of the lines of the principal spectrum due to those of the secondary. Since in the case of h *Centauri* under the conditions noted above the difference of magnitude may lie within the limit just mentioned, it is quite possible that the lines of the secondary spectrum of h *Centauri* may be affecting those of the principal spectrum. But on the plates thus far obtained no certain appearance of secondary lines is seen, although in a few instances they are suspected. These suspicions, however, must be held in reserve because of the possibility of false effects due to the grain of the photographic emulsion. Observations secured especially for the purpose might bring out the presence of the secondary spectrum.

Hypothetical Parallaxes.—A value of parallax for h *Centauri* has been derived by H. C. Plummer[19] in

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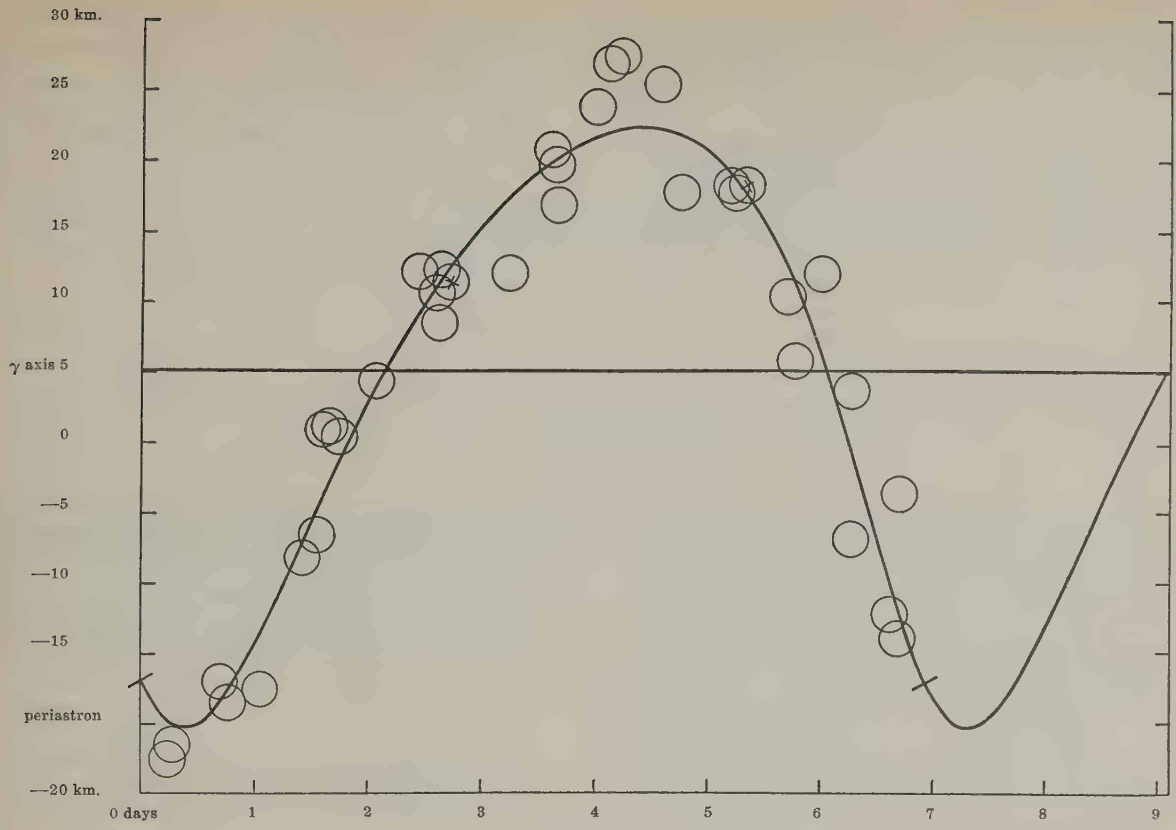
[15] *Publ. All. Obs.*, **1**, 35, 1910.

[16] *Lick Obs. Bull.*, **6**, 39, 1910.

[17] *Publ. All. Obs.*, **1**, 146, 1910.

[18] *Publ. All. Obs.*, **2**, 56, 1910.

[19] *Mon. Not.*, **73**, 180, 1913.



VELOCITY CURVE OF h CENTAURI

connection with his investigation of Class B stars on the hypothesis of motion parallel to the Galactic Plane. This star is one of the number whose velocities satisfy the hypothesis; that is, the component normal to the Galactic Plane is zero. Mr. Plummer's computation, which gave a parallax of 0″.0078, was based upon Boss's values of proper motion and a preliminary value, +8 km., for the velocity γ of the center of mass of the system. The new value of γ, +5.2 km., here derived gives the following values of hypothetical parallax, π, of velocity components *u, v, w,* toward the points 0^h and 6^h of R. A. and the North Pole, and of velocity components, *P, Q, N,* in and normal to the Galactic Plane, as defined by Plummer:[20]

$\pi = 0''.0095$ (343 light-years)
$u = +0.5$ km.
$v = +10.3$
$w = -9.2$
$P = -8.6$
$Q = +8.7$
$N = 0$

The results of Plummer's investigations lend considerable plausibility to the close approach of this value of parallax to the true value, but it may not as yet be freed from the qualification hypothetical.

Kapteyn[21] also obtained a value of parallax for this star upon the basis of stream-motion among the helium or Class B stars, using $\gamma = +8.0$ km. He gives two results, 0″.0073 and 0″.0068, the latter depending upon a systematic correction to the proper motions. The correction of the value of γ to +5.2 km. cannot affect the value of the parallax found, since it would not appreciably alter the value of the stream-velocity derived by Kapteyn from the whole group of observed radial velocities, and upon which the resulting parallax immediately depends.

Another value of parallax for h *Centauri* is given in the recent investigations of Charlier[22] on the *Distances and Distributions of the Class B Stars.* His hypothesis is that these stars have practically the same luminosity, so that the distance is a direct function of the apparent magnitude and a parameter which is derived by least squares from the radial velocities and proper motions of the whole group of observed Class B stars. His value is 0.″0071 (=458 light-years), which, since it depends immediately upon the parameter, cannot be appreciably affected by the small correction here found for the velocity of the binary system.

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[20] *Mon. Not.*, 73, 451, 1913.

[21] *Ap. Jour.*, 40, 43, 119, 1914.

[22] *Publ. Lund Obs.*, Series II, No. 14, 1916, pp. 84, 85.

These three values of parallax, all of which may for the present be called hypothetical, show a remarkable agreement. The three methods of investigation appear to be entirely independent, employing the same data but a different law or relation of certain of the quantities. They have, however, a common basis in that they assume some sort of interrelation among these stars.

G. F. Paddock.

December, 1916.

NOTE ON THE IDENTIFICATION OF TWO MORE LINES IN THE SPECTRUM OF v_4 *ERIDANI*

The writer of the *Astronomical Column* in the issue of *Nature* for January 13, 1916, volume **96**, page 547, has kindly pointed out identifications for two unidentified lines of the spectrum of v_4 *Eridani,* which are given in Table III, *Lick Observatory Bulletin* 274, volume 8, page 172. His references are: For line 4282.7A, Baxandall's *Enhanced Lines of Manganese in the Spectrum of α Andromedae, Mon. Not.,* **74**, 252, 1914; for line 4416.9A, Fowler's *Enhanced Lines of Iron . . ., Mon. Not.,* **67**, 156, 1906. These two lists escaped notice in the compilation of Table III for v_4 *Eridani.* All the lines noted as manganese in Table III, together with line 4282.7A, are given in Baxandall's list, in which the wavelength of the latter line is 4282.65A as found in the laboratory and 4282.87A in the spectrum of *α Andromedae.* Fowler finds the enhanced iron line 4416.98A to be given by the positive pole of the arc, and notes that it is masked by an air line in the spark.

G. F. Paddock.

January, 1917.

Issued January 16, 1917.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 289

EPHEMERIS OF COMET *b* 1916 (WOLF)

The ephemeris of this comet published in *Bulletin No.* **286** is holding so well that it has been decided to extend it and defer the computation of another orbit based upon longer intervals until somewhat later.

Recent observations by Barnard and Aitken show the following representation by this ephemeris:

Date, Gr. M. T.	(O – C) Δα	Δδ	Observer
1916 Dec. 31.00	$-0^s.9$	+8″	Barnard
1917 Jan. 25.55	—1.8	+6	Aitken
Feb. 10.94	—2.8	+9	Barnard

Ephemeris for Greenwich Mean Midnight

1917, Gr. M. T.	True α			True δ		log Δ	Br.
Mar. 5.5	18^h	16^m	38^s	— 1°	12′.2	0.3502	1.00
6.5		18	56	— 1	01.9		
7.5		21	14	— 0	51.4	0.3441	
8.5		23	33		40.7		
9.5		25	52		29.8	0.3381	1.09
10.5		28	11		18.7		
11.5		30	31	— 0	07.5	0.3320	
12.5		32	51	+ 0	03.9		
13.5		35	11		15.5	0.3260	1.18
14.5		37	31		27.3		
15.5		39	52		39.3	0.3199	
16.5		42	13	0	51.5		
17.5		44	35	1	03.8	0.3139	1.29
18.5		46	57		16.4		
19.5		49	19		29.1	0.3079	
20.5		51	41		42.0		
21.5		54	04	1	55.0	0.3019	1.40
22.5		56	27	2	08.2		
23.5	18	58	50		21.6	0.2959	
24.5	19	01	13		35.2		
25.5		03	36	2	48.9	0.2899	1.52
26.5		05	59	3	02.8		
27.5		08	23		16.8	0.2839	
28.5		10	47		31.0		
29.5		13	11		45.3	0.2780	1.65
30.5		13	35	3	59.8		
Mar. 31.5	19	18	00	+ 4	14.4	0.2721	

Ephemeris for Greenwich Mean Midnight—(*Continued*)

1917, Gr. M. T.	True α			True δ		log Δ	Br.
Apr. 1.5	19^h	20^m	25^s	+ 4°	29′.2		
2.5		22	50		44.1	0.2662	1.79
3.5		25	15	4	59.1		
4.5		27	41	5	14.3	0.2603	
5.5		30	06		29.6		
6.5		32	32	5	45.0	0.2545	1.94
7.5		34	58	6	00.5		
8.5		37	24		16.2	0.2488	
9.5		39	50		32.0		
10.5		42	17	6	47.9	0.2431	2.09
11.5		44	43	7	03.9		
12.5		47	10		20.0	0.2374	
13.5		49	36		36.2		
14.5		52	03	7	52.4	0.2318	2.25
15.5		54	30	8	08.8		
16.5		56	57		25.2	0.2263	
17.5	19	59	24		41.7		
18.5	20	01	51	8	58.2	0.2208	2.43
19.5		04	18	9	14.8		
20.5		06	45		31.5	0.2154	
21.5		09	12	9	48.2		
22.5		11	39	10	04.9	0.2100	2.62
23.5		14	06		21.7		
24.5		16	33		38.5	0.2047	
25.5		19	00	10	55.3		
26.5		21	28	11	12.2	0.1994	2.81
27.5		23	55		29.0		
28.5		26	22	11	45.9	0.1943	
29.5		28	49	12	02.8		
Apr. 30.5		31	16		19.6	0.1892	3.01
May 1.5		33	43		36.4		
2.5		36	09	12	53.2	0.1841	
3.5		38	36	13	09.9		
4.5		41	02		26.6	0.1791	3.21
5.5		43	28		43.2		
6.5		45	55	13	59.8	0.1742	
7.5		48	21	14	16.4		
8.5		50	47		32.9	0.1694	3.42
9.5		53	13	14	49.3		
10.5		55	38	15	05.7	0.1646	
11.5	20	58	04		21.9		
12.5	21	00	29		38.1	0.1599	3.63
May 13.5	21	02	54	+15	54.2		

Ephemeris for Greenwich Mean Midnight—(*Continued*)

1917, Gr. M. T.	True α			True δ		log Δ	Br.
May 14.5	21^{h}	05^{m}	19^{s}	+16°	10′.1	0.1553	
15.5		07	43		25.9		
16.5		10	07		41.6	0.1507	3.85
17.5		12	31	16	57.2		
18.5		14	54	17	12.6	0.1462	
19.5		17	17		27.9		
20.5		19	39		43.1	0.1418	4.06
21.5		22	01	17	58.1		
22.5		24	23	18	12.9	0.1374	
23.5		26	44		27.5		
24.5		29	05		41.9	0.1330	4.28
25.5		31	26	18	56.1		
26.5		33	46	19	10.2	0.1287	
27.5		36	05		24.1		
May 28.5	21	38	24	+19	37.7	0.1245	4.49

Ephemeris for Greenwich Mean Midnight—(*Concluded*)

1917, Gr. M. T.	True α			True δ		log Δ	Br.
May 29.5	21^{h}	40^{m}	42^{s}	+19°	51′.1		
30.5		43	00	20	04.3	0.1203	
May 31.5		45	17		17.3		
June 1.5	21	47	34	+20	30.0	0.1162	4.71

Brightness 1917 May 5.5 = 1.00

In the ephemeris of *Bulletin No.* **286** the unit of brightness is that of 1916 May 10. With this as the unit, the brightness for 1917 June 1.5 is 79.9.

R. T. Crawford.

Berkeley Astronomical Department,
February 17, 1917.

Issued February 21, 1917.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 290

ELEMENTS AND EPHEMERIS OF COMET *a* 1917 (MELLISH)

The following results were derived from the observations numbered 1, 12, 15 in the list given below.

PARABOLIC ELEMENTS

$$T = 1917 \text{ April } 10.6206 \text{ Gr. M. T.}$$

$$\left.\begin{aligned} \omega &= 120^\circ\ 32'\ 54'' \\ \Omega &= 87\ \ \ 25\ \ \ 45 \\ i &= 32\ \ \ 18\ \ \ 41 \end{aligned}\right\} 1917.0$$

$$\log q = 9.285270$$

These elements leave a residual O — C, $\Delta\delta = -8''$ in the first declination. Removal of this residual by a general solution was deemed inadvisable.

CONSTANTS FOR THE EQUATOR 1917.0

$$x = [9.927113]\ r \sin (297^\circ\ 30'\ 27'' + v)$$
$$y = [9.970160]\ r \sin (221\ \ 32\ \ 8\ \ + v)$$
$$z = [9.808257]\ r \sin (158\ \ 44\ \ 2\ \ + v)$$

EPHEMERIS FOR GREENWICH MEAN MIDNIGHT

1917	True α	True δ	log Δ	Br.
April 18.5	$0^h\ 39^m\ 39^s$	— 0° 25′.9	0.0020	11.80
20.5	42 34	2 19.1		
22.5	46 18	3 49.2	0.0435	4.80
April 24.5	0 50 26	— 5 02.5		
April 26.5	$0^h\ 54^m\ 45^s$	— 6° 03′.3	0.0779	1.55
28.5	0 59 07	6 54.7		
April 30.5	1 03 28	7 38.9	0.1067	0.97
May 2.5	07 45	8 17.6		
4.5	11 57	8 51.9	0.1312	0.66
6.5	16 03	9 22.8		
8.5	20 02	9 51.0	0.1521	0.47
10.5	23 54	10 17.0		
12.5	27 38	10 41.3	0.1702	0.36
14.5	31 16	11 04.3		
16.5	34 47	11 26.2	0.1860	0.28
18.5	38 11	11 47.3		
20.5	41 28	12 07.9	0.1999	0.22
22.5	44 39	12 28.0		
24.5	47 43	12 47.9	0.2120	0.18
26.5	50 41	13 07.7		
28.5	53 32	13 27.4	0.2227	0.15
May 30.5	56 17	13 47.2		
June 1.5	1 58 56	—14 07.2	0.2320	0.13

Brightness, March 1 = 1.

The available observations are represented as follows, O — C:

Date		Gr.M.T.	App. α	App. δ	cosδΔα	Δδ	Observer
March	1	20.5775	$2^h\ 09^m\ 40.8^s$	+14° 24′ 52″	$+0^s2$	— 8″	Barnard, Yerkes
	2	20.5932	2 09 40.13	14 25 13.0	—0.16	— 7.5	Wilson, Northfield
	3	20.6704	2 09 36.2	14 26 41	—0.2	—16	Shane, Lick
	4	21.5681		14 45 27.5		—13.9	Wilson
	5	21.5731	2 09 41.95		+0.53		Wilson
	6	21.6479	2 09 12.2	14 47 14	0.0	— 8	Shane
	7	22.3093	2 08 51.1	15 01 04	—0.8	— 9	Stroemgren, Copenhagen
	8	22.6440	2 08 40.4	15 08 11	—0.1	— 6	Shane
	9	23.3140	2 08 16.4	15 22 19	+0.3	— 2	Stroemgren
	10	23.5779	2 08 05.6	15 28 11	0.0	+15	Wilson
	11	23.6399	2 08 02.8	15 28 32	—0.2	—42	Shane
	12	24.5339	2 07 23.4	15 48 02	0.0	0	Burton, Washington
	13	27.5752	2 04 13.76	16 50 53.2	—0.38	+ 5.5	Wilson
	14	28.6400	2 02 41.66	17 11 46.5	+0.06	+ 2.9	Shane
	15	29.5742	2 01 05.67	17 24 15.3	—0.05	— 4.4	Wilson

These results were derived from preliminary elements computed from one-day intervals on the basis of the observations numbered 3, 7, 11 (*Harvard College Observatory Bulletin,* No. 630). The elements and constants for the equator 1917.0 of this preliminary orbit are:

$$\begin{aligned} T &= 1917 \text{ April } 11.81 \text{ Gr. M. T.} \\ \left.\begin{aligned} \omega &= 129^\circ\ 50'.1 \\ \Omega &= 83\ \ \ 08.7 \\ i &= 43\ \ \ 34.1 \end{aligned}\right\} & 1917.0 \\ \log q &= 9.17787 \\ \text{O} - \text{C}, \Delta\delta_1 &= -17'' \end{aligned}$$

$$\begin{aligned} x &= [9.86286]\, r \sin\, (300^\circ\ 26'.3 + v) \\ y &= [9.96921]\, r \sin\, (231\ \ \ 55.8 + v) \\ z &= [9.88926]\, r \sin\, (160\ \ \ 29.6 + v) \end{aligned}$$

The accuracy of the preliminary orbit suffered on account of a discrepancy in the third declination as revealed by the residual $\Delta\delta = -42''$ with reference to the new orbit. The constants of the intermediate orbit, for which in addition to the observed α_2 and δ_2 the geocentric distance ρ and the heliocentric velocities x', y', z', derived from the preliminary orbit for the new middle date, are chosen, therefore gave the following large initial residuals for the new first and third places:

	I	III
$\Delta\alpha$	$-12''$	$-912''$
$\Delta\delta$	-101	-104

A linear differential correction on the basis of Leuschner's closed expressions for the parabola resulted in corrections $\Delta\rho$, $\Delta x'$, $\Delta y'$, $\Delta z'$, which reduced these residuals to the following:

	I	III
$\Delta\alpha$	$+17''$	$+130''$
$\Delta\delta$	$+2$	$+30$

On account of the insufficiency of the linear differential relations employed, $\Delta\alpha$ for the third place changed by 1042″ instead of 912″. A new representation was therefore made by correcting the ρ, x', y' of the intermediate orbit by seven-eighths of the computed values of $\Delta\rho$, $\Delta x'$, $\Delta y'$ and adopting a value of z' so as to satisfy the expression for the parabolic velocity $x'^2 + y'^2 + z'^2 = \frac{2}{r}$. This representation gave:

	I	III
$\Delta\alpha$	$+11''$	$-13''$
$\Delta\delta$	-66	$+83$

Since the intervals are approximately in the ratio 4 to 5, inspection of these residuals indicated that they could be removed at once by arbitrarily changing z' so as to remove the residuals in declination and then changing y' so as to satisfy the expression for the parabolic velocity as above. This gave the final residuals:

	I	III
$\Delta\alpha$	$+3''$	$-1''$
$\Delta\delta$	-9	-4

Further reduction of these residuals would have been unwarranted in this case.

It is interesting to note that the final results were produced by a single differential correction of the intermediate orbit followed by some simple arbitrary changes, although the initial residuals were unusually large. The arbitrary changes referred to involve merely the simple formulae of representing observations, many of the numbers remaining unchanged in successive attempts.

HAMILTON MOORE JEFFERS,
ELIZABETH JANET EASTON.

BERKELEY ASTRONOMICAL DEPARTMENT,
April 15, 1917.

Issued April 21, 1917.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 291

THE QUARTZ SPECTROGRAPH OF THE LICK OBSERVATORY AND SOME OBSERVATIONS RECENTLY SECURED WITH IT[1]

The construction of the instrument which is about to be described was rendered possible by generous grants made to Director Campbell by the Rumford Committee of the American Academy of Arts and Sciences and from the Draper fund of the National Academy of Sciences. These allowances have covered the greater part of the expense, but very substantial contributions toward it have been made by the Lick Observatory.

The spectrograph is designed for use with the Crossley reflecting telescope. The optical parts are two 60° quartz prisms of the Cornu type, and simple quartz lenses for the collimator and camera. The latter are about 11 inches in focal length and 2 inches in effective aperture. The prisms are of sufficient size to accommodate the 2-inch beam delivered by the collimator lens. The spectrograph was built in the Observatory shops from detailed drawings by the writer.[2]

The instrument is so constructed as to allow its use in either of two forms: first, as a slit spectrograph of the usual type; and second, as a slitless spectrograph forming with the reflecting telescope a prismatic camera. For use in the latter form, the collimator is removed and the convex collimator lens is replaced by a concave one of equivalent focal length. The whole instrument is then placed in the telescope between the great mirror and its focus in such a position that light converging from the mirror is intercepted by the concave collimator and rendered parallel. After that the beam suffers refraction at the prism surfaces and convergence by the camera in the usual manner. It will be seen that, arranged in this fashion, the spectrograph acts simply as a dispersive agent in the optical path of the telescope. A small instrument embodying this principle has been in use for a number of years at the Lick Observatory.[3]

The spectrograph was planned for use in the direct or primary focus of the reflector. This fact introduced two important considerations into the matter of the design, in addition to the many that are determined by the general requirements of a modern spectrograph. It should intercept as little as possible of the incident radiation, and should be as light as is consistent with a high degree of rigidity, since it must be carried at the extreme end of the telescope. The general requirements of a spectroscope, such as stability, accessibility of parts, ease of manipulation and so forth, are well understood and need not be discussed here. The two factors mentioned have determined a somewhat unusual form, but the requirements have been quite satisfactorily met. The weight of the spectrograph, including supporting frame and all accessories for control of temperature, is forty pounds, or just a balance for the double slide photographic plate-holder which it displaces. The mounting is made almost entirely of aluminum.

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[1] Read before the twentieth meeting of the American Astronomical Society

[2] The project of an ultra-violet spectrograph for the Crossley reflector had its inception in the purchase in 1909 by Director Campbell, with a fund of $300 provided by the Rumford Committee of the National Academy of Arts and Sciences, of the two convex lenses and one of the prisms referred to. These were handed to the writer with instructions to adapt them to use with that telescope, particularly with reference to the spectroscopic observation of Halley's comet. The high inclination of the field given by this combination suggested the use of higher dispersion, and the additional prism was purchased by the Director. The parts were assembled in a temporary wooden mounting and considerable experimental work was done which demonstrated, among other things, the need of a more stable and convenient mounting. The funds necessary for securing this were provided, at the request of Director Campbell, by a grant of $500 from the Draper fund of the National Academy of Sciences, and by expenditures by the Lick Observatory. The writer wishes to express his appreciation of this financial assistance and of the readiness with which Director Campbell has made provision for various needs as, from time to time, they have arisen. He also wishes to thank the Director for great interest shown in the work, and Dr. H. D. Curtis for co-operation referred to elsewhere in this article.

[3] A complete account of this small instrument will be found in a paper entitled "An Application of the Crossley Reflector of the Lick Observatory to the Study of Very Faint Spectra," by H. K. Palmer, *L. O. Bulletin*, 2, 46, 1902.

As originally designed and constructed by Professor Keeler this early instrument consisted of a single small quartz prism introduced into the optical path of the telescope, near the focus. The arrangement was found when first used, shortly after Professor Keeler's death, to give an excessive amount of aberration, due to the fact that the optical pencil is of too great an angle to be treated in this manner—emerging from the prism it is no longer approximately homocentric. On the suggestion of the present writer a concave, collimator lens was placed in front of the prism and a convex lens back of it and, thus modified, the spectrograph proved effective. A similar optical arrangement, it appeared later, had already been adopted by Professor Wadsworth in the design of a spectroscope for the Leander McCormick Observatory (*Misc. Scien. Papers Allegheny Obs.*, new series, no. 7). In his paper Professor Wadsworth gives a theoretical discussion of some of the optical properties of such a system.

The prism box forms the foundation of the spectrograph proper, as distinguished from its supporting frame. It is shown in figure 1 with one side removed, and consists of an aluminum casting well reinforced with ribs and fins, to each side of which are fastened aluminum plates. One of these carries the prisms, and may be easily removed to allow of their inspection. The camera is shown in place, but the collimator has been detached. When in use it is screwed to the lower right hand corner of the box and projects downward. The prisms are set at minimum deviation for wavelength 3600A.

The box is held in its supporting frame by two pivots which project visibly above and below, and by a radial slot. The upper pivot is threaded, which allows the whole spectrograph to be adjusted vertically for focus by means of a hand nut on the frame. The pivots allow one degree of freedom in the motion of the box, namely rotation about the axis which they form, and this is restrained by means of a pin on the frame, which engages the slot, already referred to, cut in the small lug exactly at the lower left hand corner of the box. A flotation support is thereby secured which ensures freedom from strain.

A vertical rectangular space in the supporting frame (figure 2) receives the box. The pivots engage at the upper and lower ends of this aperture. The hand nut for focusing is at the top. The base of the box extends along the short horizontal arm of the frame which appears to project toward the reader. When in use the frame is bolted at the extremities of its arms to the end of the tube of the telescope.[4]

In figure 3 the spectrograph is shown mounted in its frame, and ready for attachment to the telescope. The collimator is stiffened by diagonal braces to the prism box. A column is seen projecting downward just to the left of the collimator tube. This carries the comparison arc and various other accessories. Light from the arc passes through roughly ground quartz windows in the column and strikes a small reflecting quartz prism in front of the slit. The following of the object whose spectrum is being photographed is accomplished by means of a reflecting slit, the reflected beam being caught by the prism outlined against the small white card just below the slit. From here the light passes through the hollow column, and eventually emerges from the main eyepiece of the telescope.

The optical parts, which are by Hilger, are, so far as can be judged, perfect. Excellent definition is obtainable from λ3000 to λ5000, provided the curvature and inclination of the field are allowed for. A glance at figure 3 shows the rather precipitous slope of the plate-holder. High inclination and field curvature are inseparable from prism spectroscopes of which the optical material is uniform throughout. The radius of curvature of the field is, in the present case, a matter of six inches, and it has been found impracticable to secure photographic plates on glass which will take this amount of bending. Recourse was therefore had, at first, to films, but these proved unsatisfactory by reason of certain changes in length which they were found to undergo even during the course of an exposure. It is not uncommon for a film to alter its dimensions one-half of one per cent in less than two hours, a degree of instability not to be tolerated in any photographic operation where accuracy is required. We have been attempting for two years to secure achromatic lenses which will give a better field, and thus allow the use of glass plates, but the disturbed commercial conditions in Europe have so far prevented the accomplishment of this purpose.

In view of all these difficulties it was decided some time ago to use the spectrograph almost entirely in its slitless adaptation, at least until the return of normal times, when suitable lenses could be secured. Figure 4 shows it so arranged. It is to be observed that, with this disposition of the optical elements, the irregularities introduced into the beam by the concave collimator are almost exactly nullified by the action of the convex camera lens, so that the resulting field is at right angles to the camera axis, and only slightly curved. Ordinary glass photographic plates can therefore be used. The cut shows the instrument ready for use in this manner. During observation it is surrounded by a heavy blanket under which is a lining of

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[4] In order to attach the spectrograph to the telescope it is necessary first to remove the regular photographic accessories from the latter. This is made possible through certain changes made in the telescope by Dr. H. D. Curtis who is in immediate charge of that instrument. The adaptation of the telescope to two classes of work has necessarily resulted in a certain amount of inconvenience to Dr. Curtis, and the writer desires to express his appreciation of the very considerate spirit in which the added burden has been assumed.

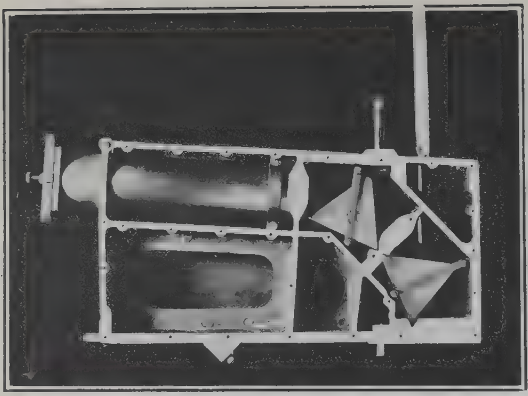

Fig. 1.

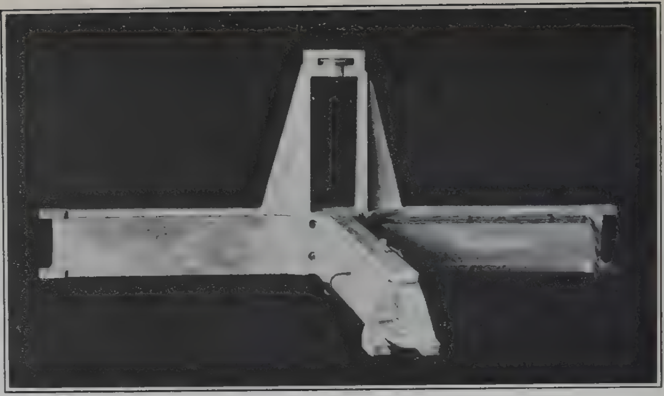

Fig. 2.

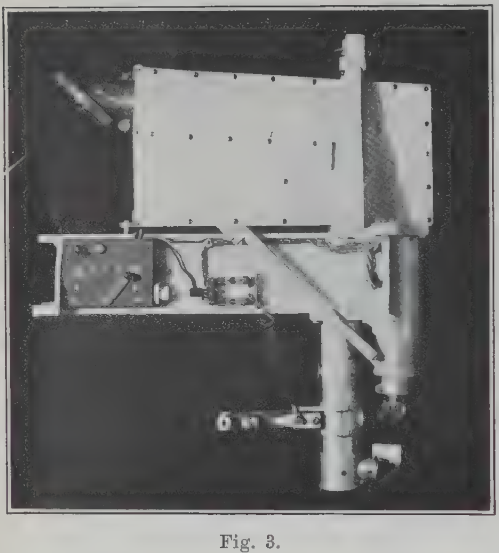

Fig. 3.

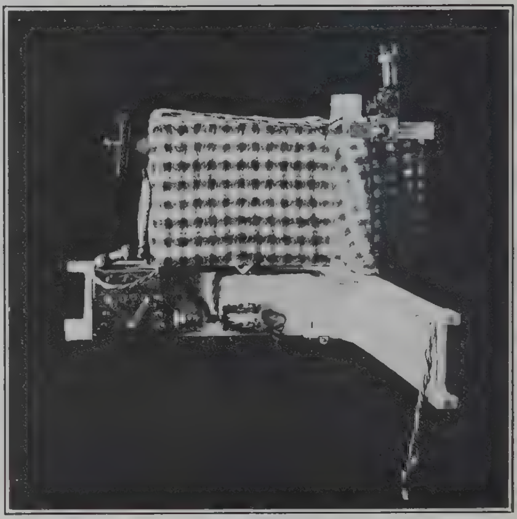

Fig. 4.

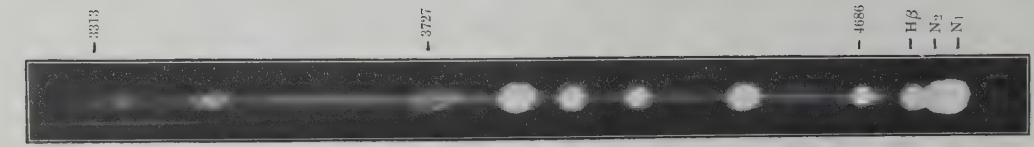

Fig. 5. Prismatic Photograph of Nebula N. G. C. 7662.

cotton cloth wired for the heating circuit. A rheostat and thermostat regulate the current.

The apparatus being now, in certain essential respects, a camera, provision must be made for following the object to be photographed in some such way as is done in direct photography. For this purpose crosswires are attached to the spectrograph close to the axis of collimation, and these are made to bisect a guiding star in the manner commonly adopted in astronomical photography. The mechanism carrying the crosswires and accessories may be seen attached near the upper right hand corner of the spectrograph.

Used as a prismatic camera the instrument is proving a very effective aid in a research in which the writer is at present engaged, that is, the investigation of the distribution throughout the nebulae of the various radiations which make up their spectra. A photograph of one of these objects consists of a series of many images, each one impressed by the light of a line in its spectrum. If there is a nucleus this is usually represented by a streak of continuous spectrum threading the centers, the whole record presenting the appearance of beads on a string. As will be seen from an inspection of figure 5, the monochromatic images may exhibit a great variety of forms for the same object. That this is true to a certain extent has been known for a number of years, the fact having been established chiefly through rather laborious observations with slit spectroscopes.[5] The prismatic camera, however, will furnish by a single observation a wealth of detail which even an extended investigation with a slit spectroscope could hardly supply. For the study of line distribution in any source the prismatic camera is, where conditions permit its use, probably the most effective form of instrument available.

In addition to depicting the nebular images as recorded by the various lines, the slitless spectrograph has revealed a number of strong lines in the ultra-violet which appear to have escaped previous notice. Among these are 3313, 3342, and 3444A. Three of the ultra-violet images of N. G. C. 7662, shown in the cut, are due to these lines. Another interesting fact brought to light is that most of the planetary nebulae possess what appears to be a faint continuous spectrum, emitted throughout the nebula, which begins quite abruptly at about 3652A and extends, with great uniformity, far into the ultra-violet. The spectrum should be investigated with a slit spectrograph for the purpose of determining its character and limit with greater certainty. On account of the well understood limitations of the slitless spectrograph in the matter of resolving power, when applied to the study of a body having a sensible diameter, one cannot be certain that the spectrum is, in fact, continuous, though its extent and great uniformity favor the assumption that it is. The peculiarity of this spectrum is not its continuity, since many nebulae present more or less continuous radiation, but the fact that it comes to an abrupt end, and furthermore that end is very close to the theoretical limit of the Balmer hydrogen series at 3647A. From this position the continuous spectrum extends in the direction of shorter wave-lengths while the lines of the Balmer series reach toward the red. Evershed has observed a similar spectrum in the solar prominences and lower chromosphere, which begins suddenly at 3668A, and suggests a hydrogen origin for it.[6] He refers to an observation by Sir William and Lady Huggins[7] to the effect that in such stars as *Vega* the continuous spectrum suffers a sudden fall in intensity at the end of the series of dark hydrogen lines, beyond which it continues without further enfeeblement far into the ultra-violet. In the later stars this peculiarity is absent. The writer's observations of *Vega* and other stars confirm these statements, the description by Sir William and Lady Huggins applying with great accuracy to plates secured with this spectrograph.[8] We have then in *Vega* an *absorption* spectrum corresponding closely with the *emission* spectrum in the chromosphere and in the nebulae. In the stellar spectrum the absorption appears to begin at about λ3700. Evershed's limit for the chromosphere emission is 3668, while in the nebulae the measured position is roughly 3650. The limit for the Balmer series is about 3647, so there is apparently some overlapping of the two spectra, the amount probably depending on the conditions in the source. A corresponding continuous spectrum appears to have been actually observed by Dufour[9] in the laboratory, during the course of his investigation of the spectrum of hydrogen, though he seems not to have associated it with the Balmer series.

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[5] The cut shows only a small part of the detail recorded on the original negative, most of it having been lost in the reproduction. As a matter of fact even a single original negative constitutes only a partial record, since the range in intensity of the different images is so great that suitable impression of all of them cannot, in general, be obtained on the same plate. The practice has been adopted of taking a series of plates of the same object with exposure times running from a few minutes to several hours. The images given by strong and weak lines are thus made fairly comparable.

[6] *Phil. Trans.*, **197A**, 399, 1901.

[7] *An Atlas of Representative Stella Spectra*, p. 85.

[8] It is to be observed that the slitless spectrograph is particularly suited to observations of this character since, when observing with the use of a slit, one part or another of the spectrum will in general be displaced from the slit by atmospheric dispersion, unless the slit should happen to lie in a vertical plane. Estimates of relative intensity cannot, under these circumstances, safely be made over wide ranges of spectrum. The usual observing practice is to follow with the aid of the visible image of the star, and, under extreme conditions, the whole ultra-violet end of the spectrum may be occulted. This difficulty of course disappears with the elimination of the slit.

[9] *Ann. Chim. et Phys.* (8), **9**, 361, 1906.

In this connection it will be recalled that Wood,[10] in his investigation of the absorption of sodium vapor, observed a continuous absorption spectrum beginning just where the Balmer series for sodium leaves off, and extending far into the ultra-violet; that is, the sodium spectrum presents a phenomenon closely paralleling the apparent behavior of hydrogen in the early stars.

All these facts seem to lend plausibility to the idea that the spectrum here discussed belongs to the Balmer series, and that its production is in some way involved in the mechanics of line series radiation. Laboratory work undertaken in this connection might lead to interesting results. To anyone contemplating such an investigation the writer would suggest the use, for purposes of preliminary investigation, of comparatively low dispersion.

The observations here touched upon will be discussed in greater detail in a forthcoming paper.

W. H. Wright.

Mount Hamilton, December, 1916.

Issued May 19, 1917.

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[10] *Ap. Jour.*, 29, 100, 1909.

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THE REFRACTION AT MOUNT HAMILTON

The diurnal variation of the correction for atmospheric refraction at Mount Hamilton was discussed first in *Lick Observatory Bulletin,* **7**, 130, 1913. The discussion was based upon observations made during the prosecution of fundamental meridian circle work, in the years 1905 and 1906. In that period of a full year transits were taken at nearly every hour of day and night. But the stars culminating at low altitudes, from which the corrections to the refraction can be best derived, were not visible generally during the brightest period of the day, within three hours either side of noon, and consequently but eighteen hours of the full twenty-four were represented in the first discussion. The second magnitude stars, *Polaris* and *β Ursae Minoris,* can be observed at any hour of the day that they cross the meridian, at either culmination, when the sky is clear; and one hundred transits of each were observed in the period of a year, including some close to noon. But, as the principal object in view, at that time, was the investigation of fundamental right ascensions, by successive transits of the circumpolar stars at alternate culminations, the zenith distances were not usually measured at the midday transits, when the bad quality of the star images and their excessive unsteadiness made it evident that such measures would not add much weight to determinations of fundamental declinations.

Since the existence of a diurnal variation in the refraction has become evident, some measures of the correction near noon are desired, and the observations of the year 1916 were designed to fill this gap. Confirmation of the change, which is rather radical in nature, is also to be desired. It is essential to include determinations of the correction in the night hours, in the same series, in order to obtain strictly differential values of the correction, and for the elimination of other sources of systematic error.

Reference can be made to the earlier discussion for evidence of the diurnal change, and for the outline of the methods of reduction. Similar methods are followed here. Emphasis should be laid again upon the necessity of eliminating such systematic errors as might produce an effect similar to that ascribed to a change in the refraction. While individual corrections to the refraction can be derived by comparing single observations of a star at low altitude with the zenith point, determined either by nadir, or by observations of standard stars near the zenith, greater reliance can always be placed upon the combinations of stars on both sides of the zenith, in which the actual determination of the zenith point has no effect. Thus the accidental variations of the nadir readings are eliminated, as well as the variation of latitude, which might enter with maximum effect in observations separated by six months' interval. If individual values of the correction are to be derived, the comparison with observations of zenith stars is preferred here, since it appears to have about three times the weight of a comparison with the nadir, owing to small and unexplained changes in the readings of the latter, from night to night.

Errors in the adopted declinations of the stars and the corresponding graduation errors of the circles are eliminated in the comparison of the same stars, observed at different epochs. Flexure is very small in this instrument, less than 0.″1, and the measures show no sensible effect depending upon temperature, which might have been assumed as likely to influence the results of observations at different hours and seasons. Bisection error is of prime importance, for, if this were of a different nature for the different characters of the images by day and by night, the indications of change in the refraction might possibly be attributed to this source. The bisection error is small, fairly constant, and the measures made by day in the past year, and in the course of the earlier fundamental work, show no sensible variation from the night values. The effect of atmospheric dispersion upon the images, at low altitudes, was discussed in the earlier *Bulletin,* and it would only become perceptible in extreme cases.

My determinations of bisection error with this instrument are summed up below, to indicate the amount and the consistency of the observed correction. The effect upon transits, due to the observer's position, is also tabulated, with the respective numbers of the determinations of each error. One determination is derived from the observations of a pair of stars upon two nights, four observations in all. The effects are tabulated in the sense, position facing south minus position facing north.

Year	$\Delta\alpha$	No.	$\Delta\phi$	No.
1894–1898	$-0^{\mathrm{s}}.030$	39	$+0''.12$	56
1905–1906	$-\ .031$	33	$+\ .15$	34
1911	$-\ .005$	60	$+\ .02$	60
1916	$-\ .034$	4	$+\ .16$	26
Means	-0.020		$+0.10$	

The probable errors of a single determination are, respectively, $\pm 0^{\mathrm{s}}.024$, and $\pm 0''.23$; which correspond to the probable errors of a single observation at the zenith. In the application of these corrections, the transits observed facing north would require the correction $-0^{\mathrm{s}}.020$, to reduce them to the equivalent of transits facing south, in which position the clock stars are generally observed. In practice, the determination of the right ascension of a star has almost invariably been made with the use of standard stars on the same side of the zenith, and the effect due to the observer's position has been eliminated in the difference of the transits. There is no reason to doubt that the bisection error in zenith distance would be the same in both positions of the observer, and consequently observed zenith distances on both sides of the zenith should be diminished $0''.05$, to correct for bisection error. This correction has, consequently, the contrary sign to that of the general correction for refraction. The observations of 1911 were largely of stars of about seventh magnitude, while all the other determinations were made with the use of brighter standard stars. This may account for the small values of both the observed corrections in that year, especially since my magnitude equation is apparently slightly different for stars observed in the two positions. In the interval 1908 to 1911 a large amount of observing was done with the eight inch telescope of the Dudley Observatory transit circle at San Luis, Argentine, and the different character of the conditions there might have produced a slight change in my observing habit. When one takes into account that the correction for large zenith distances may amount to several seconds of arc, in the differential effect due to the diurnal variation of the refraction, the bisection error can evidently be treated as negligible, unless subject to extraordinary changes of a systematic character.

In 1916 two lists of stars were observed, and the epochs of culmination cover the full period of day and night. List B includes five north stars at lower culmination, giving zenith distances from 70° to 80°. With these were combined eighteen south stars, of which six are between 60° and 70° zenith distance, eleven are between 70° and 80°, and one is at 84°. The north stars have a mean tangent zenith distance of 3.96, and the south stars of 4.14. The total arc covered is over 164°, and the mean differential tangent factor is 8.10, for the full list. Observations were continued from the first dates, when the culminations all occurred in the night, until the greater part of the stars had become invisible in daylight. The entire range is from three hours after noon, or four hours before sunset, up to nearly midnight. The brighter stars of List C were observed throughout the daylight hours, with corresponding night observations. It is not always feasible to obtain observations of the character desired to fulfill all the conditions of such investigations. In addition to the loss of effective brilliancy, by contrast with the sky in full daylight, there is the atmospheric absorption at low altitudes, which are the most effective in refraction determinations. Worst of all difficulties is the unsteadiness of the images of the stars in the daylight hours, which often makes it impossible to see a star, of which an observation could be secured if the eye could once seize the dancing point of light. By reason of their slower motion in transit, the north stars are more easily seen, and they can be observed when an equal amount of unsteadiness would prevent the detection by the eye of a rapidly moving south star. By my estimates, full daylight at noon cuts down the effective brilliancy of a star at the zenith by five magnitudes. At sunset or sunrise the loss is three magnitudes. Absorption will cut off another magnitude at 70° zenith distance, and the loss at 80° is two or more magnitudes. A daylight list must then be confined to stars of second magnitude, or at most third magnitude, and even the brightest stars cannot be observed at very low altitudes.

Light haze and thin clouds are much more prevalent at Mount Hamilton in the daytime than had been anticipated, it is likely, by anyone, previous to the prosecution of meridian circle work in daylight. The slightest amount of either may completely hide a star, even at moderate zenith distance. With respect to unsteadiness, the daytime conditions at Mount Hamilton are the worst encountered in my experience at four important observing stations.

The first tabulation of List B gives the correction to the Poulkova refractions, in terms of the factor of the tangent zenith distance, for each of sixteen dates, when both south stars and lower culmination stars were used in combination. The intervals from noon and from sunset are given for each date, for the mean of the culminations. At the foot of the table the mean values of the corrections are given for day and night observations. The mean of the night observations includes the results on all those dates for which the mean culmination follows sunset; but the correction to the refraction appears to have a progressive change, adopted in all the computed corrections, which extends to one hour after sunset. Similarly, the change adopted in the later tables extends from one hour before sunrise, into the daylight hours. A mean value has been adopted for the night, from one hour after sunset to one hour preceding sunrise, though the observations of 1905 and 1906 indicated a small progressive change during the night. The column ΔN gives the difference between the correction for each date and the mean of the night corrections reckoned from sunset. In this list of stars the mean night correction is close to that provisionally adopted from all the meridian circle observations made here, and which has been discussed in our *Publications*. That correction was —0″2 tan. Z. D. While we are concerned here only with the change in the correction between day and night, the mean value, subject to the effect of errors of declination and of graduation errors, is a satisfactory check upon both the refraction correction and the data employed.

The column of computed corrections is derived from the adopted rate of variation, +0″10 per hour, from one hour after sunset to four hours before sunset. The column giving the difference between the observed and computed values of ΔN, has an average which is less than one half the average residual of the observed values, an indication of the validity of the change adopted. The mean temperature of the observing period of each date is recorded in the last column. The nine night temperatures have an average of 56°3, and a range of 23°; the seven day temperatures have an average of 55°3 and a range of 20°.

LIST B. SOUTH STARS AND LOWER CULMINATIONS

1916	S. Stars	L.C.	Noon	Sunset	Diff. Corr.	v.	ΔN.	Comp.	O–C	Temp.
Feb. 15	18	5	$+9^{\mathrm{h}}4$	$+4^{\mathrm{h}}1$	—0″17	—0″10	+0″05	0″00	+0″05	58.1
16	18	5	9.3	4.0	— .22	— .15	.00	.00	.00	56.9
Mar. 9	18	4	8.3	2.5	— .29	— .22	— .07	.00	— .07	64.1
10	18	5	7.9	2.1	— .31	— .24	— .09	.00	— .09	65.4
15	18	5	7.6	1.7	— .11	— .04	+ .11	.00	+ .11	58.1
16	18	5	7.5	1.6	— .13	— .06	+ .09	.00	+ .09	58.5
21	17	4	7.6	1.6	— .24	— .17	— .02	.00	— .02	42.3
29	17	4	7.1	1.0	— .37	— .30	— .15	.00	— .15	48.5
Apr. 6	13	4	6.7	+0.3	— .19	— .12	+ .03	+ .07	— .04	54.5
20	11	4	5.9	—0.7	— .08	— .01	+ .14	+ .17	— .03	54.4
26	9	4	5.6	1.1	+ .10	+ .17	+ .32	+ .21	+ .11	53.1
May 4	6	1	4.3	2.8	+ .25	+ .32	+ .47	+ .38	+ .09	69.9
10	5	3	4.9	2.0	+ .10	+ .17	+ .32	+ .30	+ .02	54.1
11	4	3	4.9	2.1	+ .23	+ .30	+ .45	+ .31	+ .14	55.2
16	4	2	5.0	2.1	+ .02	+ .09	+ .24	+ .31	— .07	49.8
29	3	1	+3.8	—3.4	+0.22	+0.29	+0.44	+0.44	0.00	50.7
Sums.	197	59								
Means (16)					—0.07	±0.17			±0.07	
9 Night					—0.22					
7 Day					+0.12					
Day–Night					+0.34					

Hourly rate +0″11. Adopted rate +0″10.

The observations of List B have also been combined in hourly groups, where each group includes all the observations of any of the stars that culminated within the period of each hour, reckoned from sunset. The corrections to the refraction in this table are derived by comparing the individual observations with those of a pair of zenith stars each night. The groups will contain different stars, in varied proportions, and the effect of errors in the adopted declinations and of graduation errors might obscure the evidence of the variation in the refraction. Yet the corrections tabulated below show the same evident trend, in the hourly values, as was indicated in the tabulation by dates. The observations of south stars only on two additional dates are included here. The average of the last column, O–C, is again less than half the average residual of the hourly values before the application of the rate of variation. The declinations of the stars have been taken partly from the *Berliner Jahrbuch,* and partly from the *Preliminary General Catalogue* of Lewis Boss.

List B. Hourly Values of Refraction Correction

Sunset	S. Stars	L.C.	Corr.	*v.*	ΔN.	Comp.	O–C
$+5^{h}.9$	4	4	—0″28	—0″18	—0″03	0″00	—0″03
4.3	13	5	— .26	— .16	— .01	.00	— .01
3.5	21	9	— .29	— .19	— .04	.00	— .04
2.3	32	5	— .34	— .24	— .09	.00	— .09
1.5	35	9	— .22	— .12	+ .03	.00	+ .03
+0.5	33	7	— .10	.00	+ .15	+ .05	+ .10
—0.5	22	9	— .08	+ .02	+ .17	+ .15	+ .02
1.3	17	5	+ .04	+ .14	+ .29	+ .23	+ .06
2.4	18	5	+ .28	+ .38	+ .53	+ .34	+ .19
—3.1	8	1	+0.24	+ .34	+ .49	+ .41	+ .08
Sums.	203	59					
18 Sets							
Means (10)			—0.10	±0.18			±0.07
6 Night			—0.25				
4 Day			+0.12				
Day–Night			+0.37				

Hourly rate +0″13 Adopted rate +0″10.

The accumulation of the same algebraic signs in the column O–C is due in part to a small rate in the observed night values, and to the adoption of a smaller rate in the day values than the one computed from this table. The seven tabulations listed here give individual hourly rates, by least squares solutions, varying from +0″08 to +0″13. The value 0″10 has been adopted throughout, as a close approximation, which will be consistent for all parts of this discussion. The average O–C would usually be diminished by the use of the rate computed from any one of the tables under treatment. The day values of ΔN have been used, in each table, for the computation of the rates, but these are always applied for the interval included between an hour preceding sunrise to four hours after; or, from an hour following sunset, to four hours preceding.

List C was made from ten bright stars which can be observed when in transit at any hour of the day. Either nadir readings or the pair of zenith stars, or seven of the ten stars, at moderate zenith distances, can be employed for the determination of the zenith point. The three stars, *β Ceti* at zenith distance 55°8, *Polaris* at 51°5, and *β Ursae Minoris* at 68°2, have been used in computing the corrections to the refraction, and observations of the remaining stars have been employed as checks upon the values derived. Only the condensed results need be given here.

The tabulation below gives the resulting correction to the Poulkova refraction, derived by two methods:

First, by comparing each star with the pair of zenith stars. The tabulation gives the weighted mean correction for each date, the weights adopted for the three stars mentioned above being 2, 1, and 4, respectively. When any stars were not observed, the corrections derived from the others were reduced to the weighted mean of all three.

Secondly, the column of differential corrections results from the combination of the three stars in pairs, and is independent of the observed zenith point. For the combination of *β Ceti* and *Polaris* the weight 5 was assigned, 10 for *β Ceti* and *β Urs. Min. l. c.,* and for the two north stars 1 only. Two stars on the same side of the zenith have small weight in combination, since in that case the difference of the tangents of zenith distance is the available factor. Each combination, where only two stars were observed, has been reduced to the weighted mean of all three combinations. The temperature for each date is included, to make the record complete. The average of the day temperatures is 58°9, with a range of 27°; the average of the night temperatures is 60°6, with a range of 36°. There are twenty-four corrections in the table, between sunrise and sunset, and twenty corrections during the night hours.

The separate weighted values of C are also combined in groups, with the respective mean dates and the epochs reckoned from noon, and the intervals from sunset and sunrise. The residuals *v* of the groups are given, then the difference ΔN of each from the mean

List C. Corrections to Poulkova Refractions

1916	Stars	Noon	Sunset	Sunrise	Mean	Differential	Temp.
Feb. 24	1	$+ 3^{\mathrm{h}}1$	$—2^{\mathrm{h}}4$		$+1''09$	....	$52^{\circ}5$
25	1	3.0	2.5		—0.28	....	50.7
Mar. 7	3	2.5	3.2		— .47	$—0''50$	50.6
8	3	2.4	3.3		— .13	+ .18	62.6
13	2	2.6	3.2		— .16	+1.44	59.1
14	3	2.0	3.8		+ .08	+ .21	66.8
30	2	1.6	4.6		— .18	.00	48.3
31	2	1.5	4.7		— .61	+ .18	56.5
Apr. 3	2	1.4	4.9		+ .14	—1.10	50.8
4	2	+ 1.3	5.0		+ .07	— .55	51.4
24	2	0.0	—6.7		+ .86	— .87	58.8
25	2	0.0		$+6^{\mathrm{h}}7$	— .11	—1.05	54.9
May 11	3	— 1.6		5.4	— .48	— .58	48.2
12	3	1.7		5.3	— .22	— .42	50.1
June 5	3	3.2		4.0	— .58	— .71	64.4
6	3	3.3		3.9	+ .59	+ .63	71.9
13	2	4.4		2.9	+ .54	+ .52	74.2
14	2	4.5		2.8	+ .59	+ .58	75.8
July 6	3	5.4		1.8	— .10	— .18	63.8
7	3	5.5		1.7	+ .06	— .01	54.7
18	3	6.2		0.9	— .05	— .29	67.0
19	3	6.3		0.8	— .22	— .36	70.1
25	3	6.7		0.3	— .55	— .55	53.7
26	3	6.8		+0.3	— .20	— .28	57.7
Aug. 1	3	7.1		—0.2	— .28	— .36	70.0
2	3	7.2		0.3	— .27	— .34	71.0
23	3	8.5		2.0	— .38	— .47	74.6
24	3	8.6		2.1	— .72	— .80	74.2
29	3	8.9		2.4	— .33	— .30	68.4
30	3	9.0		2.5	— .25	— .33	68.7
Sept. 6	3	9.4		3.1	— .30	— .37	54.9
7	3	9.5		3.1	— .79	— .78	47.9
19	3	10.2		4.1	— .47	— .48	68.1
20	3	10.2		4.2	— .37	— .43	66.3
27	3	10.6		4.7	— .67	— .75	64.7
Oct. 13	3	11.6		6.0	— .46	— .51	53.1
16	3	—11.8		—6.2	— .40	— .44	60.0
23	3	+11.8	+6.4		— .51	— .65	47.1
30	3	11.4	6.1		— .32	— .49	44.0
Nov. 9	3	10.7	5.6		— .46	— .50	55.5
13	3	10.4	5.3		— .47	— .52	38.3
20	3	9.9	5.0		— .63	— .68	48.4
28	3	9.3	4.5		— .08	— .09	43.4
Dec. 15	3	+ 8.2	+3.4		— .36	— .52	48.7
Sums 44	119						

of the night values. The computed value has been reckoned with the rate, $+0''10$ per hour, from one hour after sunset, to four hours before; and with the same rate from one hour before sunrise, to four hours after. The final column gives the difference O–C between the two preceding columns. Its mean is about two-thirds the average residual of the observed values. The combination of differential corrections has three determinations in the first two tabulated values; all the other groups of both tables have four each. The third value in the tabulation of differential corrections was based upon the combination of the two north stars, of low weight, and it might be rejected by the criterion of probable error. In that case, the average O–C would be $\pm 0''14$, or one-half the average residual v. The corresponding value in the tabulation of mean corrections is in conformity with the other figures.

Some explanation of the difference in the two methods seem necessary, to account for the apparent contradiction in sign in this case and in others. Both of the north stars may give plus corrections to the refraction, treated individually, while the difference of the two observed zenith distances may give a minus sign. When two stars on opposite sides of the zenith are combined, the results will be more nearly the weighted mean of the individual results. It is partly for this reason that the main dependence for evaluating the correction is placed upon the reduction, in combination, of observations both north and south of the zenith. But it has been deemed proper to include all the computed values.

The precision of individual results, and of those in the groups, is seen to be very different by day and by night. The probable error of observation in full daylight is at least three times as large as that under ordinary conditions at night. Even the circle readings are less precise, owing to the interference of daylight with the artificially lighted fields of the microscopes. But the evidence of the change in the refraction is plain, in every method of combination, notwithstanding the difficulty of securing observations in the daytime.

List C. Groups of Four Mean Corrections

1916	Noon	Sunset	Sunrise	Mean	*v.*	ΔN.	Comp.	O–C
Mar. 1	+ 2^{h}8	—2^{h}8		+0″05	+0″25	+0″48	+0″38	+0″10
Mar. 22	+ 1.9	—4.1		— .22	— .02	+ .21	+ .50	— .29
Apr. 14	+ 0.7	—5.8		+ .24	+ .44	+ .67	+ .50	+ .17
May 24	— 2.4		+4^{h}7	— .17	+ .03	+ .26	+ .50	— .24
June 25	— 4.9		+2.3	+ .27	+ .47	+ .70	+ .33	+ .37
July 22	— 6.5		+0.6	— .26	— .06	+ .17	+ .16	+ .01
Aug. 12	— 7.8		—1.2	— .41	— .21	+ .02	.00	+ .02
Sept. 2	— 9.2		—2.8	— .42	— .22	+ .01	.00	+ .01
Sept. 27	—10.6		—4.8	— .49	— .29	— .06	.00	— .06
Oct. 27	+11.5	+6.2		— .42	— .22	+ .01	.00	+ .01
Nov. 26	+ 9.4	+4.6		— .39	— .19	+ .04	.00	+ .04
Sum. 44								
Means (11)				—0.20	±0.22			±0.12
6 Day				—0″02				
5 Night				—0.43				
Day–Night				+0.41				
Hourly rate +0″10.								

List C. Groups of Differential Corrections

1916	Noon	Sunset	Sunrise	Diff. Corr.	*v.*	ΔN.	Comp.	O–C
Mar. 9	+ 2^{h}5	—3^{h}2		+0″37	+0″66	+0″86	+0″42	+0″44
Mar. 25	+ 1.7	—4.4		+ .13	+ .42	+ .62	+ .50	+ .12
Apr. 14	+ 0.7	—5.8		— .89	— .60	— .40	+ .50	— .90
May 24	— 2.4		+4^{h}7	— .27	+ .02	+ .22	+ .50	— .28
June 25	— 4.9		+2.3	+ .23	+ .52	+ .72	+ .33	+ .39
July 22	— 6.5		+0.6	— .37	— .08	+ .12	+ .16	— .04
Aug. 12	— 7.8		—1.2	— .49	— .20	.00	.00	.00
Sept. 2	— 9.2		—2.8	— .44	— .15	+ .05	.00	+ .05
Sept. 27	—10.6		—4.8	— .54	— .25	— .05	.00	— .05
Oct. 27	+11.5	+6.2		— .52	— .23	— .03	.00	— .03
Nov. 26	+ 9.4	+4.6		— .45	— .16	+ .04	.00	+ .04
Sum. 42								
Means (11)				—0.29	±0.30			±0.21
6 Day				—0.13				
5 Night				—0.49				
Day–Night				+0.36				
Hourly rate +0″08. Adopted rate, +0″10.								

List A gives the corrections to the Poulkova refractions as originally reduced and tabulated in the earlier *Bulletin* to which reference has been made. They are reproduced here in a form similar to that of the corrections derived in 1916, and the application of the computed rate of variation has been introduced for the first time. In the tabulation, twelve values have been reckoned in the night correction, though the rate of change is applicable in three of these. The average of the last column, O–C, is about half the average residual *v* of the group values. All three lists, A, B, and C, give similar indications of the variation in refraction, with such differences as can be attributed to the probable errors of observation.

Since the shortest day here is 9½ hours in length from sunrise to sunset, the rate of variation has been reckoned in all cases up to four hours only from sunset or sunrise. The indication of the tabulations, and of graphical plots as well, is that the rate progresses up to an hour following sunset, or from an hour pre-

LIST A. 1905 AND 1906

Date	Noon	Sunset	Corr.	v.	ΔN.	Comp.	O–C
1906 Aug. 8	+ 4ʰ9	—2ʰ2	+0″.36	+0″.47	+0″.58	+0″.32	+0″.26
1905 July 16	6.4	—1.1	+ .15	+ .26	+ .37	+ .21	+ .16
1906 July 3	7.2	—0.4	+ .03	+ .14	+ .25	+ .14	+ .11
1906 Jan. 30	5.4	+0.2	+ .24	— .13	— .02	+ .08	— .10
1906 June 16	8.4	+0.8	— .19	— .08	+ .03	+ .02	+ .01
1906 Jan. 4	7.1	+2.3	— .09	+ .02	+ .13	.00	+ .13
1906 May 13	10.6	+3.5	— .12	— .01	+ .10	.00	+ .10
1905 Oct. 27	+11.6	+6.5	— .20	— .09	+ .02	.00	+ .02
		Sunrise					
1905 Oct. 13	—11.4	—5.5	— .25	— .14	— .03	.00	— .03
1905 Oct. 2	10.8	—4.7	— .28	— .17	— .06	.00	— .06
1905 Sept. 16	9.7	—3.4	— .32	— .21	— .10	.00	— .10
1906 Mar. 8	9.0	—3.4	— .33	— .22	— .11	.00	— .11
1905 Aug. 31	8.6	—2.0	— .18	— .07	+ .04	.00	+ .04
1906 Jan. 23	6.2	—1.5	— .36	— .25	— .14	.00	— .14
1905 Aug. 13	7.4	—0.5	— .12	— .01	+ .10	+ .05	+ .05
1905 July 29	6.5	+0.6	— .10	+ .01	+ .12	+ .16	— .04
1905 July 16	5.5	+1.8	+ .07	+ .18	+ .29	+ .28	+ .01
1906 June 28	— 4.4	+3.0	+ .13	+ .24	+ .35	+ .40	— .05
Sum. 72 sets.							
Means (18)			—0.11	±0.15			±0.08
12 Night			—0.22				
6 Day			+0.11				
Day–Night			+0.33				

Hourly rate +0″.12. Adopted rate +0″.10.

ceding sunrise. In the longest days of 14½ hours, a mean value can be adopted for the period near noon. The night values are nearly constant, there being little evidence in 1916 confirmatory of the small observed night rate of the years 1905 and 1906.

The observations of the three stars of List C, which were made in 1905 and 1906, have also been collected, to check the variation of the correction to the refraction. There are forty dates on which all three stars were observed, and the results are tabulated below in a condensed form similar to that of the preceding tables. The groups correspond to the epochs of observation, with an average of three values in a group. The methods and the weights are precisely the same as used for List C, but each date includes all three stars, and no single value needs reduction to the weighted mean.

In that year of fundamental work, both the circles were read at each observation of zenith distance, and the movable circle was shifted 1° after each date.

LIST *a*. THREE STARS, 1905–06

Date	No.	Noon	Sunrise	Mean Corr.	ΔN.	Comp.	Diff. Corr.	ΔN.
1906 June 21	2	— 4ʰ4	+2ʰ9	+0″.40	+0″.46	+0″.39	+0″.28	+0″.34
1906 July 4	3	— 5.3	+2.0	+ .25	+ .31	+ .30	+ .30	+ .36
1905 July 15	4	— 6.0	+1.2	+ .29	+ .35	+ .22	+ .26	+ .32
1905 July 24	3	— 6.6	+0.5	+ .24	+ .30	+ .15	+ .15	+ .21
1905 Aug. 6	4	— 7.5	—0.6	+ .07	+ .13	+ .04	+ .04	+ .10
1905 Aug. 23	3	— 8.5	—1.9	— .02	+ .04	.00	+ .01	+ .07
1905 Sept. 6	4	— 9.4	—3.0	— .01	+ .05	.00	— .06	.00
1905 Sept. 18	3	—10.1	—4.0	+ .03	+ .09	.00	+ .07	+ .13
1905 Oct. 5	3	—11.1	—5.3	— .10	— .04	.00	— .14	— .08
1905 Oct. 16	3	—11.8	—6.3	— .03	+ .03	.00	— .06	.00
			Sunset					
1905 Oct. 30	3	+11.3	+6.0	— .11	— .05	.00	— .07	— .01
1906 Jan. 5	2	+ 6.6	+1.9	— .20	— .14	.00	— .14	— .08
1906 Jan. 30	3	+ 4.8	—0.2	— .15	— .09	+ .12	— .12	— .06
Sum. 40								
Means (13)				+0.05			+0.04	
5 Day				+0.20			+0.17	
8 Night				—0.05			—0.04	
Day-Night				+0.25			+0.21	
Average residual				±0.15			±0.13	
Average O–C				±0.08			±0.07	

Hourly rate +0″.11. Adopted rate 0″.10.

The errors of graduation of the fixed circle, for the mean of the two positions, affect each measure but are eliminated in the comparison of observations at different hours of the day. For the movable circle, the differential measures are affected by the mean of the graduation errors, a small quantity when the number of measures is sufficiently large, as the division errors appear to be of accidental character. The means of the results derived from the two circles have been used in the tabulation. The average O–C, computed with the adopted rate, is half the average residual *v* of the original observed corrections.

The probable errors of a single determination of the refraction correction in List *a* have been computed from each individual value, and from the weighted means. They are as follows:

		Day	Night	All 40
	β Ceti	±0″.28	±0″.17	±0″.23
	Polaris	.37	.24	.30
	β Urs. Min. l. c.	.18	.11	.14
	Weighted means	.13	.10	.11
Pairs:	*β Ceti* and *Polaris*	±0″.23	±0″.14	±0″.18
	β Ceti and *β Urs. Min. l. c.*	.13	.10	.12
	Polaris and *β Urs. Min. l. c.*	.32	.25	.28
	Weighted means	.12	.10	.11

The computed weights of the determinations from single stars, corresponding to these probable errors, would be: 1.7, 1.0, and 4.3. These are in good accord with the weights adopted on the basis of the respective tangent factors. The computed weights of the pairs in combination would be: 2.4, 5.6, and 1.0.

The computed probable error of day-night in either method would be ±0″.036 from this table. The probable errors of a determination by night and by day are in the ratio of 1 to 1.5 but, as constant sources of error affect both, the actual accidental errors of observation are in a larger ratio, more nearly 1 to 3. It will be noted that the weaker values add but little weight, in the means above, to that of the strongest determinations from the stars in each case, if the respective probable errors were to be accepted as the only criteria.

The difference of refraction between day and night is in no sense a seasonal one, in this case; as the stars observed in 1905 and 1906 give the comparison of values by night in summer with those by day in winter, and the contrasted effects of exactly the opposite combination. The day and night values do not differ individually, at any season, by variations which cannot properly be ascribed to the legitimate accidental errors of observation. They are not affected by the range of recorded temperatures, as shown in the discussion in the earlier *Bulletin;* nor by the changes of temperature, rising or falling, during the periods of observation.

No abnormal changes of refraction need to be accounted for in these results, which extend to measures of zenith distance beyond 80°. The evidence of a diurnal change appears to be clear, and such variation in individual measures as has been encountered in reducing the results can be quite properly ascribed to the accidental errors of observation, under the difficult conditions of daytime transits. It appears to be a more consistent course to accept the existence of such handicaps to the precision of observations than to make vague assumptions of systematic changes in the atmospheric refractions at different hours or for different seasons, as has been done in much current discussion of results of observation, to account for deviations from the anticipated agreement of separate values. Such assumptions appear of especially doubtful nature, when the arcs measured are very small, in contrast with the extensive arcs covered by meridian circle observations.

Adopting the night correction to the Poulkova refractions as equivalent to a correction to observed zenith distances of —0″.20 tan. Z. D., the following table of corrections has been formed. The correction is zero at one hour preceding sunset, or one hour following sunrise, and a uniform progressive change has been adopted for a period of five hours, as tabulated below. This table gives a working basis for any observations to be made here in the future, and is not likely to be changed sensibly by the accumulation of the results of observation.

CORRECTIONS TO THE POULKOVA REFRACTIONS

	Night, —0″.20 tan. Z.D.	
Sunset, +1ʰ.0	—0.20 tan. Z.D.	Sunrise, —1ʰ.0
Sunset, 0.0	—0.10 tan. Z.D.	Sunrise, 0.0
Sunset, —1.0	0.00	Sunrise, +1.0
Sunset, —2.0	+0.10 tan. Z.D.	Sunrise, +2.0
Sunset, —3.0	+0.20 tan. Z.D.	Sunrise, +3.0
Sunset, —4.0	+0.30 tan. Z.D.	Sunrise, +4.0
	Noon, +0.30 tan. Z.D.	

Some partial results of this discussion were presented at the meeting of the Pacific Division of the American Association for the Advancement of Science, at San Diego, in August last.

R. H. TUCKER.

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MEASURES OF COMETS IN 1915 AND IN 1916

The majority of the following measures were made with the 12-inch refractor, $\Delta\alpha$ being measured directly with the micrometer in every instance but one. Comet *d* 1915, on September 21, 1915, was so far from a comparison star that the measures in both co-ordinates had to be made by transits. The measures of Comet *a* 1915 on February 26, 1915, of Comet *e* 1915, on December 15 and 18, 1915, and the four measures of Comet *a* 1916 were made with the 36-inch telescope.

One measure was secured of the brighter companion to Comet *a* 1915. On May 14, 1915, at 17^{h} 24^{m} 51^{s} Mt. Hamilton sidereal time, this companion was 35″2 in position angle 285°2 from the principal nucleus of the comet. The fainter companion was barely visible and was not measured. Stormy weather prevented further observations before the comet passed below the southern horizon. In September the companions were not visible.

COMET *a* 1915 (MELLISH)

1915	Mt. H. M. T.	Star	No. of Comp.	Comet–Star		Comet's Apparent		Log ρΔ	
				$\Delta\alpha$	$\Delta\delta$	α	δ	α	δ
Feb. 12	17^{h} 22^{m} 31^{s}	2	10, 8	+ 7^{s}40	+5′ 37″7	17^{h} 06^{m} 01^{s}93	+ 2° 59′ 01″1	9.410*n*	0.699
Feb. 13	17 03 21	3	10, 8	—12.95	+4 22.5	17 07 17.48	+ 2 54 26.9	9.454*n*	0.701
Feb. 26	16 21 14	4	10, 8	— 3.90	—3 38.1	17 23 43.72	+ 1 54 50.2	9.472*n*	0.710
Mar. 18	16 23 17	5	12, 8	+17.26	+7 48.5	17 48 48.98	+ 0 06 23.8	9.302*n*	0.724
May 14	13 30 29	6	9, 9	—22.24	—4 56.4	19 11 24.08	—23 27 17.1	9.445*n*	0.858
Sept. 11	16 01 45	7	12, 12	—24.35	+0 11.0	6 27 37.95	—20 59 44.7	9.556*n*	0.826
Sept. 19	16 47 56	8	8, 8	+ 6.00	—5 16.1	6 24 02.98	—19 28 59.2	9.332*n*	0.852
Oct. 15	14 56 43	9	13, 10	+10.74	—4 05.2	5 54 08.41	—14 16 51.9	9.227*n*	0.830

MEAN PLACES OF COMPARISON STARS FOR 1915.0

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
1	17^{h} 05^{m} 43^{s}41	+0^{s}44	+ 2° 48′ 55″6	—16″6	A. G. Albany 5674.
2	17 05 54.09	+0.44	+ 2 53 40.0	—16.6	Micrometer comparison with (1).
3	17 07 29.96	+0.47	+ 2 50 21.1	—16.7	A. G. Albany 5684.
4	17 23 46.88	+0.74	+ 1 58 46.1	—17.8	A. G. Albany 5782.
5	17 48 30.48	+1.24	— 0 01 07.7	—17.0	A. G. Nicolajew 4435.
6	19 11 43.39	+2.93	—23 22 17.1	— 3.6	C. G. C. 26400.
7	6 28 00.39	+1.91	—21 00 14.3	+18.6	Cinn. Zone Cat. 1044.
8	6 23 54.74	+2.24	—19 24 02.0	+18.9	Cinn. Zone Cat. 1023.
9	5 53 54.33	+3.34	—14 13 05.7	+19.0	A. G. Washington 1845.

COMET *d* 1915 (MELLISH)

1915	Mt. H. M. T.	Star	No. of Comp.	Comet–Star		Comet's Apparent		Log ρΔ	
				$\Delta\alpha$	$\Delta\delta$	α	δ	α	δ
Sept. 19	16^{h} 21^{m} 49^{s}	1	12, 8	— 12^{s}64	+6′ 29″9	10^{h} 37^{m} 03^{s}58	+26° 13′ 17″1	9.714*n*	0.705
Sept. 20	16 18 42	2	10, 8	— 37.39	—2 33.1	10 44 54.83	+25 47 31.0	9.711*n*	0.713
Sept. 21	16 35 42	4	4, 4	+115.84	+6 50.8	10 52 58.60	+25 18 51.6	9.712*n*	0.700
Sept. 22	16 33 33	5	8, 6	— 36.10	+6 48.5	11 01 00.09	+24 47 47.8	9.708*n*	0.708

Mean Places of Comparison Stars for 1915.0

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
1	10^h 37^m 14^s18	$+2^s04$	+26° 06′ 57″8	—10″6	A. G. Cambridge 5461.
2	10 45 30.20	+2.02	+25 50 15.0	—10.9	Micrometer comparison with (3).
3	10 45 34.22	+2.02	+25 59 41.7	—10.9	A. G. Cambridge 5515.
4	10 51 00.75	+2.01	+25 12 11.8	—11.1	Boss, P. G. C. 2909.
5	11 01 34.21	+1.98	+24 41 10.6	—11.3	½ (A. G. Cambridge 5628 + A. G. Berlin B 4182).

COMET *e* 1915 (TAYLOR)

1915	Mt. H. M. T.	Star	No. of Comp.	Comet–Star		Comet's Apparent		Log ρΔ	
				Δα	Δδ	α	δ	α	δ
Dec. 7	13^h 13^m 50^s	1	10, 8	+ 7^s20	—1′ 53″8	5^h 22^m 04^s69	+0° 14′ 36″6	9.353	0.723
Dec. 9	10 47 40	3	12, 10	+27.09	—4 16.0	5 20 59.58	+0 37 16.3	9.212*n*	0.719
Dec. 15	9 45 19	4	12, 10	+ 2.90	+3 32.3	5 17 24.35	+2 02 11.0	9.364*n*	0.646
Dec. 18	10 40 42	6	12, 10	—12.27	—3 57.9	5 15 32.07	+2 53 30.7	8.982*n*	0.695

Mean Places of Comparison Stars for 1915.0

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
1	5^h 21^m 52^s53	$+4^s96$	+0° 16′ 19″5	+10″9	Micrometer comparison with (2).
2	5 21 24.84	+4.96	+0 26 41.6	+10.9	A. G. Nicolajew 1304.
3	5 20 27.48	+5.01	+0 41 21.0	+11.3	A. G. Nicolajew 1300.
4	5 17 16.34	+5.11	+1 58 27.8	+10.9	Micrometer comparison with (5).
5	5 17 07.98	+5.11	+1 46 50.6	+10.9	A. G. Albany 1671.
6	5 15 39.17	+5.17	+2 57 17.8	+10.8	A.G. Albany 1661.

COMET *a* 1916 (NEUJMIN)

1916	Mt. H. M. T.	Star	No. of Comp.	Comet–Star		Comet's Apparent		Log ρΔ	
				Δα	Δδ	α	δ	α	δ
Mar. 7	7^h 41^m 41^s	1	12, 12	$—16^s66$	—1′ 33″3	9^h 01^m 03^s47	+9° 53′ 20″5	9.430*n*	0.627
Mar. 11	9 43 15	3	10, 10	+20.74	—0 58.2	9 03 32.17	+7 48 21.4	7.636*n*	0.634
Apr. 1	10 35 28	5	12, 10	+14.01	—2 17.7	9 28 36.80	—1 31 16.1	9.326	0.738
Apr. 20	9 25 24	6	12, 10	—13.04	—5 23.2	10 04 4.24	—7 41 04.9	9.198	0.788

Mean Places of Comparison Stars for 1916.0

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
1	9^h 01^m 17^s47	$+2^s66$	+ 9° 55′ 02″4	— 8″6	Micrometer comparison with (2).
2	9 01 28.94	+2.66	+10 00 55.6	— 8.6	A. G. Leipzig I 3650.
3	9 03 08.82	+2.61	+ 7 49 28.9	— 9.3	A. G. Leipzig II 4965.
4	9 28 15.46	+2.37	— 1 28 10.6	—12.9	A. G. Nicolajew 2885.
5	9 28 20.42	+2.37	— 1 28 45.5	—12.9	Micrometer comparison with (4).
6	10 04 15.03	+2.25	— 7 35 25.3	—16.4	Micrometer comparison with (7).
7	10 04 15.03	+2.25	— 7 32 17.2	—16.4	A. G. Wien-Ottakring 3905.

February, 1917.

R. G. Aitken.

PHOTOGRAPHIC MEASURES OF SATELLITES AND OF COMETS

The following observations were obtained photographically with the Crossley reflector. For the Ninth Satellite of *Jupiter* it was necessary to set off the satellite's motion, and the stars therefore trail. The Sixth Satellite of *Jupiter* happened to be upon one of these plates.

In order to measure the plate of Wolf's Comet for July 17, 1916, it was necessary to determine the positions of some stars on the plate of July 18, 1916. Images of two nebulae were plainly visible upon these short exposures and their positions were also obtained. The positions of these stars and nebulae for 1916.0 are given together.

IX SATELLITE OF JUPITER

Date	Gr.M.T.	α 1915.0	δ 1915.0	Exposure	Comparison Stars	Remarks
1915 Aug. 19^d	21^h 34^m 25^s	23^h 56^m 46^s.97	— 2° 54′ 39″.9	3^h	Algiers Astr. Cat. Cliche 1551, Nos. 81, 91, 98, 122. Cliche 1914, No. 137.	Image elongated E & W.
Sept. 8	18 31 0	23 47 22.72	— 3 56 1.4	3	Bonn. A. G. 8150, 8153, 8154, 8155, 8157, 8160.	Image elongated E & W. Star 8160 overexposed.
Sept. 8	21 37 38	23 47 18.51	— 3 56 28.4	2 45^m	Bonn. A. G. 8150, 8153, 8154, 8155, 8157, 8160.	Image elongated E & W. Star 8160 overexposed.
Sept. 11	19 47 34	23 45 43.88	— 4 6 11.0	3	Bonn. A. G. 8147, 8149, 8150, 8153, 8154, 8155.	
Sept. 15	19 25 53	23 43 32.98	— 4 19 27.4	3	Bonn. A. G. 8136, 8139, 8147, 8149.	Image elongated.

VI SATELLITE OF JUPITER

Date	Gr.M.T.	α 1915.0	δ 1915.0	Exposure	Comparison Stars	Remarks
1915 Sept. 11	19 47 34	23 44 32.88	— 3 47 29.7	3	Bonn. A. G. 8147, 8149, 8150, 8153, 8154, 8155.	A trial.

COMET *a* 1915 (MELLISH)

Date	Gr.M.T.	α 1915.0	δ 1915.0	Exposure	Comparison Stars	Log pΔ α	Log pΔ δ
1915 Oct. 21	0 51 22	5 44 37.23	—13 1 51.9	3^m	Camb. A. G. 1612, 1613, 1619, 1620, 1626, 1635.	7.992*n*	0.830

COMET *e* 1915 (TAYLOR)

Date	Gr.M.T.	α 1915.0	δ 1915.0	Exposure	Comparison Stars	Log ρΔ α	Log ρΔ δ
1915 Dec. 16	19 6 12	5 16 40.02	+ 2 19 12.5	10^m	Albany A. G. 1663, 1671, 1681, 1683.	8.876*n*	0.701
Dec. 18	18 26 58	5 15 27.57	+ 2 53 7.2	3	Albany A. G. 1653, 1656, 1659, 1661, 1663, 1664.	9.136*n*	0.696
Dec. 20	18 46 30	5 14 14.20	+ 3 29 52.4	3	Albany A. G. 1646, 1652, 1656, 1661, 1662.	8.902*n*	0.687

COMET *b* 1916 (WOLF)

Date	Gr.M.T.	α 1916.0	δ 1916.0	Exposure	Comparison Stars	Log pΔ α	Log pΔ δ
1916 June 17	18 34 57	12 28 57.52	+ 4 45 27.9	10^m	Leipzig A. G. II 6150, 6151, 6166, Albany A.G. 4510.	9.590	0.696
July 17	17 1 50	12 37 53.05	+ 4 13 43.3	20	Albany A. G. 4550, 4553, and three stars determined from following plate.	9.608	0.692
July 18	17 10 3	12 38 22.85	+ 4 11 28.6	25	Albany A. G. 4550, 4552, 4553, 4555, 4557, 4559.	9.619	0.704

Mean Places for 1916.0 of Stars and Nebulae Measured on Wolf's Comet Plate of July 18, 1916

α 1916.0	δ 1916.0	Remarks
12 37 48.08	+ 4 12 35.9	B.D. +4°.2638 Mag. 9.5.
12 38 14.41	+ 4 25 10.7	Nebula N. G. C. 4630.
12 38 25.50	+ 4 35 17.5	Anon. star. Mag. 9.5 (estimated).
12 38 39.61	+ 4 8 12.4	Unidentified nebula. Somewhat fainter than N. G. C. 4630.
12 38 58.19	+ 3 39 38.6	B.D. +4°.2640. Mag. 9.5

MICROMETER MEASURES WITH THE 12-INCH TELESCOPE

Comet *e* 1915 (Taylor)

Date	Gr.M.T.	Star	No. of Comp,	Comet–Star Δα	Comet–Star Δδ	Comet's Apparent α	Comet's Apparent δ	Log ρΔ α	Log ρΔ δ
1915 Dec. 30^d	20^h 31^m 33^s	1	10, 19	+0^m 3^s.06	—1′ 10″.0	5^h 9^m 4^s.07	+7° 3′ 51″.2	9.340	0.634
1916 Jan. 4	17 44 50	2	10, 11*	—0 19.22	+0 13.6	5 7 26.31	+9 1 41.5	8.859*n*	0.616

* Observations interrupted by dampness slackening micrometer wires.

Mean Places of Comparison Stars

Star	Epoch	α	Red. to App. Place	δ	Red. to App. Place	Authority.
1	1915.0	5^h 8^m 55″.67	+5^s.34	+7° 4′ 50″.6	+10″.56	Leipzig A. G. II 2062.
2	1916.0	5 7 43.05	+2.48	+9 1 21.9	+ 5.96	Leipzig A. G. II 2052.

January, 1917.

R. F. Sanford.

OBSERVATIONS OF COMETS

COMET *e* 1914 (CAMPBELL)

1914	Gr.M.T.	Star	No. of Comp.	Comet–Star Δα	Comet–Star Δδ	Comet's Apparent α	Comet's Apparent δ	Log ρΔ α	Log ρΔ δ
Oct. 6	$17^{h}\ 45^{m}\ 21^{s}.2$	1	15, 11	$-15^{s}.53$	—0′ 2″.0	$22^{h}\ 18^{m}\ 31^{s}.86$	—15° 22′ 36″.8	8.633	0.843
Oct. 13	19 6 17.6	2	11, 12	—32.33	+3 1.3	21 58 31.80	— 6 15 4.1	9.457	0.770

COMET *a* 1915 (MELLISH)

1915	Gr.M.T.	Star	No. of Comp.	Comet–Star Δα	Comet–Star Δδ	Comet's Apparent α	Comet's Apparent δ	Log ρΔ α	Log ρΔ δ
May 6	$22^{h}\ 00^{m}\ 5^{s}.8$	4	10, 7	$-5^{s}.93$	+6′ 28″.2	$18^{h}\ 54^{m}\ 49^{s}.83$	—15° 16′ 3″.2	9.400*n*	0.826

COMET *e* 1915 (TAYLOR)

1915	Gr.M.T.	Star	No. of Comp.	Comet–Star Δα	Comet–Star Δδ	Comet's Apparent α	Comet's Apparent δ	Log ρΔ α	Log ρΔ δ
Dec. 30	$19^{h}\ 54^{m}\ 33^{s}.4$	5	17, 11	$+4^{s}.01$	—1′ 49″.8	$5^{h}\ 9^{m}\ 5^{s}.02$	+ 7° 3′ 11″.4	9.172	0.649

COMET *a* 1916 (NEUJMIN)

1916	Gr.M.T.	Star	No. of Comp.	Comet–Star Δα	Comet–Star Δδ	Comet's Apparent α	Comet's Apparent δ	Log ρΔ α	Log ρΔ δ
May 25	$18^{h}\ 00^{m}\ 2^{s}.1$	6	8, 10	$-20^{s}.59$	—4′ 17″.8	$11^{h}\ 22^{m}\ 39^{s}.02$	—15° 15′ 29″.3	9.504	0.817

Mean Places of Comparison Stars for 1914.0

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
1	$22^{h}\ 18^{m}\ 43^{s}.34$	+4″.05	—15° 22′ 51″.7	+16″.9	A. G. Washington 8357.
2	21 59 00.41	+3.72	— 6 18 22.9	+17.5	A. G. Wien-Ottakring 7909.

Mean Places of Comparison Stars for 1915.0

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
3	$18^{h}\ 54^{m}\ 52^{s}.18$	+2″.61	—15° 25′ 47″.0	— 9″.7	A. G. Washington 7021.
4	18 54 53.15	+2.61	—15 22 41.1	— 9.7	Micrometer comparison with (3).
5	5 8 55.67	+5.34	+ 7 4 50.6	+10.6	A. G. Leipzig II 2062.

Mean Places of Comparison Star for 1916.0

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
6	$11^{h}\ 22^{m}\ 57^{s}.34$	+2″.27	—15° 10′ 50″.5	—21″.0	A. G. Washington 4495.

The foregoing observations were secured with the 12-inch refractor of the Lick Observatory. The observation of May 25, 1916, may be in error by as much as 10″ owing to the faintness of the comet, very poor seeing, and trouble with the micrometer. I am indebted to Mr. R. F. Sanford for recording and checking the reduction of the measure on December 30, 1915.

Warren K. Green.

PHOTOGRAPHIC POSITIONS OF COMETS

COMET *c* 1914 (NEUJMIN)

Plate	Exposure	Gr. M.T. of Mid-Exposure	Mean α	Mean δ
3272	1^{h}	1915 July 13 $18^{h}\ 11^{m}\ 26^{s}.0$	$19^{h}\ 45^{m}\ 38^{s}.91$	+54° 54′ 43″.0
3273	1	1915 July 13 21 01 26.0	19 43 10.81	+54 54 38.0
3274	1	1915 July 14 18 42 55.3	19 42 09.51	+54 59 52.5
3285	15^{m}	1915 Aug. 9 17 40 57.3	19 44 29.13	+56 04 50.7
3286	20	1915 Aug. 9 18 9 28.7	19 44 29.56	+56 04 52.9

COMET *e* 1915 (TAYLOR)

Plate	Exposure	Gr. M.T. of Mid-Exposure	Mean α	Mean δ
3456	10^{m}	1916 Apr. 5 $15^{h}\ 59^{m}\ 30^{s}.0$	$7^{h}\ 35^{m}\ 36^{s}.96$	+33° 03′ 47″.3
3457	30	1916 Apr. 5 16 22 45.0	7 35 39.88	+33 03 48.0

COMET *a* 1916 (NEUJMIN)

Plate	Exposure	Gr. M.T. of Mid-Exposure	Mean α	Mean δ
3551	10^{m}	1916 May 28 $16^{h}\ 36^{m}\ 49^{s}.4$	$11^{h}\ 29^{m}\ 27^{s}.17$	—15° 45′ 04″.7

The foregoing positions were secured from plates taken with the Crossley reflector of the Lick Observatory. The plates were measured with the Stackpole engine, and the reductions made by a modification of Jacobi's method. These were checked by re-reduction by Turner's method. In every case at least four reference stars were used. The residuals of the star places were such that a least-squares solution was not performed for any of the plates.

Warren K. Green.

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THE RADIAL VELOCITIES OF ELEVEN SOUTHERN LIGHT-VARIABLES

The data and results contained in this paper have been derived from observations made at the D. O. Mills Observatory, Santiago, Chile, in 1909. The purpose of the work was to discover spectroscopic orbital motion in the light-variables observable in the southern hemisphere, especially those of the *Cepheid* class. From the *Second Catalogue of Variables*,[1] eleven stars were selected for the observation of their velocities in the line-of-sight, as follows:

Variable	α		δ		Class of Variable		Magnitude	Spectral Class	Period
U *Hydrae*	10^h	32^m6	—12°	52′	III	Irreg.	4.5 to 6.3	N	
T *Carinae*	10	51.3	—59	59		Susp.	6.7 to 7.0	Ko	
S *Muscae*	12	7.4	—69	36	IV	*Cepheid*	6.4 to 7.3	F8p	9.6+
T *Centauri*	13	36.0	—33	6	II	Long	6.5 to 9.2	Md	90.3
R *Apodis*	14	46.5	—76	15		Susp.	5.5 to 6.2	Ko	
ι *Librae*	15	6.5	—19	25	III		4.3 to 5.0	Aop	
R *Tri. Austr.*	15	10.8	—66	8	IV	*Cepheid*	6.7 to 7.4	K5 to G5	3.3+
S *Tri. Austr.*	15	52.2	—63	30	IV	*Cepheid*	6.4 to 7.4	G5	6.3+
S *Normae*	16	10.6	—57	39	IV	*Cepheid*	6.6 to 7.6	Go	9.7+
RV *Scorpii*	16	51.8	—33	27	IV	*Cepheid*	6.7 to 7.4	F 5 to G5	6.0+
RS *Sagittarii*	18	11.0	—34	8	V	*Algol*	5.9 to 6.3	B5	2.4+

The above assignments of spectral class have kindly been furnished by Professor Pickering and Miss Cannon. They write that the spectra of R *Trianguli Australis* and RV *Scorpii* have been found to vary from F5 at maximum to G5 at minimum. The present series of spectrograms show no changes unless possibly slight changes in the relative intensity of the hydrogen absorption.

Observations.—The observations, following below, were all made with the single-prism spectrograph attached to the 37-inch reflector. A cemented doublet lens by Brashear with a focal length of 16 inches was used in the camera. The angle of the prism is 63½° and the dispersion is 41A per millimeter at Hγ. The iron arc for the comparison spectrum and Seed plates of emulsion 27 were employed. Three plates marked L in the fourth column below were of the Lumiere Σ emulsion. Slit width was 0.044 inch.

Date Gr.M.T.	Julian Day 241—	Exposure h	m	Quality of plate	Velocity km./sec.
		U *Hydrae*			
1909 March 31	8397.564	2	0	fair	—20
		T *Carinae*			
1909 March 31	8397.687	2	50	good	—23
April 15	8412.635	2	50	good	—25
April 22	8419.619	3	0	good	—26
April 27	8424.608	3	2	good	—28
		S *Muscae*			
1909 March 26	8392.780	3	0	good	— 9
March 29	8395.743	3	0	good	+10
April 6	8403.753	3	20	good	— 1
April 19	8416.711	3	0	good	+24
April 29	8426.651	3	0	fair	+23
June 5	8463.567	2	30	fair	+19

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[1] *Annals H. C. O.*, 55, 7, 1907.

Date Gr.M.T.	Julian Day 241—	Exposure h	m	Quality of plate	Velocity km./sec.
	T Centauri				
1909 March 30	8396.669	3	30	poor	+38
				br. Hγ	+36
May 9	8436.726	3	40	fair	+19
				br. Hγ	+20
May 24	8451.667	3	30	poor L	+16
May 26	8453.633	6	30	fair	+25
	R Apodis				
1909 March 29	8395.861	1	50	good	—37
April 4	8401.800	1	50	good	—34
April 22	8419.769	2	40	good	—30
May 1	8428.736	2	10	good	—29
May 25	8452.681	2	30	good	—32
June 7	8465.697	2	0	fair	—32
	ι Librae				
1909 March 26	8392.687	0	26	good	—10
March 30	8396.780	0	24	good	—37
March 31	8397.840	0	20	good	+17
	R *Trianguli Australis*				
1909 April 7	8404.640	2	48	good	—34
April 7	8404.825	2	40	good	—31
April 8	8405.627	3	10	fair	—28
April 17	8414.615	2	40	good	—33
April 20	8417.599	3	0	fair	—19
April 25	8422.581	3	14	fair	—27
April 28	8425.574	3	10	fair	—31
May 1	8428.569	2	58	good	—32
May 28	8455.726	4	0	good	—30
May 29	8456.713	4	0	good	—17
May 30	8457.739	2	52	fair	— 1
June 5	8463.709	4	0	fair	—10
June 6	8464.707	5	0	fair	— 7
	S *Trianguli Australis*				
1909 April 5	8402.867	2	34	fair	+ 4
April 6	8403.874	3	0	good	+11
April 8	8405.847	2	30	good	— 7
April 17	8414.853	3	0	good	+ 1
April 27	8424.799	2	40	good	— 6
April 29	8426.817	3	2	good	— 7
May 2	8429.852	3	40	fair	+21
	S *Normae*				
1909 April 5	8402.748	2	40	good	—12
April 14	8411.722	3	0	good	— 4
April 19	8416.844	3	0	good	—10
April 30	8427.701	3	36	fair	— 3
May 2	8429.683	4	0	fair	+12
	RV *Scorpii*				
1909 April 17	8414.733	2	20	fair	—41
April 25	8422.722	2	50	poor	—30
April 26	8423.711	4	20	poor	—25
April 30	8427.874	3	30	fair	—33
May 3	8430.702	4	20	poor	— 9
May 10	8437.764	6	0	fair	—25
May 24	8451.806	2	0	useless L	
May 30	8457.703	3	0	fair	—37

Date Gr.M.T.	Julian Day 241—	Exposure h	m	Quality of plate	Velocity km./sec.
	RS *Sagittarii*				
1909 March 31	8397.792	1	0	poor	+27
April 15	8412.812	1	16	good	—73
April 25	8422.820	1	32	good	—43
May 9	8436.858	1	30	good	—46
May 24	8451.873	0	52	good L	—35
May 25	8452.818	1	44	good	+73
May 26	8453.808	1	24	good	—51
May 28	8455.860	1	50	fair	—20
May 29	8456.838	1	40	good	—20
June 6	8464.865	2	20	good	+84

Mars

Date Gr.M.T.	Julian Day 241—	Computed velocity	Observed velocity	O—C
1909 April 14	8411.812	—16.53	—19.11	—2.58
April 15	8412.849	—16.42	—20.69	—4.27
April 15	8412.858	—16.41	—18.89	—2.48
April 25	8422.865	—15.91	—15.20	+0.71
April 29	8426.891	—15.68	—19.56	—3.88
May 1	8428.793	—15.71	—21.92	—6.21
May 10	8437.903	—14.99	—18.17	—3.18
				—3.13

Measures and Reductions.—It was at first expected that the measures would be made on the Toepfer measuring machine and reduced by means of a table based on measures of a sky plate. In the meantime the Hartmann comparator became available and its advantages were made use of. No standard comparison plates of stars of known velocity having been taken for the purpose along with the observations of the variables, it has been necessary to adopt a sky plate as the basis of the velocities to be derived. Since the apparent velocity shown by a sky spectrum is never greater than one-half kilometer this becomes essentially a zero-velocity standard for low dispersion. This procedure was practically the only reliable one in view of the difficulties of constructing tables for dissimilar spectra, particularly of low dispersion.

The detailed procedure of measurement is as follows: All the plates of a variable were measured twice against the best plate of that variable. The eight comparison plates thus employed were measured three times against a sky plate selected to be the basis of all the derived velocities. This sky plate was exposed May 6, 1909, the temperature in the prism box having been carefully kept constant. Ten regions of this sky spectrum, more or less uniformly distributed from 4100A to 4500A, were selected as the places for making the comparisons. By means of the Hartmann-Cornu formula a short table was constructed for the measurement of the isolated lines of the spectra of Classes A and B shown by the variables *ι Libræ* and RS *Sagittarii*.

The Mars plates.—Each of the seven *Mars* plates was compared three times against the standard sky plate, these measures being interspersed among the measures of the variables. The differences O–C between the mean of the three measures and the computed values of *Mars'* velocity indicate a systematic error in the measurements. These differences are practically all negative, their range being nearly 7 km. and their mean —3 km. Similar differences are found in comparing the *Mars* plates with other sky plates, indicating that the discrepancy is not peculiar to the adopted sky plate alone. Introducing a systematic correction of +3 km., the probable error of a single measure would be 1.4 km., which would indicate fairly reliable measures for these one-prism plates. It is difficult, however, to assign the cause of a systematic error, —3 km., otherwise than that the character of the lines of the sky or *Mars* spectrum on these low-dispersion plates produces it; but it is likewise difficult to say that such an effect does not exist equally in both sky and *Mars* spectra, or that either one rather than the other actually represents its theoretical velocity. The application of a systematic correction to all measures with the sky plate is therefore of doubtful propriety and will not be performed since accidental error of measurement is much greater, and relative measures only are to be considered at this time. A proper precaution and check for this work would have been to observe at the time of observing the variables several bright stars whose velocities had been determined with three prisms. But the difficulty and need were not forseen.

U Hydrae.—One plate only was secured of U *Hydrae* whose spectrum is of Class N. The exposure was sufficient to secure only the two short regions of spectrum lying between 4400 and 4600A, and 4700 and 4900A. Between and on either side of them are the absorption bands characteristic of this spectral class. These regions contain numerous lines, but the corresponding regions of the sky plate are useless. However, on comparing the plate of U *Hydrae* with one of T *Centauri* of Class Md the lines of the two spectra were found to correspond closely and a value of velocity was easily derived. Similarly, the plate was compared with one of R *Apodis* of Class K. These indirect results, —20 km. and —26 km., might be distrusted without confirmation by another process of measurement, but it is not practicable to measure the plate otherwise, partly because of the difficulty of identifying the lines. Corroboration, however, is found in the Yerkes[2] three-prism observations of this same star, 132 Schjellerup, by Hale, Ellerman, and Parkhurst, who obtained twenty-one very satisfactory line identifications, mostly with iron and titanium, from which they derived a velocity of —28 km.

T *Centauri.*—The usual variable bright hydrogen lines associated with Class Md appear in the present observations. On the plates of 1909 March 30 and May 9, Hβ, Hγ, and Hδ are bright, but on the plates of May 24 and 26 the bright lines are not seen. The bright-line radiation seemed more intense on March 30 than on May 9. Hγ is the strongest line on each plate and is more or less three times stronger than Hδ, while the intensity of Hβ lies between that of Hδ and Hγ. These lines are narrow and apparently single. The velocities derived from Hγ are given in the table of observations and are seen to agree with those of the absorption lines of the general spectrum. This result differs from those of Campbell[3] and Stebbins[4] for *o Ceti* and of Merrill[5] for eleven Class Md stars. They found displacements of the bright lines of 11 to 30 km. to the violet of the dark lines and no evidence of variable velocity. There appears to be a variation of 15 to 20 km. in the present observations but this cannot be considered certain without further observation. The nearest date of maximum given by Roberts' elements,[6] J.D. 2415076.1 +90.30 E, is 1909 April 20 = J.D. 2418417.1, which is nearer to the second date of observation, May 9, than to the first. But the elements given in Hartwig's catalogue,[7] J.D. 2415068 + 90.21 E, give the maximum on 1909 April 8 = J.D. 2418405.77, which is nearer the first date, March 30. From the deduction with regard to the intensity of the bright lines it would be expected that the maximum was nearer the first date. However, Professor Pickering writes that the elements given in Hartwig's catalogue do not satisfy the Harvard photographic observations so well as those of Roberts. The shortness of the interval, less than 15 days, within which the bright lines vanished is to be noted.

T Carinae and R Apodis.—These two stars are not definitely known to be variable. They are classed only as *suspected,* and in the Harvard notes their variability is considered very doubtful, although Chandler's *Third Catalogue*[8] announces them confirmed by independent observers. The very aproximate uniformity of the radial velocities found in the present observations helps considerably to substantiate the doubt of the variability of the light of these stars, particularly in view of the fact that *long period, Cepheid,* and *Algol* variables are in general not of Class K.

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[2] *Publ. Yerkes Obs.*, **2**, 339, 1903.

[3] *Ap. Jour.*, **9**, 31, 1899.

[4] *Lick Obs. Bull.*, **2**, 78, 1903.

[5] *Ap. Jour.*, 41, 247, 1915; *Publ. Astr. Obs. Univ. Mich.*, **2**, 69, 1916. Sanford found L_2 *Puppis* and Merrill W *Cygni*, in which the bright and dark lines agree.

[6] *Annals H. C. O.*, 55, 45, 1906, note 133633.

[7] *Vierteljahrsschrift der Astr. Gesell.*, 49, 322, 1914.

[8] *Astr. Jour.*, 41, 145, 1896.

ι *Librae.*—This star, suspected of variability from Harvard observations, was simultaneously and independently observed at both Mt. Hamilton and Santiago for radial velocity. The northern observations showed definitely that the velocity was variable and that both spectra, one fainter than the other, were present on the spectrograms. It was therefore announced as a spectroscopic binary[9] in 1911. In view of the weakness and ill-defined aspect of the lines on the northern plates, the evidence of the southern plates is not superfluous, although the uncertainties of the one-prism plates are large. On the latter, four lines, Si 4128.204A, Si 4131.040,[10] Hγ 4340.634, Mg 4481.400, are satisfactory for measurement but yield large differences. The two silicon lines together show a range of 13 km., the Hγ 24 km., and the magnesium 53 km. The only line appearing to be double is Hγ on the plate of 1909 March 31, where the doubling is not certain but measurement gives —106 km. and +15 km. for the components. In the table of observations only the values obtained from the magnesium line are given, since it is the faintest line and probably least affected by the still fainter component. The variability of the light of this star was not definitely established at the date of the *Second Catalogue of Variables.*[1] We may expect that it will be found to be an *Algol* variable.

RS Sagittarii.—Preliminary measures of the plates of this *Algol* variable at the time of observation showed it to be a spectroscopic binary, as was to be expected from the character of its light variation. A preliminary report was made in *Lick Observatory Bulletin,* **9**, 39, 1916. The usual lines of Class A spectra are seen on these plates, but only four lines were selected for measurement, the adopted wavelengths being the same as for h *Centauri.*[11] Hγ 4340.634A, He 4388.100, He 4471.663, were measured on all the plates, and Mg 4481.400 on only four. No indication of the secondary spectrum is seen. Roberts derived the following times of minimum:[12] J.D. 2415023.085 + 2.415702 E. These elements place a minimum, after the elapse of 1397 periods, at the epoch J.D. 2418397.821. Using this latter epoch, we obtain the following phases corresponding to the observed velocities:

Phase	Velocity
0.062 days	—20 km.
0.386	—46
0.426	—51
0.497	—73
0.842	—43
0.907	—35
1.040	—20
1.820	+84
1.852	+73
2.387	+27

From a plot of these values it is seen that light minimum occurs when the observed velocity is at its mean value, in this case about zero, and when the velocity is decreasing. These facts agree with the theory of the *Algol* variable, and establish that the light fluctuation of this variable is caused by an eclipsing body. The Harvard light curve gives evidence of a secondary minimum, indicating that the eclipsing body has a luminosity. It is thus not impossible that the secondary spectrum may be obtainable with higher dispersion. The plot indicates that the eccentricity and the velocity of the system with respect to the Sun are approximately zero and the amplitude about 150 km. Further deductions will not here be drawn since additional observations and study of this star are being made in Chile.

Cepheid Variables.—Five of the eleven variables selected are placed with δ *Cephei* in Class IV; namely, S *Muscae,* R *Trianguli Australis,* S *Trianguli Australis,* S *Normae,* RV *Scorpii.* Only these five were within reach at the epoch of the observations and bright enough to be photographed in the four or five hours continuous exposure possible without reversal of the apparatus. The characteristic of special interest in the variables of the δ *Cepheid* class thus far observed is the evidence, as usually interpreted, of spectroscopic orbital motion and of the approximate coincidence of maximum light with maximum velocity of approach. In view of the small range of values for the two stars, T *Carinae* and R *Apodis,* the evident range, 33, 33, 28, 24, 32 km., respectively, observed for these five *Cephids* offers considerable satisfaction in spite of the large uncertainty of one-prism determinations of velocity. It is fairly safe to conclude then that these five *Cepheid* variables possess variable velocity. To discover the relation between the light- and velocity-variation each is considered separately, as follows:

S Muscae.—The elements for maximum light were found by Roberts to be,[13] J.D. 2415029.18 + 9.657 E, which give, after 348 periods, a maximum at J. D. 2418389.816 and the following phases for the observed velocities:

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[9] *Lick Obs. Bull.,* 6, 146, 1911.

[10] Hartmann, *Ap. Jour.,* 18, 65, 1903.

[11] *Lick Obs. Bull.,* 9, 44, 1917.

[12] *Annals H. C. O.,* 55, 51, note 181134.

[13] *Annals H. C. O.,* 55, 43, note 120769.

Phase	Velocity
2.964 days	— 9 km.
4.280	— 1
5.927	+10
6.152	+19
7.581	+24
7.864	+23

A plot of these values shows that the maximum light thus calculated occurs during the interval of approach. The interval maximum minus minimum according to Roberts is 3.45 days, or min. — max. = 6.2 days, but the phase of the maximum velocity would appear to be greater than 6.2 days. No observation falls at the epoch of greatest negative velocity, but it also would appear to be one day or more later than maximum light. A very rough estimate of the velocity of the system is +3 km. The eccentricity would appear to be appreciable and the amplitude about 35 km./sec.

R Trianguli Australis.—Chandler's elements for times of maximum light are[14] J. D. 2404623.71 + 3.38922 E, and M—m = 1.01 days. A maximum following 4066 periods occurs at J.D. 2418404.279, from which we get the following phases for the velocities in hand:

Phase	Velocity
0.169 days	—33 km.
0.361	—34
0.546	—31
0.566	—32
0.609	—30
0.960	—31
1.348	—28
1.356	—27
1.596	—17
1.814	—10
2.622	— 1
2.812	— 7
3.153	—19

These values are fairly well distributed over the period and when plotted show with certainty a velocity curve of considerable eccentricity, with an amplitude about 35 km., and a velocity of the system of about —20 km. Observations are lacking at the maximum of the curve, but the minimum is well represented and indicates the occurrence of greatest approaching velocity about 0.3 of a day after light maximum. The phase of maximum velocity may possibly be slightly greater than the interval between maximum and minimum light, 2.38 days.

S Trianguli Australis.—From Roberts' elements for the maxima,[15] J. D. 24150234.41 + 6.3231 E, M—m = 2.10, we find, after an interval of 534 periods, a maximum at J. D. 2418399.945 and the following phases:

Phase	Velocity
1.579 days	— 7 km.
2.261	+ 1
2.922	+ 4
3.929	+11
4.614	+21
5.884	— 6
5.902	— 7

The plot of these velocities shows orbital motion with considerable eccentricity, an amplitude of about 30 km., and a velocity of the system of one or two kilometers positive. There are no observations at the minimum of the curve, but the interval of 4.2 days from maximum to minimum light can easily correspond to the interval between greatest negative and greatest positive velocity, these latter values, however, probably occurring a few tenths of a day later than the times of maximum and minimum light.

S Normae.—The phases corresponding to the observed velocities of this star, deduced from Roberts' elements,[16] J.D. 2415029.45 + 9.7525 E, M—m = 4.4, which give a maximum at J.D. 2418394.062 after 345 periods, are:

Phase	Velocity
3.276 days	—10 km.
4.381	— 3
6.363	+12
7.907	— 4
8.686	—12

Results similar to those in the preceding cases are evident, but deductions must be still less definite. The greatest negative velocity was not caught in the series of observations, and only one value stands near the greatest positive velocity, but the form of the velocity curve can be discerned. The phase of maximum velocity would appear to be larger than the interval of 5.3 days between maximum and minimum light, but the time of minimum velocity can hardly be estimated. The eccentricity is appreciable, the amplitude probably between 25 and 30 km., and the velocity of the system between —5 and —10 km.

RV Scorpii.—The phases with respect to the epoch of maximum at J.D. 2418414.810 are:

Phase	Velocity
0.458 days	—37 km.
0.940	—33
1.850	—30
2.839	—25
3.768	— 9
4.768	—25
5.985	—41

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[14] *Annals H. C. O.*, 55, 46, note 151066.

[15] *Annals H. C. O.*, 55, 47, note 155263.

[16] *Annals H. C. O.*, 55, 48, note 161057.

which follow by 559 periods from the epoch of Robert's elements,[17] J.D. 2415026.04 + 6.0622 E, M—m =1.41. Both maximum and minimum velocity appear to be approximately observed, but their phases would seem to be somewhat small. The phase of the greatest positive observed velocity which apparently marks the maximum of the velocity curve is 0.6 to 0.8 days less than the interval m—M = 4.65 days, while minimum velocity probably precedes maximum light by one or two tenths of a day. The orbital eccentricity is evidently large, the amplitude more or less 30 km., and the velocity of the system about —28 km.

Results for the Cepheids.—Collecting the above derived data, and indicating maximum light by *Ml*, minimum light by *ml*, greatest positive velocity by *Mv*, and greatest negative velocity by *mv*, we have:

Variable	days *mv—Ml*	days *Mv—ml*	Eccentricity	Amplitude km.	Velocity of system km.
S *Muscae*	+1.0+	+1.0+	0.1 to 0.2	35±	+ 3±
R *Tri. Austr.*	+0.3±	+0.1 or 0.2	0.2 to 0.3	35±	—20±
S *Tri. Austr.*	+0.2, 0.3, or 0.4	+0.2, 0.3, or 0.4	0.3±	30±	+1 or 2
S *Normae*	....	+1.0±	0.2±	25 to 30	—5 to —10
RV *Scorpii*	—0.1 or —0.2	—0.6 to —0.8	0.3±	30±	—28±

These estimated values are uncertain and cannot be used for further deductions, but they serve to indicate the orbital character of the velocity-variation and the relation between the light- and velocity variation. It can safely be concluded that these *Cepheid* variables possess the distinguishing characteristics found in others of their class, namely: considerable eccentricity; similar order of amplitude of velocity, 20 to 40 km. for the majority; greatest negative velocity occurring a few tenths of a day later than maximum light; and close correspondence between times of maximum velocity and minimum light.

G. F. Paddock.

March, 1917.
Issued May 19, 1917.

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[17] *Annals H. C. O.*, 55, 49, note 165133.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 295

SECOND ELEMENTS AND EPHEMERIS OF COMET *b* 1916 (WOLF)

This second set of elements was derived by differentially correcting the first elements (*Bulletin No.* **282**) so as to represent the following observations, all of which were made by Barnard:

Gr. M. T.	Comet's Apparent α	Comet's Apparent δ
1916 April 24.6365	$12^h\ 41^m\ 42^s.4$	$+2^\circ\ 10'\ 23''$
1916 Dec. 31.00279	16 01 45.71	—6 04 35.5
1917 April 21.87685	20 09 55.92	+9 54 17.2

ELEMENTS

$T = 1917$ June 16.5373 Gr. M. T.

$$\left.\begin{aligned}\omega &= 120^\circ\ 36'\ 01''.4\\ \Omega &= 183\ \ 17\ \ 00.0\\ i &= 25\ \ 40\ \ 11.3\end{aligned}\right\}1916.0 \qquad \left.\begin{aligned}\omega &= 120^\circ\ 36'\ 01''.2\\ \Omega &= 183\ \ 17\ \ 50.4\\ i &= 25\ \ 40\ \ 10.8\end{aligned}\right\}1917.0$$

$$\log q = 0.227067$$

(O–C)	I	III
$\cos\delta\,\Delta\alpha$	$+0''.2$	$+0''.5$
$\Delta\delta$	+0.4	+1.3

CONSTANTS FOR THE EQUATOR 1916.0

$$\begin{aligned}x &= r\,[9.999867]\sin(\ 33^\circ\ 33'\ 37''.1 + v)\\ y &= r\,[9.999684]\sin(303\ \ 36\ \ 52.2 + v)\\ z &= r\,[8.657553]\sin(\ 90\ \ 30\ \ 14.7 + v)\end{aligned}$$

CONSTANTS FOR THE EQUATOR 1917.0

$$\begin{aligned}x &= r\,[9.999865]\sin(\ 33^\circ\ 34'\ 22''.3 + v)\\ y &= r\,[9.999684]\sin(303\ \ 37\ \ 38.2 + v)\\ z &= r\,[8.658019]\sin(\ 90\ \ 23\ \ 54.7 + v)\end{aligned}$$

In view of the fact that the orbit is not based upon normal places it was deemed inadvisable to attempt further improvement by removing the small residuals which are certainly less than the accumulated error of observation.

The observations at present available (other than those upon which these elements are based) are represented by this orbit as follows:

1916 Gr. M. T.		Comet's Apparent α			Comet's Apparent δ			$\cos\delta\,\Delta\alpha$	$\Delta\delta$	Observer
May	4.8565	12^h	36^m	$55^s.6$	$+2^\circ$	$59'$	$51''$	$0^s.0$	$0''$	Van Biesbroeck, Yerkes.
	6.6559	12	36	10.3	3	07	46	+0.1	+1	Barnard, Yerkes.
	6.6846	12	36	09.5	3	07	53	0.0	+2	Burton, Washington.
	8.6889	12	35	20.9	3	16	21	—0.1	0	Hubble, Yerkes.
	10.6331	12	34	36.0	3	24	13	—0.1	0	Barnard.
	18.6680	12	31	55.7	3	53	09	0.0	—1	Van Biesbroeck.
	22.6721	12	30	51.9	4	05	19	—0.1	0	Barnard.
	23.7135	12	30	37.2	4	08	13	—0.2	0	Van Biesbroeck.
	27.6893	12	29	48.6	4	18	17	—0.3	0	Barnard.
	27.7933	12	29	47.4	4	18	30	—0.3	—2	Aitken.
	28.6536	12	29	38.8	4	20	28	—0.1	—2	Van Biesbroeck.
May	30.7009	12	29	19.9	4	24	50	—0.1	—1	Van Biesbroeck.
June	4.7197	12	28	47.9	4	33	47	—0.1	+1	Van Biesbroeck.
	17.6493	12	28	59.2	4	45	12	—0.2	+2	Barnard.
	21.6709	12	29	30.2	4	45	27	—0.1	0	Barnard.
	24.6368	12	30	01.3	4	44	43	—0.2	+1	Barnard.
June	28.6400	12	30	54.2	4	42	27	—0.3	+1	Barnard.
July	1.6326	12	31	42.2	4	39	49	—0.3	—1	Barnard.
	1.6559	12	31	42.7	4	39	49	—0.2	+1	Barnard.
July	5.6396	12	32	57.3	+4	35	11	0.0	0	Barnard.

1917 Gr. M. T.	Comet's Apparent α	Comet's Apparent δ	cos δ Δα	Δδ	Observer
Jan. 25.0616	16h 50m 43s.9	—5° 30′ 52″	+0s.2	—1″	Aitken.
Jan. 26.0492	16 52 45.0	5 27 41	+0.1	0	Aitken.
Feb. 10.9384	17 26 12.8	4 14 24	+0.2	+3	Barnard.
Feb. 22.9476	17 52 37.1	2 49 22	+0.2	0	Barnard.
Mar. 1.9732	18 08 28.1	1 47 03	0.0	—1	Barnard.
Mar. 3.9682	18 13 01.4	—1 27 37	+0.1	+1	Barnard.
April 5.8884	19 30 53.6	+5 35 25	+0.4	—2	Barnard.

Ephemeris for Greenwich Mean Midnight

1917 Gr.M.T.		True α			True δ			log Δ	Br.
May	29.5	21h	40m	24s.7	+19°	51′	16″	0.1224	1.76
	30.5		42	42.3	20	04	29		
May	31.5		44	59.3		17	28	0.1183	
June	1.5		47	15.7		30	12		
	2.5		49	31.4		42	42	0.1142	1.84
	3.5		51	46.5	20	54	57		
	4.5		54	00.9	21	06	57	0.1101	
	5.5		56	14.7		18	41		
	6.5	21	58	27.7		30	09	0.1061	1.92
	7.5	22	00	40.0		41	21		
	8.5		02	51.5	21	52	16	0.1021	
	9.5		05	02.2	22	02	54		
	10.5		07	12.1		13	15	0.0982	2.00
	11.5		09	21.1		23	18		
	12.5		11	29.2		33	02	0.0943	
	13.5		13	36.4		42	28		
	14.5		15	42.7	22	51	35	0.0904	2.08
	15.5		17	48.0	23	00	23		
	16.5		19	52.3		08	51	0.0865	
	17.5		21	55.5		16	59		
	18.5		23	57.7		24	46	0.0827	2.15
	19.5		25	58.8		32	12		
	20.5		27	58.7		39	17	0.0789	
	21.5		29	57.5		46	00		
	22.5		31	55.1		52	22	0.0751	2.22
	23.5		33	51.5	23	58	21		
	24.5		35	46.7	24	03	58	0.0714	
	25.5		37	40.6		09	12		
	26.5		39	33.3		14	03	0.0677	2.29
	27.5		41	24.6		18	30		
	28.5		43	14.6		22	33	0.0640	
	29.5		45	03.2		26	12		
June	30.5		46	50.5		29	27	0.0604	2.36
July	1.5		48	36.3		32	17		
	2.5		50	20.7		34	42	0.0567	
	3.5		52	03.6		36	42		
	4.5		53	45.1		38	16	0.0531	2.42
	5.5		55	25.0		39	24		
	6.5		57	03.4		40	07	0.0496	
	7.5	22	58	40.2		40	23		
	8.5	23	00	15.4		40	13	0.0461	2.48
	9.5		01	48.9		39	36		
	10.5		03	20.8		38	31	0.0426	
	11.5		04	51.0		36	58		
	12.5		06	19.4		34	58	0.0391	2.53
	13.5		07	46.0		32	29		
	14.5		09	10.9		29	32	0.0357	
	15.5		10	33.9		26	06		
	16.5		11	55.1		22	10	0.0324	2.57
July	17.5	23	13	14.4	+24	17	45		
July	18.5	23h	14m	31s.9	+24°	12′	50″	0.0291	
	19.5		15	47.5		07	25		
	20.5		17	01.2	24	01	30	0.0259	2.61
	21.5		18	13.0	23	55	04		
	22.5		19	22.8		48	08	0.0228	
	23.5		20	30.7		40	41		
	24.5		21	36.7		32	43	0.0198	2.64
	25.5		22	40.7		24	14		
	26.5		23	42.7		15	13	0.0168	
	27.5		24	42.8	23	05	42		
	28.5		25	40.9	22	55	39	0.0140	2.67
	29.5		26	37.0		45	05		
	30.5		27	31.2		33	59	0.0114	
July	31.5		28	23.3		22	22		
Aug.	1.5		29	13.5	22	10	13	0.0088	2.68
	2.5		30	01.7	21	57	33		
	3.5		30	47.9		44	21	0.0065	
	4.5		31	32.2		30	38		
	5.5		32	14.5		16	23	0.0043	2.68
	6.5		32	54.9	21	01	37		
	7.5		33	33.3	20	46	20	0.0023	
	8.5		34	09.8		30	32		
	9.5		34	44.3	20	14	14	0.0005	2.67
	10.5		35	16.9	19	57	25		
	11.5		35	47.7		40	06	9.9989	
	12.5		36	16.6		22	18		
	13.5		36	43.6	19	04	00	9.9976	2.65
	14.5		37	08.8	18	45	13		
	15.5		37	32.3		25	58	9.9965	
	16.5		37	54.0	18	06	15		
	17.5		38	14.0	17	46	05	9.9957	2.61
	18.5		38	32.3		25	28		
	19.5		38	49.0	17	04	26	9.9953	
	20.5		39	04.2	16	42	59		
	21.5		39	17.8	16	21	08	9.9951	2.55
	22.5		39	29.9	15	58	54		
	23.5		39	40.6		36	17	9.9953	
	24.5		39	50.0	15	13	20		
	25.5		39	58.0	14	50	02	9.9959	2.48
	26.5		40	04.8		26	25		
	27.5		40	10.3	14	02	31	9.9968	
	28.5		40	14.7	13	38	21		
	29.5		40	18.1	13	13	55	9.9981	2.39
	30.5		40	20.4	12	49	15		
Aug.	31.5		40	21.7	12	24	23	9.9998	
Sept.	1.5		40	22.1	11	59	19		
	2.5		40	21.6		34	05	0.0020	2.29
	3.5		40	20.3	11	08	42		
	4.5		40	18.2	10	43	12	0.0045	
Sept.	5.5	23	40	15.4	+10	17	36		

1917 Gr.M.T.		True α			True δ				log Δ	Br.
Sept.	6.5	23^{h}	40^{m}	$12^{s}0$	+	9°	51′	55″	0.0074	2.17
	7.5		40	08.0			26	11		
	8.5		40	03.5		9	00	25	0.0108	
	9.5		39	58.5		8	34	39		
	10.5		39	53.1		8	08	54	0.0146	2.05
	11.5		39	47.4		7	43	12		
	12.5		39	41.5		7	17	34	0.0188	
	13.5		39	35.3		6	52	01		
	14.5		39	29.0			26	35	0.0234	1.91
	15.5		39	22.6		6	01	17		
	16.5		39	16.2		5	36	09	0.0284	
	17.5		39	09.9		5	11	12		
	18.5		39	03.6		4	46	27	0.0338	1.77
	19.5		38	57.5		4	21	55		
	20.5		38	51.6		3	57	38	0.0396	
	21.5		38	45.9			33	37		
	22.5		38	40.6		3	09	52	0.0457	1.63
	23.5		38	35.6		2	46	25		
	24.5		38	31.1			23	17	0.0522	
	25.5		38	27.0		2	00	29		
	26.5		38	23.5		1	38	01	0.0590	1.49
	27.5		38	20.5		1	15	55		
	28.5		38	18.0		0	54	11	0.0661	
	29.5		38	16.1			32	49		
Sept.	30.5		38	14.9	+	0	11	50	0.0735	1.35
Oct.	1.5		38	14.3	—	0	08	45		
	2.5		38	14.4			28	55	0.0812	
	3.5		38	15.2		0	48	41		
	4.5		38	16.8		1	08	02	0.0891	1.22
	5.5		38	19.1			26	57		
	6.5		38	22.2		1	45	27	0.0972	
	7.5		38	26.1		2	03	31		
	8.5		38	30.8			21	09	0.1055	1.10
	9.5		38	36.4			38	20		
	10.5		38	42.8		2	55	05	0.1141	
	11.5		38	50.2		3	11	23		
	12.5		38	58.4			27	14	0.1227	0.99
	13.5		39	07.5			42	39		
	14.5		39	17.6		3	57	37	0.1315	
	15.5		39	28.6		4	12	08		
	16.5		39	40.6			26	13	0.1404	0.89
	17.6		39	53.6			39	51		
	18.5		40	07.5		4	53	03	0.1495	
	19.5		40	22.4		5	05	49		
	20.5		40	38.2			18	09	0.1586	0.79
	21.5		40	55.0			30	04		
	22.5		41	12.8			41	33	0.1678	
	23.5		41	31.6		5	52	37		
	24.5		41	51.3		6	03	16	0.1770	0.71
	25.5		42	12.0			13	31		
	26.5		42	33.6			23	21	0.1863	
	27.5		42	56.2			32	48		
	28.5		43	19.8			41	51	0.1956	0.63
	29.5		43	44.4			50	31		
	30.5		44	09.8		6	58	49	0.2049	
Oct.	31.5		44	36.2		7	06	44		
Nov.	1.5		45	03.4			14	17	0.2142	0.56
	2.5		45	31.6			21	29		
Nov.	3.5	23	46	00.6	—	7	28	20	0.2235	
Nov.	4.5	23^{h}	46^{m}	$30^{s}5$	—	7°	34′	50″		
	5.5		47	01.3			40	59	0.2327	0.50
	6.5		47	33.0			46	48		
	7.5		48	05.5		7	52	18	0.2420	
	8.5		48	38.8		7	57	28		
	9.5		49	13.0		8	02	20	0.2512	0.45
	10.5		49	48.0			06	53		
	11.5		50	23.9			11	08	0.2603	
	12.5		51	00.6			15	05		
	13.5		51	38.0			18	44	0.2694	0.40
	14.5		52	16.3			22	06		
	15.5		52	55.3			25	11	0.2785	
	16.5		53	35.1			28	00		
	17.5		54	15.7			30	33	0.2874	0.36
	18.5		54	57.0			32	50		
	19.5		55	39.1			34	52	0.2964	
	20.5		56	21.8			36	39		
	21.5		57	05.3			38	12	0.3052	0.32
	22.5		57	49.5			39	30		
	23.5		58	34.3			40	35	0.3139	
	24.5	23	59	19.8			41	26		
	25.5	0	00	06.0			42	05	0.3226	0.29
	26.5		00	52.8			42	31		
	27.5		01	40.2			42	44	0.3312	
	28.5		02	28.2			42	44		
	29.5		03	16.9			42	33	0.3397	0.26
Nov.	30.5		04	06.1			42	10		
Dec.	1.5		04	55.9			41	36	0.3481	
	2.5		05	46.3			40	51		
	3.5		06	37.2			39	55	0.3564	0.23
	4.5		07	28.7			38	49		
	5.5		08	20.7			37	33	0.3646	
	6.5		09	13.2			36	07		
	7.5		10	06.2			34	31	0.3727	0.21
	8.5		10	59.7			32	46		
	9.5		11	53.7			30	52	0.3807	
	10.5		12	48.2			28	49		
	11.5		13	43.2			26	37	0.3887	0.19
	12.5		14	38.7			24	17		
	13.5		15	34.6			21	48	0.3965	
	14.5		16	31.0			19	11		
	15.5		17	27.8			16	26	0.4042	0.17
	16.5		18	25.0			13	34		
	17.5		19	22.7			10	35	0.4118	
	18.5		20	20.8			07	28		
	19.5		21	19.3			04	15	0.4193	0.16
	20.5		22	18.2		8	00	55		
	21.5		23	17.4		7	57	29	0.4267	
	22.5		24	17.0			53	56		
	23.5		25	17.0			50	18	0.4340	0.14
	24.5		26	17.3			46	34		
	25.5		27	18.0			42	44	0.4412	
	26.5		28	19.0			38	49		
	27.5		29	20.3			34	48	0.4483	0.13
	28.5		30	21.9			30	42		
	29.5		31	23.9			26	32	0.4553	
	30.5		32	26.2			22	17		
Dec.	31.5	0	33	28.7	—	7	17	57	0.4622	0.12

Brightness, 1917 April 21 = 1.00.

The comet will be at opposition September 17.

In communicating his observation of April 21, Professor Barnard wrote: "The comet was not brighter than 12^m in the 40-inch telescope. It has a diffuse tail ±2′ long, and a strong condensation to perhaps a 13^m nucleus." It will attain theoretical maximum brilliancy the first week in August, at which time it will be only 2.7 times brighter than when Professor Barnard observed it on April 21. It is quite improbable, therefore, that this comet will become visible to the unaided eye as was hoped.

R. T. Crawford,
Dinsmore Alter.

Berkeley Astronomical Department,
May 14, 1917.

Issued May 19, 1917.

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ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 296

SEARCH EPHEMERIS FOR FAYE'S COMET

The following opposition ephemeris was computed from the elements by Meyer and Miss Levy, given in *Bulletin* 187. These elements are based on an arc extending from November 9 to December 12, 1910. Perihelion passage will occur 1918, April 10. The comet will be very unfavorably situated for observation when near perihelion, since it will then be near conjunction.

The uncertainty in the adopted mean daily motion is such that the ephemeris may be in error by a degree or more.

Ephemeris for Greenwich Mean Midnight

1917 Gr. M. T.	True α h m	True δ	log Δ	Br.
June 1.5	20 51.7	—4° 06′	0.411	0.04
9.5	52.3	3 30		
June 17.5	20 51.5	—2 59	0.378	
June 25.5	20 49.5	—2° 35′		
July 3.5	46.0	2 18	0.318	0.07
11.5	41.4	2 10		
19.5	35.7	2 13	0.280	
July 27.5	29.2	2 26		
Aug. 4.5	22.2	2 49	0.254	0.10
12.5	15.3	3 22		
20.5	08.8	4 03	0.243	
Aug. 28.5	20 03.3	4 49		
Sept. 5.5	19 59.3	—5 37	0.247	0.12

Brightness Dec. 30, 1910 = 1.00. The comet will be between the 13th and 14th magnitude during opposition.

Hamilton M. Jeffers.

Berkeley Astronomical Department.
May 25, 1917.

ELEMENTS AND EPHEMERIS OF COMET *b* 1917 (SCHAUMASSE)

From observations of April 28, 29, and May 2 (the first two of which were made by Shane and the third by Neubauer) the following elements were computed by Dr. Einarsson and Miss Young:

Elements

$$T = 1917\ \text{May}\ 18.8868\ \text{Gr. M. T.}$$
$$\left.\begin{array}{l} \omega = 119^\circ\ 05'.1 \\ \Omega = \ \ 8\ \ 49.0 \\ i = 158\ \ 34.9 \end{array}\right\} 1917.0$$
$$\log q = 9.88217$$

These elements and an ephemeris from May 6 to May 18 were telegraphed to Harvard College Observatory (*H. C. O. Bull.* 637). Later an extension of this ephemeris to June 3 was also telegraphed (*H. C. O. Bull.* 639). The residuals (*O–C*) were increasing so rapidly, however, because of the close approach of the comet to the earth (log Δ = 9.556 on May 22) that a second orbit became necessary.

The new orbit is based on the following observations:

1917 Gr. M. T.	Comet's Apparent α	Comet's Apparent δ	Observer
April 28.0055	23ʰ 07ᵐ 39ˢ.8	+11° 36′ 57″	Shane
May 6.89468	23 21 25.97	19 03 09.2	Barnard, Yerkes
May 17.89104	0 43 50.39	+43 39 36.2	Barnard

Elements

$$T = 1917\ \text{May}\ 18.23322\ \text{Gr. M. T.}$$
$$\left.\begin{array}{l} \omega = 119^\circ\ 03'\ 39''.0 \\ \Omega = \ \ 9\ \ 31\ \ 54.0 \\ i = 158\ \ 43\ \ 47.0 \end{array}\right\} 1917.0$$
$$\log q = 9.883119$$

$$(O\text{–}C),\ \Delta\delta_1 = +37''$$

Constants for the Equator 1917.0

$$x = r\,[9.999215]\sin(200^\circ\ 10'\ 03''.8 + v)$$
$$y = r\,[9.999604]\sin(290\ \ 18\ \ 53.5 + v)$$
$$z = r\,[8.867239]\sin(235\ \ 36\ \ 11.1 + v)$$

On account of the close approach to the earth during the period covered by the observations upon which this orbit is based, and the resulting large geocentric motion, small changes in the elements will make large changes in the geocentric positions, and vice versa. It was therefore deemed inadvisable to improve the orbit, although no residuals would remain for an ellipse with a period of 1031 years.

Ephemeris for Greenwich Mean Midnight

1917 Gr. M. T.	True α	True δ	log Δ	Br.
June 1.5	8^h 35^m 35^s	+36° 01′.2	9.7381	4.73
2.5	43 32	33 48.8		
3.5	50 10	31 49.7	9.7894	
4.5	8 55 45	30 02.6		
5.5	9 00 31	28 26.1	9.8374	2.79
6.5	04 37	26 59.0		
7.5	08 11	25 40.0	9.8816	
8.5	11 18	24 28.3		
9.5	14 02	23 22.9	9.9223	1.74
10.5	16 28	22 23.1		
11.5	18 38	21 28.2	9.9597	
12.5	20 35	20 37.6		
13.5	22 21	19 51.0	9.9941	1.14
14.5	23 56	19 07.7		
15.5	25 24	18 27.5	0.0258	
16.5	26 44	17 50.0		
17.5	27 57	17 15.0	0.0551	0.79
18.5	29 06	16 42.2		
19.5	30 09	16 11.4	0.0822	
20.5	31 08	15 42.4		
21.5	32 03	15 15.0	0.1075	0.56
22.5	32 55	14 49.1		
23.5	33 44	14 24.6	0.1309	
24.5	34 31	14 01.3		
25.5	35 15	13 39.1	0.1529	0.42
26.5	35 58	13 17.9		
27.5	36 38	12 57.7	0.1734	
28.5	37 17	12 38.4		
29.5	37 54	12 19.8	0.1926	0.32
June 30.5	9 38 30	+12 02.0		

Brightness April 28 = 1.00.

Berkeley Astronomical Department,
May 29, 1917.

Issued June 2, 1917.

Ephemeris for Greenwich Mean Midnight—*(Continued)*

1917 Gr. M. T.	True α	True δ	log Δ	Br.
July 1.5	9^h 39^m 05^s	+11° 44′.9	0.2106	
2.5	39 39	11 28.4		
3.5	40 12	11 12.6	0.2276	0.24
4.5	40 44	10 57.2		
5.5	41 16	10 42.4	0.2436	
6.5	41 46	10 28.0		
7.5	42 17	10 14.0	0.2587	0.19
8.5	42 46	10 00.5		
9.5	43 16	9 47.3	0.2730	
10.5	43 44	9 34.5		
11.5	44 13	9 22.0	0.2865	0.16
12.5	44 41	9 09.9		
13.5	45 08	8 58.0	0.2992	
14.5	45 36	8 46.4		
15.5	46 03	8 35.0	0.3113	0.13
16.5	46 30	8 23.9		
17.5	46 57	8 13.0	0.3228	
18.5	47 23	8 02.3		
19.5	47 50	7 51.8	0.3337	0.11
20.5	48 16	7 41.5		
21.5	48 42	7 31.4	0.3440	
22.5	49 08	7 21.4		
23.5	49 34	7 11.6	0.3538	0.09
24.5	49 59	7 02.0		
25.5	50 25	6 52.5	0.3631	
26.5	50 50	6 43.1		
27.5	51 15	6 33.8	0.3720	0.08
28.5	51 40	6 24.7		
29.5	52 05	6 15.6	0.3804	
30.5	52 30	6 06.7		
July 31.5	9 52 54	+ 5 57.9	0.3884	0.07

Jessica May Young,
Hamilton M. Jeffers.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

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ON SOME RELATIONS OF THE SPIRAL NEBULAE TO THE MILKY WAY

There exist data for the stars, sufficiently accurate, systematic, and exhaustive, to enable astronomers to make comparatively reliable discussions of their brightness, number, and distribution; and these discussions have added materially to the knowledge of our stellar universe. It should be remembered, too, that even with the very much cruder material of earlier days, much information of value about stellar distribution was obtained. This in itself justifies the attempt to use the admittedly crude material which we have for the nebulae to a like end for them. There is further stimulus in the present activities of the various observatories which are devoting powerful telescopes with improved auxiliary apparatus to several lines of research upon these objects. It was deemed worth while to undertake a further study of nebular distribution, based upon available data and supplemented with additional observations to be secured. Because the spiral nebulae form so large a proportion of all the known nebulae, and because the most firmly established and longest accepted feature of spiral nebular distribution is the apparent dependence of their numbers upon galactic latitude, this study is an effort to trace, if possible, some plausible cause inherent in, or connected with the Galaxy, to explain the observed distribution.

It has seemed expedient to present the material under the following general heads:

A. Sources available for studies of nebular distribution.

B. Studies in nebular distribution.

C. Nebular distribution as a function of galactic coördinates.

D. Nebular distribution as a function of nebular brightness.

E. Nebular distribution as a function of nebular apparent size.

F. Nebular distribution as a function of star density.

G. Conclusions.

A. SOURCES AVAILABLE FOR STUDIES OF NEBULAR DISTRIBUTION

There are two quite natural divisions of the sources, the one based upon visual observations, for the most part, and the other based upon photographic observations. They are explained in order below.

1. The first important catalogue was made by Messier, not to facilitate the observation and study of the nebulae, but to avoid mistaking them for comets. The Herschels formed the first great lists of nebulae, the results of their sweeps over the sky, both north and south. Data concerning the Herschel objects are to be found today in most satisfactory form in the Dreyer lists.[1] They contain the Herschel objects, and such similar objects found by other observers as had sufficiently complete descriptions published at the time each list was issued. The Dreyer lists form the nearest approach that we have at the present time to a *durchmusterung* of the nebulae, and they may be regarded as depending entirely upon visual data, in contradistinction to the other group.

2. The spectroscope, early in its use as an astronomical instrument of value, demonstrated a marked constitutional difference for the different kinds of nebulae, that was not apparent from previous observations; and, when the photographic dry plate came into general use, and it was possible to get a record of the finer details of these objects, it was apparent that, in outward form as well, there were striking differences which might indicate fundamental differ-

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[1] *New General Catalogue of Nebulae and Star Clusters: Memoirs R. A. S.*, 49, 1888. *First Index Catalogue: Ibid.*, 51, 185, 1895. *Second Index Catalogue: Ibid.*, 59, 105, 1908.

ences in constitution. Keeler[2] showed that a surprising proportion of the nebulae are spirals. These differences led to separate treatment of their distributions. Even the first crude efforts to obtain a knowledge of their distribution revealed a contrast that made more systematic classification imperative. For such a classification it is essential that the entire sky be treated uniformly; and, since these objects are extremely faint, photography supplies the only known means of doing this. Such treatment with large-scale photographs is impracticable for the whole sky, and today we have but one source that approaches uniformity of treatment on such a scale. It is furnished by the one-hour exposures made by Fath with the Mt. Wilson 60-inch reflector, upon the centers of Kapteyn's Selected Areas,[3] each plate covering an area of 1.88 square degrees with definition sufficiently good for this purpose. The areas are distributed over the sky with approximate regularity, from the north pole of the sky to declination —22°.5. The lists of nebulae secured from this program may be said to furnish material that is uniform over the part of the sky covered; but these nebulae are for the most part unclassified. Two other lists satisfy the requirements of completeness, uniform exposure and classification. One list[4] comprises all the nebulae found upon the Harvard plates taken with a 1-inch Cooke lens of thirteen inches focal length. It contains, necessarily, only the large bright nebulae. The other list[5] was made up of all nebulae found upon the Franklin-Adams star photographs made with a 10-inch lens of forty-five inches focal length. The Harvard plates cover an area 30 degrees square and the Franklin-Adams plates less than 4 degrees square. Hence the latter plates are much more efficient than the former, and they supply a considerably larger number of objects. Our second division of the sources of nebular data is exclusively photographic, and comprises the Mt. Wilson lists, the Harvard bright nebulae from Cooke-lens plates, and the nebulae found on the Franklin-Adams plates. Several other valuable lists exist, but they do not cover the sky uniformly and are therefore not included.

B. STUDIES IN NEBULAR DISTRIBUTION

Discussions based upon the Herschel lists and later upon the Dreyer lists made it apparent that, so far as numbers go, without classification as to type, the regions near the poles of the Galaxy are relatively very rich in nebulae, and the neighborhood of the Milky Way, the reverse. That this distribution was not alone a galactic-latitude function was apparent to two investigators at least. Sir John Herschel, according to Proctor,[6] pointed out that the nebulae might be in a sort of belt perpendicular to the Galaxy, but he seems only to have speculated upon this idea. Stratanoff,[7] who made some elaborate charts based mostly upon the N. G. C. of Dreyer, pointed out several places where the nebulae show concentrations toward the Milky Way. Furthermore, Cleveland Abbe[8] was able to present very good evidence for assigning planetary nebulae to our Galaxy. The need of stricter classification and more careful treatment became apparent as these different discussions appeared.

Hinks[9] was led, therefore, to doubt some conclusions which seemed to prevail. He undertook to derive an idea of the distribution from the nebulae found upon the Cooke-lens plates, taking care that those which he used were reliably classified. He also pointed out the desirability of treating the nebulae found upon the Franklin-Adams plates in a similar fashion, a task that was done a little later by Hardcastle.[10] Since we are concerned with the spiral nebulae, Hink's remarks about them may be appropriately quoted here: "It appears therefore that the brighter spirals are not disposed symmetrically in galactic latitude. The great patch of spirals in N G has no counterpart in S G."[11] In a note added to Hardcastle's paper Hinks also states, "The old view that this distribution of these spirals and spindles, the 'white' nebulae, is determined by mere avoidance of the galactic plane cannot be maintained. Whatever may be the law of distribution, it is not a function of galactic latitude only. Galactic longitude is equally important." He also notes that Stratanoff's distribution map for the bright nebulae fits that shown for the spirals alone from the Harvard plates.

The more than a thousand nebulae found upon the one-hour Mt. Wilson 60-inch reflector photographs[12] of the Selected Areas should be considered here, despite the fact that definite and direct evidence of the spiral form of but few of these objects has been found. By far the greater number of all the nebulae in the Mt. Wilson lists are very small, in fact too small to enable one to say anything definite as to whether they are spiral or not. Their occurrence in large numbers where we also find quantities of large spirals leads to a presumption of spiral form for the great majority outside of the galactic structure. It is to be noted that, upon the photographs which show large numbers of the small nebulae, their appearance does not conflict with what might be expected from very distant and therefore apparently-small spiral nebulae seen at various angles. We scarcely need to inquire whether they may be distant planetary nebulae, since the Mt. Wilson nebulae have a distribution contrary to that of the planetaries. The latter, with few exceptions, are within or near the Milky Way structure. What few are not superimposed upon it have a tendency to be in angular diameter the largest, as Curtis[13] has shown. When we consider that our Galaxy is probably roughly lens-shaped, greater angular size and large angular distance from the Milky Way are consistent with location within its structure; and, presumably, the exceptions are only apparent. Moreover, it is well established that the planetary type is relatively rare; fewer than 150 are known. Therefore, while we may not presume to state positively that the great majority of the small nebulae of the Mt. Wilson lists are spirals, there is some evidence to support such a hypothesis, and we include them upon the supposition that, except for galactic regions, we are chiefly concerned with spiral nebulae.

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[2] *Astr. Nach.*, 151, 1, 1899.

[3] *Astr. Jour.*, 28, 75, 1914.

[4] *H. A.*, 60, 199, 1908.

[5] *Mon. Not. R. A. S.*, 74, 699, 1914.

[6] *Mon. Not R. A. S.* 29, 337, 1869.

[7] *Publ. Taschkent Observatory*, No. 2, p. 46.

[8] *Mon. Not. R. A. S.*, 27, 257, 1867.

[9] *Mon. Not. R. A. S.*, 71, 588, 1911.

[10] *Mon. Not R. A. S.*, 74, 699, 1914.

[11] By N G is meant the region of the north galactic pole and by S G the region of the south galactic pole.

[12] *Astr. Jour.*, 28, 85, 1914.

A glance at the figure[14] published with the Mt. Wilson lists is sufficient to demonstrate the general similarity to the distribution shown by Hardcastle for the spirals. The figure's weakness lies in the fact that each selected small area is located 15° from the next area, and in some cases the unobserved intervals are markedly important in their density distribution, as for example in the long branch shown on Stratanoff's charts of nebular distribution. This branch approaches fairly close to the Milky Way at R. A. 2 hrs. in the south galactic hemisphere, but is so located with reference to the centers of the Selected Areas that it is not covered by their photographs. This is made apparent by four Crossley photographs, of about two hours exposure each, that had already been taken in this region, ranging from R. A. 2 hrs. to 3 hrs. and from declination +36° to +42°. It was further confirmed by four other exposures of similar length, extending the limits covered by the other four plates to +50° in declination. All of the eight plates were found to have nebulae upon them, there being as many as 60 on one plate and no fewer than 12 on any plate. In order to make sure that there might not also be some other places in the sky where a near approach of nebulae to the Galaxy had been missed by the wide separation of the samplings furnished by the Selected Areas, all the Crossley reflector plates that fall between +25° and —25° galactic latitude and north of declination —15°, and that have an exposure of two hours or more, have been examined. These plates cover by no means all the part of the sky specified, but are scattered over it in haphazard fashion and therefore do not furnish complete testimony on this question. At two places only within this zone, so far as it can be reached from Mt. Hamilton, is the distribution different from that indicated by Fath's chart. One of these places has been referred to already. At one other place two plates show considerable numbers of nebulae. One of the regions is adjacent to a Selected Area that yielded no nebulae, and the other falls between two Selected Areas which show a few nebulae. Both plates are on the edge of the zone covering 25 degrees on each side of the galactic plane, which was taken as the limiting area for the plates to be examined. From about eighty Crossley plates within the above limits there is evidence for a marked difference of distribution from what the Selected Areas yield at only one point, close to the Milky Way structure. This point coincides roughly with the near approach shown by Stratanoff for the bright nebulae. The nebulae found upon these plates are for the most part very small, and in appearance, similar to those found in such great numbers in the regions near the poles of the Galaxy. It may be inferred that they follow the distribution of the large bright spirals, even in their departure from the general tendency to avoid the Milky Way.

Hinks also calls attention to the scarcity of spirals in the vicinity of the Milky Way at R. A. 18 hrs., a region rich in large gaseous nebulae. The Mt. Wilson lists may be said to offer no contradiction to this state of affairs, although its positive evidence is not striking. Part of this may be due to the abrupt limit at declination —22°5, which marks the southern extent of the sky for which the areas are representative.

From the foregoing discussion we gather the following points of importance concerning the distribution of the spiral nebulae:

(1) The well known increase in numbers towards the galactic poles, especially toward the northern galactic pole, is unmistakable.

(2) The polar concentration is markedly greater in the northern galactic hemisphere.

(3) There is in general a shunning of regions near the Galaxy.

(4) There is a near approach to the Galaxy at R. A. 2 hrs. in the south galactic hemisphere.

(5) There is a region of maximum shunning of the Galaxy around R. A. 18 hrs.

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[13] *Publ. Astr. Soc. Pac.*, **29**, 52, 1917.

[14] *Astr. Jour.*, **28**, 85, 1914.

These features suggest two main lines of inquiry: the one an attempt to trace some functional relation to galactic co-ordinates, and the other a special attention to the two contrasted sections of the Milky Way, to which notice is called in (4) and (5) above.

The second of the two lines of inquiry will be taken up first. Investigation has been carried on by means of further observations at the contrasted sections of the Milky Way. In order that there might be a chance to make comparisons with work already done, it was decided to reöbserve the Selected Areas of these parts of the sky, as far as possible, by making the longest exposures practicable with the Crossley reflector. The aim was to secure evidence as to whether the non-appearance of nebulae in these regions is due to their extreme faintness, and whether very long exposures would record any such nebulae. To determine how long an exposure could be made with this instrument, without too great a blackening of the photographic film from the sky background, such long-exposure photographs as were at hand were examined. They showed that with Seed 27 plates twelve hours would not produce too great a blackening on moonless nights. Such an exposure is roughly equivalent to four times that given to the Mt. Wilson plates.[15] During the summer it was necessary to give two nights to such an exposure. The best seeing did not seem an essential to mere detection of nebulae, and exposures were made on all available nights that were transparent and not of unreasonably bad seeing. It was necessary to wait until the fall was well advanced before starting on the region around R. A. 2 hrs. Unfortunately, bad weather cut into the programme here, and few plates were obtained. The plates that were secured and the data concerning them are described in Table I. Column one has the Crossley plate number; column two, the number of the Selected Area whose center was observed; in columns three and four are the right ascensions and declinations of the centers of the plates, for 1875; in five and six are the corresponding rough values of the galactic coördinates; and column seven gives the length of exposure. Column eight gives the density of the Milky Way background, as estimated by Pickering from the drawings of the Milky Way by Heis and others.[16] Column nine contains the number of nebulae found upon each plate, and column ten contains the logarithm of the number of stars per square degree found upon each plate. This number is based upon the assumption that a section of about one-ninth of a square degree in the center of the plate is representative of the entire plate.

TABLE I

1	2	3	4	5	6	7	8	9	10
3746	45	1^h24^m	+30° 0′	99°	—32°	11^h30^m	0	23	3.34
3735	22	2 36	+45 0	109	— 8	11 22	3	7	3.75
3714a	9	3 1	+60 0	106	+ 2	16 18	5	0	3.82
3783	48	4 21	+30 0	135	—13	9 51	5	1	
3620	133	17 14	—15 7	334	+12	6 2	5	0	4.25
3621	109	17 41	— 0 7	351	+14	8 55	3	0	4.07
3603	134	18 9	—14 58	341	0	10 55	7	0	4.08
3622	86	18 12	+15 2	8	+15	11 25	6	3	4.28
3561	110	18 36	0 0	358	+ 1	5 32	7	0	
3626	63	19 1	+30 3	28	+10	11 45	8	0	4.39
3600	87	19 10	+15 0	14	+ 1	11 29	8	0	4.44
3624	135	19 14	—14 57	349	—13	11 18	4	1	double images
3623	111	19 34	+ 0 9	15	—11	12 1	7	0	4.44
3601	64	19 56	+29 52	35	— 1	11 12	8	0	4.55
3625	88	20 12	+15 10	23	—11	13 6	6	0	4.49
3674	Coal Sack	20 56	+51 30	58	+ 3	9 0	8	0	2.99

The only area close to the Milky Way that may be said to show more nebulae than the Mt. Wilson exposures is area No. 22. This falls in the midst of Crossley reflector regions that show goodly numbers of nebulae at R. A. 2 hrs. One other area, No. 45, galactic latitude —32°, yields 23 nebulae, whereas the Mt. Wilson plate gave 16. Since the Crossley plate utilizes scarcely 0.75 square degree, and the area on the 60-inch reflector plates is 1.88 square degrees, it is apparent that a marked increase has been attained by lengthening the exposure upon this area. It may be significant, however, that one of the most striking, large spirals that we know of, *Messier 33,* is almost at the center of this area; in fact, some of the outer portions of this spiral fall upon the Crossley plate.

The purpose of these observations was to find out whether a powerful instrument and long exposures could bring to light any new nebulae in the regions of the Milky Way which have hitherto seemed barren of them. It must be said that they afford no evidence of undiscovered faint nebulae, especially of the spiral type, in the regions where they have not previously been found with much shorter exposures. Also, they have made no significant additions of small nebulae of any class to the number already known.

Estimates have been made by Keeler,[17] Perrine[18] and Fath[19] as to the probable number of the nebulae. It is quite safe to say that the estimates do not need to be modified, at least in so far as they have been based upon the regions where no nebulae were found with comparatively short exposures. However, it is probable that the number it would be possible to photograph by longer exposures made with the same instruments as before could be very materially added to in extra-galactic regions. How much of an increase might be indicated by long exposures must await settlement until more such observations have been secured.

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[15] *Astr. Jour.*, 28, 85, 1914.

[16] *H. A.*, 48, 154, 1902–03.

[17] *Publ. Lick Obs.*, 8, 28, 1908.

[18] *Lick Obs. Bull.*, 3, 47, 1904.

[19] See footnote 14.

Attention will now be directed to the other division of the proposed study. This includes the items enumerated under the headings C, D, E, F, at the beginning of the paper. As constant reference has to be made to the Hardcastle and the Mt. Wilson lists of nebulae, these will hereafter be referred to as the Hl nebulae and the MW nebulae. From the former it is possible to pick out readily the spirals and spindles, as Hardcastle calls them. I have chosen to add the spindles to those certainly spiral on the assumption that the spindles are spirals seen edgewise. The Hl and MW nebulae are treated in Table II. The first column gives the subdivisions of the sky, in galactic latitude, into which all the nebulae in the same horizontal line fall. In the second division of the table a further grouping has been made, wherein the corresponding belts in the two hemispheres have been united.

TABLE II

Zone	No. per Area	Size	Brightness		Hardcastle Spirals		
			All	Nearly Round	No. per Zone	Size	Brightness
+90° to +75°	54	14″.6	2.5		169		
+75 to +60	12	13.6	2.1		63		
+60 to +45	10.5	15.2	2.5		28		
+45 to +30	11	15.3	2.6		13		
+30 to +15	3	13.4	2.4		13		
+15 to 0	0.3	15.0	3.2		4		
0 to —15	0.3	17.5	3.2		3		
—15 to —30	2.5	14.3	2.7		11		
—30 to —45	8	13.7	2.7		20		
—45 to —60	5	9.8	2.2		44		
—60 to —75	7	9.5	2.5		50		
—75 to —90	26	11.1	2.5		46		
Combining Both Hemispheres							
±90 to ±75	45	14.0	2.5	2.4	215	5′.2	95
±75 to ±60	10.6	12.9	2.2	2.2	113	5.6	69
±60 to ±45	9.3	14.3	2.4	2.2	72	5.3	43
±45 to ±30	10.1	14.8	2.6	2.4	33	5.4	71
±30 to ±15	2.7	13.8	2.5	2.7	24	6.5	45
±15 to 0	0.3	16.5	3.2	3.7	7	11.3	40

The different subdivisions referred to in the beginning of the paragraph will now be taken up in order.

C. Nebular Distribution as a Function of Galactic Coördinates

This feature has already been referred to; but it has usually been shown graphically, by means of maps, while here a tabulation is used. It was necessary to reduce the numbers found in each zone to equivalent areas, before a comparison could be made. This was done in the case of the Hl nebulae by multiplying the numbers found in each zone by the secant of the latitude of the middle of the zone. It was more simple, in the case of the MW nebulae, to do this by taking the average number found upon each plate within the zone in question. Column two shows this for the MW nebulae, and column six has the numbers relating to the Hl nebulae. It is immediately apparent that the numbers in a given area (that of a plate) for the former are very decidedly greater in the zone about either pole than in any of the others; that intermediate latitudes, 75° to 30°, are very little different among themselves; that the zones 30° either side of the galactic plane are very sparsely supplied; and that the outer parts of each of these zones have the greater density. This seems to be a property of the two hemispheres alike. For the Hl nebulae the results given for the relative numbers show that the polar regions are decidedly the richer in numbers, but that there is not the sudden falling off for the intermediate zones that was found for the MW nebulae; there is, in fact, for the northern hemisphere, a steady decrease, while in the southern hemisphere there seems to be about the same density over the entire zone from —90° to —45°. For the rest of the southern galactic hemisphere there is very close agreement with the density for the northern hemisphere. In order to be sure that the decided difference shown in the run of the densities in the two sources did not result from a peculiar arrangement in longitude of the plates which go to make up the mean for the zone in the case of the MW nebulae, a grouping of the areas showing large numbers of nebulae, as against those which show small numbers in each zone, was made. This grouping displays a haphazard arrangement of the longitude that makes this suggestion untenable. If we combine the results for the two hemispheres, we may make the general statement that both sources show the much superior richness of the polar zones and the poverty of the zones in the immediate vicinity of the Galaxy, the Hl intermediate densities being a smooth gradation, while the MW densities in the intermediate zones are all about the same and markedly inferior to that of the polar region. We shall see later that there is a very great difference in the mean sizes of the nebulae in the two cases, and this may be significant. It is conceivable that an agency effective in blotting out the nebulae from the reach of our instruments might have a different rate of increasing effectiveness as we go farther and farther out into space, into the regions of the apparently-smallest nebulae.

In all the discussions of nebular density that have been based upon the N.G.C., the south galactic hemisphere has seemed to be less dense than the northern. Stratanoff[20] has shown that for the N.G.C. the distribution in the south galactic hemisphere is quite similar to that of the north, if we confine our attention to the part of the heavens lying north of declination —20°. He thinks the relatively inferior density of the southern hemisphere is due to the less careful character of the survey of the sky that lies outside the reach of the more numerous northern observatories. The evidence in Table II tends to support the statement that there is, in reality, greater density in the northern than in the southern hemisphere. Both the indications of density are from homogeneous material and are therefore free from the objection urged by Stratanoff against the N.G.C. results as a whole. It has been pointed out that the MW nebulae are not capable of shedding light upon their distribution in relation to galactic longitude. We have been able to show, by means of additional Crossley photographs, that the tendency to approach the Milky Way in great numbers in the neighborhood of the Andromeda nebula [Region (4), p. 82] is in reality as pronounced for the small nebulae, such as make up the MW lists, as for the specially selected lists of Hardcastle. Therefore it seems necessary to confine our statement in regard to dependence upon galactic longitude to what Hinks has said. (See page 81.)

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[20] *Publ. Taschkent Observatory*, No. 2, p. 43.

The most striking case we have of near approach to the Milky Way, that at about 100 degrees galactic longitude, which is the region (4) of page 82, needs explanation. A possible one is contained in the fact that there is, according to Easton,[21] "a lack of continuity" in the Galaxy at this place. In depicting a hypothetical spiral to represent the features of our Galaxy, he has the Sun so situated, in regard to the arms of the spiral, that we look out in the direction of galactic longitude 100° through very little of the assumed nebula. There is also a comparatively inferior star density in this part of the Milky Way for all stars down to the sixteenth magnitude, as shown upon the Franklin-Adams photographs.

D. NEBULAR DISTRIBUTION AS A FUNCTION OF NEBULAR BRIGHTNESS

In column eight of Table II we find a designation for the mean brightness of the spiral nebulae which fall in the different zones of galactic latitude. These numbers were obtained by substituting for the N.G.C. designations of brightness those which Mr. F. G. Brown[22] obtained from a comparison between the N.G.C. designations and the direct photometric measures made upon the same objects at the Harvard College Observatory.[23] His plan is meant to show the relative intensities, directly. Our table shows us that the brightest spirals, on the average, lie more than 30 degrees from the galactic plane. In the zone ±60° to ±45° there is an average brightness that falls in with that for the galactic zones, but the average brightness for the zone that lies between +30° and —30° is considerably less than that for the remainder of the sky. They stand in relation to each other in the ratio of 44 to 66.

For the MW nebulae it was necessary to assume arbitrary numerical values for the brightness designations, as they are simply given as VB (very bright), B (bright), etc. There were six of these designations, from VB, the brightest, to VF, the faintest. The value 6 was assigned to the brightest and 1 to the faintest, with steps of 1 filling out the series. The means of the results of these substitutions are given in Table II, column four, for all the nebulae of the MW lists. Column five gives a similar tabulation, wherein only the circular and nearly circular nebulae are used. This will be referred to again in another connection. It is apparent at once that we do not in the case of the nebulae considered, in either column four or five, have a marked difference in the brightness in different galactic latitudes, except a tendency to highest values in low galactic latitudes. If we divide the sky as before, the mean for galactic regions is found to be brightness 2.8, while that for the extra-galactic is 2.4. What indication there is with regard to a difference of brightness for galactic as against non-galactic nebulae in this case is in contradiction to the evidence previously presented for the Hl nebulae. But the difference in the two values of brightness given above is less than one-half the difference in brightness which separates what Fath has called the F and MF nebulae. The further fact should not be lost sight of, that the Hl nebulae are entirely made up of spirals and the MW nebulae are for the most part unclassified. While this probably makes little difference outside the regions of the Galaxy, the same need not be the case when we get close to it. Brown[24] has made use of the entire N.G.C. to inquire into the relation between size and brightness in the nebulae. By means of his tables, and of Pickering's chart[25] showing the relative intensities of the Milky Way background, it was a simple matter to derive an idea of the mean brightness of the nebulae within and without the Galaxy. Here, too, the brightness for the galactic regions is greater than that for the non-galactic. This was further checked by using the excellently arranged tables of nebular data that Charlier[26] has formed for the N.G.C. nebulae.

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[21] *Ap. Jour.*, **37**, 111, 1913.

[22] *Mon. Not. R. A. S.*, **72**, 718, 1912.

[23] *H. A.*, **33**, 135, 1894–99.

[24] *Mon. Not. R. A. S.*, **72**, 195, 1912.

[25] See footnote 16.

Now both the N.G.C. and the MW lists may be expected to contain nebulae in the region of the Milky Way of a kind that belong there alone, and which are not represented in the Hl lists of spirals. Charlier's tables make possible a subdivision of the material for this comparison of galactic and non-galactic nebular brightness, which points the way out of the apparent contradiction. Of the largest-sized nebulae, without distinction of kind, it can be shown that a greater percentage of those within the galactic structure are faint, than of those outside. As the smaller nebulae of the Milky Way are the intrinsically bright and proportionately numerous planetaries, we can see a possible reason for an apparent disagreement in results between lists of unclassified nebulae and lists of known spiral nebulae, near the Milky Way. For the galactic zone +30° to —30° it might easily be that the brightness was so far influenced by the relatively large number of strictly galactic nebulae that a condition, such as was shown for nebulae certainly spiral, might still exist for the few of the spiral nebulae found here, and yet be obliterated by the larger number of the other kind.

From the foregoing, it is apparent that our most reliable evidence with regard to the relation of the spiral-nebular brightness to the Milky Way is to be derived from the Hl lists, and we therefore conclude that there is greater average brightness for the extra-galactic than for the galactic spirals.

E. Nebular Distribution as a Function of Nebular Apparent Size

The apparent mean size of the Hl nebulae for the different galactic-latitude zones is given in column seven, Table II. Column three has similar data for the MW nebulae. The latter could be obtained directly from the lists; but for the former it was necessary to substitute the numerical values that Sir John Herschel assigned to the latter designations used by him and by Dreyer. The apparent sizes of the Hl nebulae are expressed in minutes of arc, and it is clear that the zones with the largest mean values are those near the Galaxy. Subdividing the sky into two parts, as before, we find that the subdivision which lies near to and includes the Galaxy has spirals and spindles that average 9′ in angular size. The mean size for this class of nebulae in regions outside ±30° galactic latitude is 6′. If we divide the sky by means of galactic-latitude circles at +15° and —15° the contrast is still greater; the spirals within this zone average 15′ in size, and those outside it, 6′. The MW nebulae are seen to be very small on the average in comparison with the Hl nebulae, for their sizes are expressed in seconds of arc. While there is here, too, an indication that the zones with the largest mean value of the size are the zones of low galactic latitude, these objects are too small and the measurements relatively too rough to warrant the assertion that there are any real differences in the sizes for different zones. Between the largest mean value and the smallest there is less than 4″. While the means for such large numbers of nebulae may not be much in error, even if the individual results are subject to considerable error, the difference in the extremes is so small a quantity that little weight can be attached to the results, though they conform to the results from the Hl objects. We have also described a possible influence of other than spiral nebulae in these lists, and that makes it inadvisable to push the apparent agreement too far under this heading.

We are therefore confronted with the fact that those spiral nebulae which lie nearest to the Milky Way are on the average of a larger angular size than those at greater distances from it.

There is another factor which might conceivably have some bearing upon the apparent size and the apparent brightness of a nebula, viz.: the angle at which one views these objects. This angle, it is assumed, will have a direct relation to the amount of observed ellipticity. The underlying idea is that all nebulae of the spiral type are approximately circular, if seen with their principal planes normal to our line of vision. To test for any difference of brightness depending upon this angle of vision, the nebulae that fell in two different classes of size were subdivided in respect to the ratio of minor to major axes. In both classes the number of nebulae which fell into the division, ratio of minor to major axis 0.2, was very small. For those with ratios from 0.4 to 1.0 there is practically no difference in the mean brightness dependent upon these ratios in either of the two classes. The differences between those with the greatest and those with the smallest eccentricity is but a very small part of the difference between two successive steps in the brightness scale. It is apparent that this factor, as affecting the results of the preceding discussion, is of no importance.

It is conceivable that if we have two nebulae of the same actual size, brightness and distance, one edgewise and the other flatwise to our line of vision, we might get the effect of a decidedly larger nebula for the former than for the latter, judging as we do by the major axis. If, as has been done in discussing this list, we divide the nebulae into three diameter groups, 0″ to 10″, 10″ to 20″, and 20″ to 30″, some of those nebulae which, seen flatwise, would fall in the group 0″ to 10″, might, if seen edgewise, fall into the next group and swell the numbers which are truly of that size. This assumes that the greater depth of radiating matter due to edgewise projection should cause the fainter outlying portions to be effective upon the plate, whereas they would not make an impression if the principal plane were normal to the line of sight. An examination of the list does show an increasing ratio of minor to major axis with decreasing size. Part of this, of course, will be due to the great difficulty in recognizing a departure from circular form in a very small object. This ratio is smallest for the largest objects and increases smoothly for decreasing size, until the smallest class is reached, where the ratio is nearest to unity. The progression is unmistakable. Table II gives the mean brightness for zones as it was derived in the two different ways, the one as it came from using all the nebulae of the lists, and the other as it resulted from only such nebulae as have a ratio of minor to major axis of 0.8 or greater. Both procedures show the same peculiarities, though with the nearly circular nebulae these peculiarities are the more pronounced. It is not intended to press this point, for the observations are too crude to give more than a suggestion that such a relation may prove of importance when the observations of nebular brightness shall have been better systematized.

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[26] *Archiv für Math., Astron., och Fysik,* No. 15.

Brown has shown that the larger nebulae are on the average the brighter nebulae. The following table shows that the same is true for the MW nebulae:

TABLE III

Size	Number	Brightness
> 2′	6	2.8
1′ – 2′	15	2.9
30″–60″	68	3.0
20 –30	109	2.6
10 –20	452	2.6
< 10	269	2.2
Stellar	96	3.1

Column one contains the size limits, column two the number of nebulae that make up each division, and column three gives the mean brightness upon the plan explained on page 85. It is seen that the smallest nebulae are the faintest; but we do not include those in the last line which are designated as "stellar" in the MW lists, the real nature of which may be different from the rest. There seems to be good ground for saying that, on the average, the larger nebulae are the brighter nebulae. We assume that this applies to spirals alone, because they are in the majority. It has been shown, however, that those spirals near the Milky Way are not only the largest on the average, but are very few in comparison with those in the remainder of the sky, and are actually fainter on the average. If we take as a criterion of distance the mean angular size, we are forced to the conclusion that an agency is at work which cuts off some of the light from these objects, thereby letting only the nearer ones be perceptible and then only in diminished brightness. The further assumption is here implied that we may consider the surface brightness of all the nebulae the same, when dealing with large numbers of them. Hence the inference is made that something is dimming them, when a group is found whose brightness is less than the average, although the average size of its members is such as to indicate greater proximity to us. What the nature of this agency may be, will be discussed a little later when the conditions to be satisfied are brought together.

F. NEBULAR DENSITY AS A FUNCTION OF STAR DENSITY

From a discussion of the N.G.C. objects, Brown[27] found that, on the average, those areas have the greater star density at similar galactic latitudes which have the greater average brightness for the nebulae. He made use of the Herschel star gauges to show this. I have covered the entire sky in a similar comparison of Brown's values of brightness, using the star counts upon the Franklin-Adams star plates, as given by Chapman and Mellotte,[28] and find a pretty consistent confirmation of his results. It is necessary to make two different comparisons, one for those plates taken at Mervel Hill, England, and another for those taken at Johannesburg, South Africa. It is stated in Chapman and Mellotte's paper that the South African plates go down about one magnitude fainter and give results the more consistent amongst themselves. The range of brightness considered by Brown was divided into three sections, such that about equal numbers of the areas into which he divides the sky fall in each section. The first division includes all that he calls brightness 3.00 or brighter, the second division those between 3.00 and 2.50, and the third all below 2.50. As used below they are the means, for each area, of the brightness of all the nebulae that fall in that area. If we make no latitude distinction, we find a relation between star density[29] and nebular brightness, as follows:

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[27] *Mon. Not. R. A. S.*, 24, 201–202, 1912.

[28] *Memoirs R. A. S.*, 60, part IV.

[29] Star density as given is 1000 stars per 225 square degrees.

TABLE IV

Brightness	> 3.00		3.00 to 2.50		< 2.50	
	Mean Bright.	Mean Star Density	Mean Bright.	Mean Star Density	Mean Bright.	Mean Star Density
Johannesburg	3.52	745	2.74	513	2.33	395
Mervel Hill	3.39	257	2.71	376	2.32	276

If we consider rough divisions of latitude, we have the following table:

TABLE V

Zone Gal. Lat.	No. of Areas	Mean Brightness	Mean Star Density	Galactic Latitude
JOHANNESBURG, SOUTH AFRICA				
—90° to —60°	1	3.37	339	—62°
	4	2.41	269	—72
±60 to ±30	8	3.40	475	±40
	10	2.45	395	±41
±30 to 0	7	3.60	848	±12
	8	2.67	713	±17
MERVEL HILL, ENGLAND				
+90 to +60	1	3.53	117	+82°
	2	2.50	208	+68
±60 to ±30	4	3.18	205	±44
	8	2.23	321	±39
±30 to 0	4	2.99	520	±14
	8	2.40	437	±16

In Table IV, for Johannesburg plates, we have a regular falling off in density with decrease in brightness, which is what Brown found. This is shown in Table V, too. But the Mervel Hill plates, which are necessarily in the northern galactic hemisphere for the most part, give the least star density to areas of greatest nebular brightness. This is shown in both tables, except in Table V for the zone ±30° to 0°, where the relation that holds for Johannesburg prevails. It is somewhat difficult to say whether we have here a true difference in the two parts of the sky, or whether the result is due to the difficulty of assigning the proper density to the areas considered. Possibly we penetrate about as far as it is possible in the part of the sky where the brightness is less, even in the Mervel Hill plates, and that the additional exposure necessary to make the magnitude limit of the Mervel Hill plates equal that of the Johannesburg plates would decidedly increase the density of the areas whose average nebular brightness is greatest. It is apparent that the mean density for the two extra galactic zones where the nebular brightness is small is of the same order of size as that for Johannesburg, while the areas of large average nebular brightness are very much less dense. In the foregoing one must constantly keep in mind that a part of our data is of the crudest sort, and that the adaptation of some of the rest is made difficult through the somewhat hazy nature of the limits of the areas. At best it can but indicate a tendency.

Through the kindness of the Mt. Wilson Solar Observatory I have been allowed to use the counts that have been made there by Seares upon the centers of the Selected Areas. The numbers which are given for each are the results for a circular area of 0.112 square degree at the center of each area, and include all stars down to about 17.5 magnitude. Of the 139 areas from the north pole of the heavens to declination —22°.5, some 90 areas have completed counts. Within each of the zones into which the MW lists were divided, I have made two subdivisions, so that in the one were found the areas of greater nebular density and in the other those of lesser density. In order to see what the possible relation of star density to the two subdivisions would be, the mean star densities were obtained for these areas. In each zone 15 degrees wide the mean star density for areas containing large numbers of nebulae is greater than for areas with small numbers. Combining for the whole region outside of ±30° galactic latitude, the ratio of star density of the regions rich in nebulae to those poor in them is about 120 to 90. Inside ±30° galactic latitude the ratio is reversed, being about 500 to 960. Since not all the areas inside ±30° galactic latitude are in the naked-eye Milky Way structure, I made a division to see whether outside the Milky Way but still within this zone there might not be a contrast similar to that outside the zone itself. Table VI shows the result of this comparison.

TABLE VI

RELATIVE STAR DENSITIES (SEARES, MT. WILSON)

Region	±90° to ±30°			
Areas with	Many nebulae		Few nebulae	
Relative Star Density	123		90	
Region	±30° to 0°			
Division	Outside the Milky Way structure		Inside the Milky Way structure	
Areas with	Nebulae	No nebulae	Nebulae	No nebulae
Relative Star Density	452	511	709	1090

It is readily apparent that there is a progressive change in the relative star density as we come into the Milky Way structure. Outside ±30° galactic latitude in the same zone the mean star density for the areas with many nebulae is larger than for areas with few nebulae; inside of this limit but outside the Milky Way structure the reverse is true, though to a limited extent, and inside its structure the reverse is very marked.

The material is at hand and in good form to obtain another comparison of a kind similar to the one that has just been described. Stratanoff[30] gives tables for the distribution of stars by magnitudes and by types. The former go down to the limits of the Bonner Durchmusterung and Cape Photographic Durchmusterung, one division comprising all from the brightest to the sixth magnitude, and each succeeding division covering a half magnitude. The tables for stellar type are based upon the Draper Catalogue.[31] Two divisions cover all, in which class I comprises spectral classes A, B, C, and D, and class II comprises E, F, G, H, I, K, and L. The same sort of division was made within zones of latitude, as in the preceding section. From a summarized statement which relates to three zones thirty degrees of latitude in width we have, out of twenty-four comparisons, all except three showing a greater star density where the nebulae are fewest. It is true that the difference is very small, but it seems consistent. In the same sort of subdivisions we have, except inside of ±30° galactic latitude, a greater percentage of stars with class II spectra in the regions of the areas with small numbers of nebulae. Inside this limit the reverse is true.

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[30] *Publ. Taschkent Observatory,* No. 3, Appendices II and III.

G. CONCLUSIONS

Recapitulating the features brought out, we see that any hypothesis offered as a solution for the apparent distribution of the spiral nebulae must be made compatible with the following conditions:

First, the numbers are dependent, on the average, upon the distance from the poles of the Galaxy, the largest numbers per unit of sky area being found near the poles, and the regions of few or no nebulae being in or near the Galaxy. The exceptions to this must likewise find explanation in the hypothesis.

Second, if we divide the sky so that one part is comprised between the two parallels of galactic latitude +30° and —30° and the other part embraces the rest of the sky, then the mean size of the spirals is materially larger for the former division than for the latter. Moreover, the same division yields a mean value for the brightness of the spirals in the zone +30° to —30° that is decidedly less than for the other region.

Third, if within the same galactic-latitude belt we contrast those Selected Areas which the Mt. Wilson observations show to be rich in nebulae with those poor in nebulae, there is found a greater star density for the former than for the latter, with the exception of the regions of the Milky Way. The difference between the results, when there is a difference of one magnitude in the limiting magnitude recorded on the plates, has to find explanation also. Furthermore the fact that in the Milky Way structure itself we have the strongest reversal of the conditions stated in the first sentence of this paragraph must also be borne in mind. The close approach of nebulae to the Galaxy at R. A. 2 hrs. and declination +40° seems to have significance in this connection.

The view has been held that the apparent concentration toward the poles of the Milky Way is a real one. But there is great difficulty in conceiving what is the nature of the space that these nebulae occupy, in order that the apparent may represent the actual distribution, and at the same time be adequate to explain the different peculiarities of distribution. This is equally true whether we consider the spirals as a part of our stellar system or not.

If we examine the first hypothesis, that they are a part of our system, and retain the assumption that spirals are, on the average, close or far away, according as we find them large or small angularly, we must either conclude that the large ones are relatively quite near at hand, or that the small ones are controlled to magnificent distances by the Milky Way aggregation. Van Maanen[32] has made measurements of points upon spiral nebula *Messier 101,* which seem to indicate that sensible proper motions, radial within the nebula, have occurred. If such proves to be the case in this and other spiral nebulae, there will be ground for the more positive statement that some at least of these spirals are within galactic distances.

A serious difficulty connected with any attempt to assign these objects to our system comes about because of the very high radial velocities which have been indicated for them by the spectrograph. Within our Galaxy we do not have, so far as is known, any objects that can be linked with the spirals by virtue of their high velocities in the line of sight. The gaseous nebulae come nearest to fulfilling this condition. Of the gaseous nebulae, the only group which comes nearest to having comparable radial velocities is that found in the Magellanic Clouds. There is independent evidence to show that these Clouds are not a part of our Galaxy, so that the very group which most resembles the spirals in the magnitude of their radical velocities is probably not within our stellar system. But, while the average velocities[33] for gaseous nebulae are much above the average for the stars, they are far lower than those of the spiral nebulae[34] already observed. Moreover, the latter are found only outside the Galaxy, while the former are quite generally found within it. The contrast in spectrum also furnishes a very serious difficulty in linking up the two classes. Mention might also be made of the distinct difference in form that is found between the two. The fact that spiral nebulae have a continuous spectrum and absorption lines analogous to the Fraunhofer lines suggests that they are composites of great numbers of stars, the integration of whose light builds up the spectrum that we photograph. We are not able to resolve these nebulae into component stars, and, therefore, if they are composed of stars, they must be at tremendous distances.

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[31] *H. A.*, **28**, 1897–1901.

[32] *Ap. Jour.*, **44**, 210, 1916.

[33] *Proc. Nat. Acad. Sci.*, **1**, 8, 496, 1915.

[34] *Pop. Astr.*, **23**, 21, 1915; **25**, 36, 1917.

The hypothesis that the spirals are other universes of stars, comparable with our own, is next examined. So far as is known, they are in shape much like the Milky Way system. The bifurcation that runs through a large portion of the Milky Way structure can also be paralleled in the case of such spiral nebulae as, in the edgewise position, show a marked vacant lane through their principal dimension. Another pretty well established feature of the Milky Way is that the dominion[35] of the yellow and red stars within it is much more restricted than that of the blue-type stars, and also the yellow and red stars are apparently more evenly distributed in a roughly spherical space about the center of the Galaxy. The analogy is furnished in this case by the photographic and photovisual work upon certain of the larger spirals. Seares[36] has shown that the centers of the large spirals are sending us light richer in the visible radiations than are the outer portions. In the spirals, therefore, if considered as external Galaxies, the realm of more advanced spectrum is nearer the center, and that of earlier spectral type is found in the outer portions; and in this respect the spirals resemble our own stellar system. Besides the fact that the peculiarities in the distribution of the spiral nebulae are no easier to explain on the ground that they are near us and form a part of our Galaxy, there are several reasons, such as the above, why we should be hesitant about assigning them to our own system.

Several characteristics of the spiral nebulae have already been mentioned, which fit in well with the idea that they are at distances vastly superior to the distances within our Galaxy, and that they are quite probably isolated systems. But no distribution of the spiral nebulae has yet been suggested which seems to explain satisfactorily how the apparent distribution results from the real distribution, by virtue of our peculiar position in the system. We should be forced to accept the idea that the dimensions of greatest extent for the domain of the spirals would be perpendicular to the dimensions of greatest extent for the Galaxy. It would then be necessary to ascribe a force to the latter adequate to effect such a relation. Gravitation is the only known force whose operations approach so grand a scale. If we were not prevented, for other reasons, from calling upon the gravitation of the stellar system to perform the mammoth task of ordering these tens of thousands of other systems, we would still have the difficult problem of showing that gravitation would be capable of gathering them into a figure at right angles to its own figure. But we have already presented several reasons which indicate that these systems are of the order of size of our Galaxy. Since then, the Galaxy would be of the order of size of any of the other systems, we are not justified in singling it out to perform a task whose magnitude would have to be incomparably greater than that of any other one of the systems.

The reasons thus far given have made it seem the more likely that the spirals are not only outside our own system, but that they can have no intimate connection with it dynamically. We have then to look for some agency that will convert an actual distribution of the nebulae which appeals to our reason as the most likely into the apparent distribution which we have observed.

The view has been advanced that there may be a distribution of the spiral nebulae which is actually quite indifferent to any relation to the Galaxy and that intervening absorbing or occulting matter is modifying the actual distribution into the apparent distribution. Before taking up the practicability of explaining the various attributes of spiral distribution in this manner, it would be well to see what indications there may be that an obstructing medium does actually exist. Barnard[37] has presented very strong evidence that there exist what he terms ''dark nebulae.'' They are made evident by certain so-called ''holes'' in the galactic structure, patches devoid of stars in some of the very densest parts of the Milky Way structure. They range in size from patches too small to be recognized by the unaided eye up to such striking empty spaces as the Coal Sacks. It seems highly improbable that the apparent star density of the holes could be their true star density, in view of the fact that adjacent Milky Way regions are amongst the richest in numbers of stars. We are faced with the necessity of explaining, dynamically, how such a condition could have any stability. If we are free to accept the assumption, which seems a reasonable one, that the apparent intensity of the Milky Way structure is a function of the depth from which the stars of the Galaxy are contributing light, then it follows that, in the case of such a region as the northern Coal Sack, we really see out into space in its direction to a very limited distance indeed. In other words, the obstructing matter, whatever it may be, is comparatively near, as distances go in the Galaxy. A nine-hour exposure was made with the Crossley reflector upon that part of the northern Coal Sack which seemed, from a previous four-hour exposure with the Crocker telescope, to have the least star density. This long exposure fails to reveal anything but the most sparse distribution even in comparison with short exposures on regions in high galactic latitudes. Table I has data for this exposure. It is readily seen that the density is not more than about one-twenty-fifth part as great as that of other portions of the Milky Way for which the densities are given in the same table. The difference would probably be even more striking if we could contrast a long exposure of one of the very dense regions close to the Coal Sack. The assumption that the obstructing matter which we are discussing lies well inside the galactic structure seems warranted. Furthermore, we know that the Coal Sacks and other vacant configurations are not distributed with regularity over the Galaxy. Such careful studies as have been made by Gould, Easton and others, have shown that the Milky Way is far from being a regular cloud belt across the sky, but is rich in a sort of network of dark, irregular lanes, in the cloudy structure that we all recognize as the Milky Way. If we are to ascribe even a portion of this detailed structure to the effect of obstructing matter, then it is evident at a glance that we are under the obligation of ascribing an irregular character to the distribution of this matter.

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[35] *Astr. Jour.*, **26**, 188, 1911.

[36] *Proc. Nat. Acad. Sci.*, **2**, 553, 1916.

[37] *Ap. Jour.*, **43**, 1, 1916.

It seems justifiable, therefore, to speculate upon the possibility of an obstructing medium between us and the realm of the spiral nebulae, of such a structure that it will produce the apparent distribution that we note. What shape it will assume is a matter of the purest speculation, but one can say something of its possible characteristics. There are at least three reasons for asserting that this obstructing matter lies in the galactic structure. They are: first, the progressive falling off of the numbers of nebulae as we approach the Milky Way; second, the fact that we do not find spirals in its structure; and, third, that where there is an abrupt discontinuity in the Milky Way structure there we find considerable numbers of the large spiral nebulae, and likewise of the small ones that are usually numerous near the large ones. An irregular distribution is assumed for this obstructing material, on the ground that it is analogous to, if not identical with, the so-called dark nebulae. This in general will lead to a more or less uniformly integrated effect upon objects outside the Galaxy altogether, and will likewise make possible the explanation of the greater star density for the same approximate galactic latitude where the nebular density is the greater. It makes possible the effect shown for the star counts by Chapman and Mellotte on the Franklin-Adams star plates. If, in a series of plates of regions outside the galactic structure, we prolong the exposures sufficiently to record stars an entire magnitude fainter than those in another series, also outside its structure, it should happen that the greater proportionate increase would result when the absorbing material is the less. Now the number of the nebulae is in some inverse relation to the amount of the obstructing material on the hypothesis outlined above. Therefore we should expect a greater increase on the average in the number of stars where the nebulae are more numerous. Chapman and Mellotte's results for the Johannesburg and Mervel Hill plates show this very condition.

The northern galactic hemisphere shows a more restricted distribution of the nebulae than the southern. If the solar system is south of the median plane of the Galaxy and somewhere near its short axis, it will be unnecessary to go to as great an angle from the north galactic pole as from the south, in order to be looking through an amount of the hypothetical obstructing material sufficient to occult the spirals. This position for our system has been derived by others from a study of the comparative star densities in the two hemispheres. The southerly location of our solar system will also make easier an explanation of the abnormal number of nebulae that are found near one part of the Galaxy; for, if we combine that condition with an irregular distribution of this obstructing matter, it is possible to conceive of such a falling off of this matter in one direction that our vision is little more obstructed there than in the direction of the poles.

Briefly stated, an arbitrary and general distribution of the spiral nebulae is harmonized with the distribution that presents itself to observation, by means of the hypothesis of an obstructing medium irregularly distributed throughout the Galaxy. The solar system is assumed to lie to the south of the median plane of the Galaxy and near its short axis, but within the realm of the obstructing material.

The long-exposure plates that have served for this discussion have consumed a considerable part of the most favorable observing periods for the Crossley reflector throughout an entire season. I wish to express my appreciation for the extended use of an instrument so much in demand. I also wish to thank Director Campbell and Dr. Curtis for their advice, suggestion and kindly criticism.

R. F. Sanford.

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A STUDY OF CERTAIN NEBULAE

The nebulae are a weak link in every chain of stellar evolution thus far forged. Some hypotheses place them before the stars, while others consider them as the latest stage of development. Detailed studies of their present forms and the physical conditions under which they now exist are a pressing need.

Of the several classes of nebulae, the planetary nebulae seem to be the most promising for a detailed investigation. They are in general small and bright, have fairly definite forms in space and give bright-line spectra. The extended nebulae seem to have no definite form. The spiral nebulae are quite faint, and suitable spectrograms are very difficult to obtain.

The planetary nebulae N. G. C. 6543 and N. G. C. 7009 have been chosen as the subjects of the present investigation. Up to about two years ago the work on these two objects had been of a very general nature. Holden and Schaeberle made careful drawings[1] of them with the 36-inch refractor. These interesting visual observations are of limited value in comparison with existing photographic results.

Keeler[2], Campbell[3], Wright[4] and others observed these objects to determine the wave-lengths of the light from the nebulae and their radial velocities, employing both visual and photographic methods. While data of this sort are of inestimable value in studying the nebulae as a class, they do not teach us much concerning the structure of the specific objects.

Wright has secured spectrograms of nebulae with slit spectographs[5], using long exposures and low dispersion, and also with a recently completed slitless spectograph[6]. His observations are extremely valuable in studying the distribution of the various materials of which they are composed.

In 1915 Campbell and Moore[7] found, from spectographic observations, that the two planetary nebulae in question have not merely motions of translation, but have also relative internal motions. These motions are not at random, but give evidence that the nebulae are in rotation. The rate of rotation is not the same in various parts of the objects, and the laws governing it are yet to be determined.

Our problem may be divided into

(1) A detailed study of the relative motions of various parts of the nebulae, employing spectographic methods.

(2) A detailed study of the direct photographs of the nebulae as obtained with reflecting telescopes under a variety of conditions.

The instrument used in obtaining the spectograms was essentially the original Mills Spectograph[8], except that the prisms, in a new prism box, were set for 4950A in minimum deviation. A new camera lens of 32 inches focal length and corrected for the D lines was secured and used for taking all of the plates. The definition is excellent from 4800A to 5100A. The linear scale of the spectogram is such that at 4970A, which is central on the plate, 1 mm. corresponds to about 12A, or to a radial velocity shift of 675 km./sec. The author has designed a new constant temperature case for this instrument which is lighter and more conveniently handled than the one previously employed[9].

In addition to the usual precautions for avoiding or guarding against instrumental errors in the spectrograms, plates were taken with the nebular image on the slit respectively direct and reversed 180°. No instrumental defects have been detected on any of the plates.

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[1] *Mon. Not. R. A. S.*, 48, 388, 1888.

[2] *Publ. Lick Obsy.*, 3, 162, 1894.

[3] *A. & A. P.*, 13, 494, 1894.

[4] *Lick Obsy. Bul.*, 1, 156, 1902.

[5] *Proc. Nat. Acad. Sci.*, 1, 590, 1915.

[6] *Lick Obsy. Bul.*, 9, 52, 1917.

[7] *Lick Obsy. Bul.*, 9, 1, 1916.

[8] *Ap. Jour.*, 8, 123, 1898.

[9] *Ap. Jour.*, 11, 259, 1900.

Tables I and II give the data for the spectrograms of N. G. C. 6543 and N. G. C. 7009, respectively. The last column indicates the quadrant toward which the end of the slit ordinarily toward the telescope pier was pointing.

TABLE I.—N. G. C. 6543

Plate No.	Date G. M. T.	Exposure Time	Pos. Ang. of Slit	Quadrant of "Toward" End of Slit
N 350	1916 Mar. 31.9	7h 50m	220°	SW
N 354	Apr. 9.9	8 30	40	NE
N 357	Apr. 19.8	8 40	205	SW
N 360	Apr. 21.8	8 18	25	NE
N 361	Apr. 23.8	8 10	243	SW
N 369	May 2.8	8 00	265	SW
N 376	May 10.8	7 08	175	SE
N 380	May 14.8	7 00	310	NW

TABLE II.—N. G. C. 7009

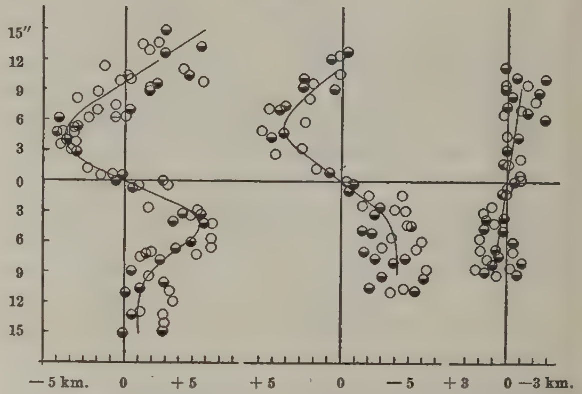
Plate No.	Date G. M. T.	Exposure Time	Pos. Ang. of Slit	Quadrant of "Toward" End of Slit
N 443	1916 Aug. 6.8	6h 10m	79°	NE
N 444	Aug. 7.8	7 00	125	SE
N 451	Aug. 10.8	7 30	169	SE
N 452	Aug. 11.8	7 30	214	SW
N 457	Aug. 17.8	7 30	102	SE
N 458	Aug. 18.8	7 30	192	SW
N 461	Aug. 20.8	7 30	79	NE
N 462	Aug. 21.8	7 30	236	SW

The spectrograms were measured and reduced in the usual manner. The eye-pieces of the microscopes employed contain two mutually perpendicular wires movable parallel to themselves by means of the usual screws. With these it is possible to obtain the rectangular co-ordinates of the measured points in the spectral lines. After applying the usual corrections for curvature, etc., any inclinations or distortions of the spectral lines were confidently assumed to be due to real changes of wave-length arising from conditions existing in the nebula itself.

Fig. 1 contains plots of the reduced measures of three plates of N. G. C. 6543 and three of N. G. C. 7009. The open circles are the results of the measures of the line 4959A, and the half-darkened circles the results on 5007A. As is evident from the figure there is no essential difference in the velocities from the two lines, and this is true of all of the plates measured. The nebular line Hβ appears on many of the plates in suitable density for measurement. The chromatic aberration of the 36-inch refractor, however, causes this radiation to be out of focus on the slit when the other radiations are in focus. Furthermore, Wright has shown[10] that the distribution of hydrogen in these nebulae is different from that of the material producing the 5007A and 4959A radiations. Accordingly, the Hβ line was not measured for relative motion on any of the plates.

Photographs of the two nebulae were taken with the Crossley reflector on Seed 30 plates, using a color screen. This combination transmits only the radiation from 4850A to 5050A, which is practically the region of the green nebular lines. These plates were carefully measured, and from the measures rough outlines of the nebulae were drawn, including the diameters in the various directions occupied by the slit. Then from the plots of the lines the velocities of points at three-second intervals were found relative to the apparent centers of the diameters. Figs. 2 and 3 are the diagrams of N. G. C. 6543 and N. G. C. 7009 obtained in this manner. The numbered lines outside the borders of the nebulae give the positions of the slit for the various plates. The velocities as set down

Fig. 1*a*

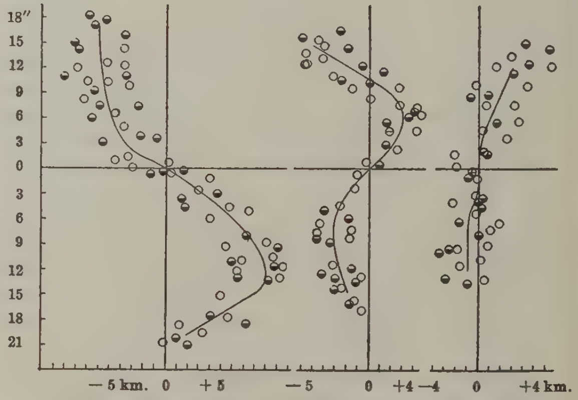


N.G.C. 6543.—N357 and N360; p. a.=25°

N.G.C. 6543—N376; p. a.=175°

N.G.C.6543.—N380; p. a.=310°

Fig. 1*b*



N.G.C. 7009.—N443 and N461; p. a. = 79°

N.G.C. 7009.—N452; p. a. = 214°

N.G.C. 7009.—N451; p. a. = 169°

Fig. 1.—Radial velocity measures of N. G. C. 6543 and N. G. C. 7009

on the diagrams are the means of two independent measures on each line. The probable error of each velocity runs from less than 1 km./sec. near the center of the nebula to 2 km./sec. near the edges. The reasons for the decreasing accuracy toward the edges are, faintness of the outer parts of the nebula and the fact that the slit may not have been uniformly illuminated at these points.

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[10] *Proc. Nat. Acad. Sci.,* 1, 590, 1915.

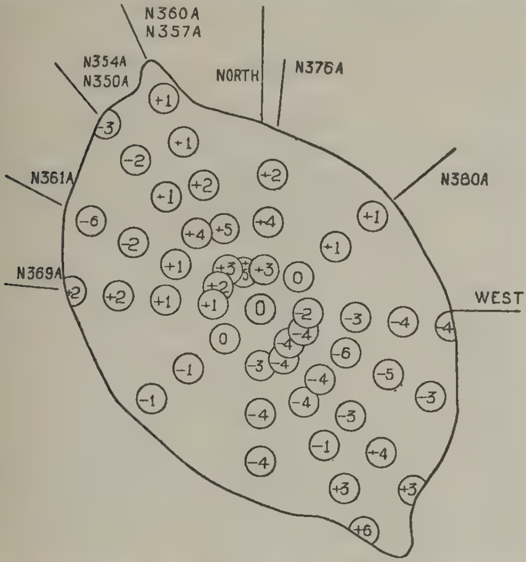

Fig. 2.—Diagram of N. G. C. 6543, with radial velocities in km./sec.

We shall now consider the direct photographs of the nebulae, as obtained with the Crossley reflector.

The image of a nebula as it appears on a photographic plate is projected on a plane, and is therefore without perspective. It is desired to determine, if possible, the shape of these objects in three dimensional space. In the case of a solid body shining by its own light, and hence without shadows, this knowledge would be difficult to obtain from a photographic plate taken from a distant point. In the case of a self-luminous body of gas, however, the problem is not so hopeless, although there are many difficulties to overcome. For the moment we shall consider only the simplest possible case: that of a gaseous body of uniform density and radiating power, shining by its own light, and neither absorbing nor occulting light. In the image of this purely hypothetical body, as projected on a plane, the intensity of the light from any part would be proportional to the equivalent depth of gas at that point. The intensity of the light from any one point in the projected object relative to that from any other may be obtained from the photographic image. We shall waive for the present the difficulties presented by the uncertainty of the law governing the darkening of a photographic emulsion by light, and see how the darkening may be measured.

The apparatus used for this section of the work is the Hartmann microphotometer[11]. The method of measurement requires a set of standard squares impressed on the same plate as the image to be analyzed. Since the data desired in the problem at hand are purely relative, this does not seem necessary. On the

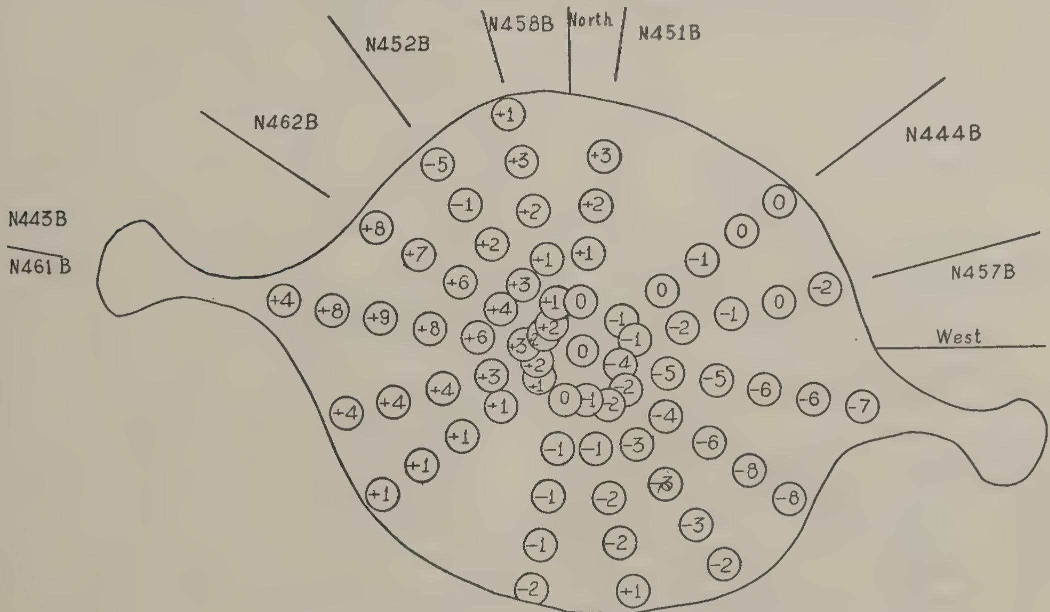

Fig. 3.—Diagram of N. G. C. 7009, with radial velocities in km./sec.

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[11] The general principles of this instrument are described in the *Astrophysical Journal*, 10, 321, 1899.

other hand, in order to obtain any accurate relative values, the secondary wedge must be calibrated. For this purpose, the photometer wedge was tested by means of several "standard squares" plates. A series of careful tests showed that the wedge of the photometer was linear throughout the range used in the measures. This fact greatly facilitates the measurement of any plate. If a point A on the photograph under analysis corresponds to a setting 23 on the wedge, and a point B to one of 27, we can say at once that B is 4 units darker than A. Since the work in this problem is purely relative, the value of these units is a matter of no consequence.

The scale of the Crossley reflector plates of the two nebulae is so small that their images are only slightly over a millimeter in length. This is obviously too small to be used on the photometer, and an enlargement of the plates was necessary. Here we were confronted with the difficulty that enlarging alters the relative darkening of areas on the plate. In order to allow for this difficulty the following procedure was adopted: Six circles about a millimeter in diameter were cut in a plate of brass. This plate was then clamped over a photographic plate of the same grain as the nebular plates, and the circles exposed to the light for various lengths of time. The times were so chosen that the range of densities of the circular images should be approximately the same as in the nebular image. These circles were enlarged to the same scale and with the same exposure time as the nebular images and on the same plate. It seems safe to assume that any changes in the relative densities were the same for the nebular image and the standard circles.

The actual measurement of the plates is a relatively simple matter. The stage which holds the plate on the microphotometer is provided with two mutually perpendicular scales and a graduated circle. By means of these the wedge setting may be found for any point on the nebular image defined by either rectangular or polar co-ordinates. Measurements are made on the standard circles before enlargement and again on the enlargement while measuring the nebula. From the change in relative intensity a curve is constructed by means of which we can reduce the relative intensities of the enlarged nebula image to those existing in the original. Five plates of N. G. C. 6543 and of N. G. C. 7009 have been enlarged and measured in both rectangular and polar co-ordinates.

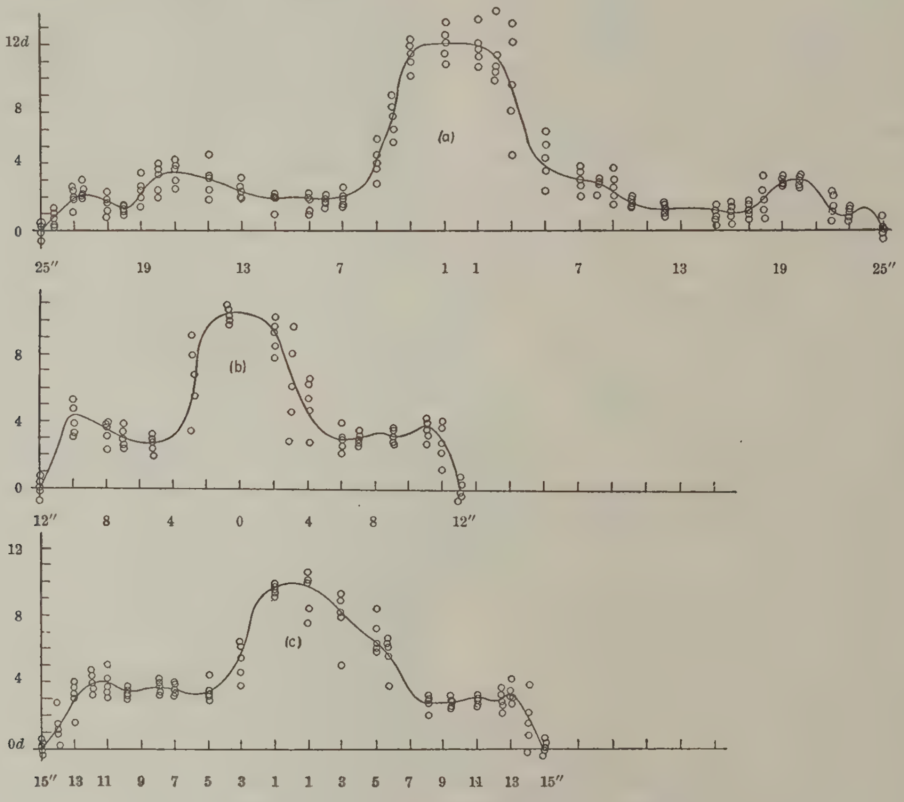

Fig. 4.—Photometric curves of N. G. C. 7009

Table III contains the data for the plates thus measured. All of the photographs were enlarged about five diameters, on lantern slide plates. The last column gives the photometric setting on that portion of the enlarged plate adjacent to the nebular image.

TABLE III

Crossley Plate Number	N. G. C. Number	Emulsion of Crossley Plates	Exposure Time of Original	Photometric Density of Background of Enlargem't
3568	6543	Lantern Slide	3^m	21
3568	6543	Lantern Slide	2	21
3597	6543	Seed 30 and Screen	2	27
3597	6543	Seed 30 and Screen	4	27
3563	6543	Seed 23	3	24
3606	7009	Seed 30 and Screen	4	19
3607	7009	Lantern Slide	3	23
3607	7009	Lantern Slide	2	23
3607	7009	Lantern Slide	4	21
3608	7009	Seed 27	4	20

Diameters of the nebulae in several directions were selected and each set of measures was plotted with distances from the center of the nucleus as abscissae against wedge-readings as ordinates. These curves were then combined and the most probable curve for each diameter on each plate was thus obtained. The curves for corresponding diameters on the various plates were then combined and the resulting curves adopted as final.

Figures 4 and 5 are typical of the curves thus obtained from measures of plates of N. G. C. 7009 and N. G. C. 6543, respectively. In the figures (*a*) is the curve along the nebulae in the direction of the ansae, (*b*) that along the minor axis, and (*c*) along the diagonal in the northwest quadrant of each object.

After the curves were thus graphically obtained, certain critical points were selected and the ordinates of these points on the individual plate-curves read off. These values have been plotted on the curves in Figures 4 and 5. The plotted points are not the original measures, but points taken at selected abscissae on the adjusted plate curves. This accounts for some of the uniformity of distribution of the circles along vertical lines in Figures 4 and 5.

The curves are far from definite, and too much dependence should not be placed on them. Each setting of the comparison wedge gives a value of the amount of darkening of the photographic emulsion for a relatively large area of the nebula, even on the enlarged plate. At the points where the darkening is varying most rapidly the settings are exceedingly difficult, owing to the fact that the section of the nebula appearing in the eye-piece is non-uniform. Even by slightly shifting the plate and making a number of settings in this vicinity a high order of accuracy is unattainable. It should be remembered that such measures must be dealt with as the average effect of a relatively large area of the object. There is apparently no method at hand for increasing the accuracy of the measures of these plates. Greater accuracy could probably be attained by photographing with an instrument of much greater focal length, or possibly by measurement with a Koch microphotometer. A greater enlargement of the Crossley plates than that employed is impracticable, owing to the effect of the grain of the emulsion.

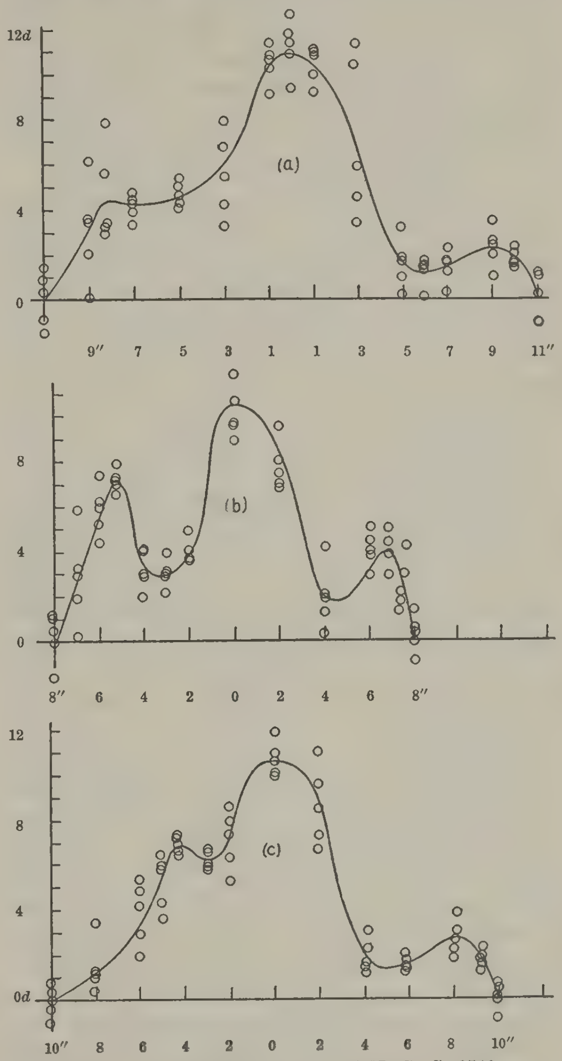

Fig. 5.—Photometric curves of N. G. C. 6543

In the photographs the darkening of the emulsion is due to light radiating from all of the materials in the nebulae. Some of these radiations are much more active photographically than others, so that the actual amount of material producing the radiation is difficult to estimate. To allow for this effect, as far as possible, plates of the nebulae were taken on different kinds of emulsion, and both with and without color screens. By means of these various combinations direct photographs in several restricted parts of the spectrum were obtained. The Seed 30 plates with the color screen (see Table III) give photographs in the region from approximately 4800A to 5200A, the lantern slide plates in the region to the violet of 4800A, and the other plates in the general photographic region. Within the accuracy of measurement the intensity curves were substantially the same regardless of the part of the spectrum employed.

Both of the nebulae have nuclei intrinsically brighter than the surrounding materials. As the exposure time on the photographs is increased to record the fainter portions of the nebular structure, the diameter of the apparent disk of the nucleus increases. This bright disk prevents any determination of the shape of the intensity curve at the center of the nebular image.

Finally, there appears the difficulty that the darkening of the photographic emulsion is not directly proportional to the intensity of the light causing it. There have been various laws advanced for the expression of this relation, and within certain limits of darkening they all lead to essentially the same results. Since the nebular plates seem to fall within these limits, it is possible to compute, with sufficient accuracy for the purpose at hand, the intensity of the light from one section of the nebula relative to that from any other.

The short exposures on planetary nebulae give the impression that many of them are elliptical rings. The lengthening of the exposure time transforms these to approximately elliptical disks. The researches of MacLaurin, Jacobi, Poincaré, Darwin, Jeans, and others have shown that these forms are not the most probable. It seems that a truly three-dimensional figure, such as an ellipsoid, spheroid, ellipsoidal shell, or some other such form is the true one. Assuming that they are composed of gaseous material, we have deduced from a number of these forms the intensity curves of the radiation from them, assuming that they are either self-luminous, or that they shine by reflected light emanating from a very bright nucleus. The conclusion that they are gaseous seems justified: the spectrum of the entire nebulosity is almost wholly the bright-line, or gaseous, type. The nuclei give strong continuous spectra, and Wright has shown quite conclusively, from the presence of absorption lines, that they are shining through a gaseous material. The existence of the bright-line spectra seems to exclude the possibility of the nebula shining by reflected light from the nucleus. The spectrum of the latter is continuous and the resonance radiation from a "white light" excitation is continuous or banded.

On the assumption that they are self-luminous and non-absorbing bodies of gas, the intensity curves that several different forms would produce have been plotted. These curves may be modified by imposing the condition of absorption in the gas, as well as radiation. The laws governing the absorption and emission of light from a body of gas are not definitely known, but for the purpose at hand the law derived by Nichols and Merritt[12]: $I=\frac{\alpha}{\beta}\left(1-e^{-\beta x}\right)$, where α is the coefficient of emission, β the coefficient of absorption, and x the thickness of the gas, is sufficiently good.

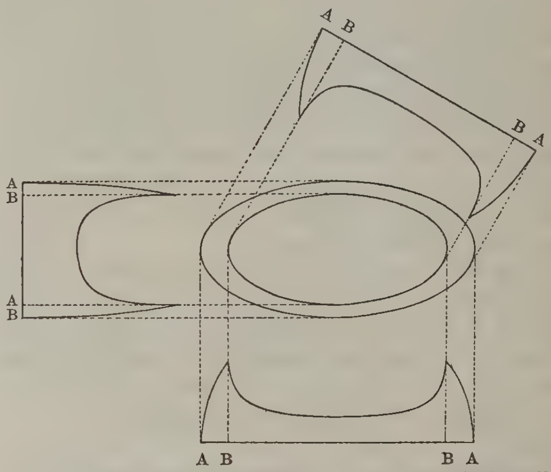

Fig. 6.—Theoretical intensity curves of ellipsoidal shell of gas

Figure 6 is the type of curve produced by an ellipsoidal shell of non-absorbing gas. These curves may be rapidly computed by the formula:

$$Y=2\left\{b_1\sqrt{1-\frac{x^2}{a_1^2}}-b_2\sqrt{1-\frac{x^2}{a_2^2}}\right\}$$

in which Y is the depth of gas, b_1, b_2 and a_1, a_2 are the semi-minor and semi-major axes of the ellipses bounding the shell, and x is the distance along the major axis.

The rough curves (Figs. 4 and 5) as obtained from the photometric data were compared with the various computed curves. The inaccuracy of the photometric observations is such that perfect agreement could not be expected. The curves produced by shells of gas (c. f. Figure 6) are the nearest approach to those obtained photometrically; but, as is evident from the figures, there are several flagrant discrepancies. The sections of the curve from A to B (Figure 6) are what we are most interested in, and it is at just such points that the photometric measures are the least accurate, by reason of the rapid change in intensity. Along the apparent minor axis of the ellipse the agreement is satisfactory. When we turn to the major axis, however, the photometric curve in the region A B disagrees and seems to be concave downward instead of upward. There are also several isolated points in each nebula where the photometric curve increases very rapidly, as though there were secondary nuclei at these points.

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[12] *Studies in Luminescence* (publ. by The Carnegie Institution of Washington), pp. 34 ff.

Turning to the relative velocity diagrams in Figures 2 and 3 we notice that the central portions of the nebulae give direct evidence of a body in rotation about the shorter axis. As we pass out along the major axis this rotation seems to break down and the outer portions of the object in these directions seem to be rotating in the opposite direction from the center. There are also several points toward the edges of the nebulae where the relative velocities seem to violate any regular law. It has been pointed out above that the accuracy of the measurement in these regions of the nebulae is much less than toward the center. The irregularities of the observed velocities, however, are in excess of any probable error which it would be logical to assign.

Recently Campbell and Moore[13] have found that the central sections of the spectrum lines in nebulae of the Harvard Pe type are doubled. The nebula N. G. C. 6543 is of the Pd type and it was deemed advisable to test it for evidence of this effect. Accordingly, an exposure of seventeen hours was made with the spectrograph described above, the slit being very narrow and set along the major axis of the nebula. The resulting plate shows traces of abnormal brightening on the violet edges of the lines at one end and on the red edges at the other. There is no doubling apparent, however, and the measurement of the general inclinations check with that previously found.

Both the spectrographic and photometric study of these two nebulae along the minor axes seem to point to a rotating ellipsoidal shell as a possible form. Along the major axes this theory seems to fail in both cases. In both nebulae there are faint ansae extending out beyond the main nebula in directions approximating those of the major axes. It is possible that if the shell were not in rotation the attractive forces from a condensed nucleus might counterbalance the repulsive forces (light-pressure, diffusion of the gas, etc.) in such a manner as to hold the shell in equilibrium. As the shell rotated this equilibrium would be destroyed, and the material might be driven out along the lines perpendicular to the axis of rotation. The researches of Darwin, Jeans, and others referred to above have dealt with problems of this sort, but their results are not conclusive as to just what the final figure would be. An effect of this kind might account for the failure of the photometric curves to fit the intensity curves produced by any simple geometrical theory, but it would offer no explanation of the apparent reversal of direction of rotation at the outer ends of the major axes.

An attempt has been made to account for this reversal on the hypothesis that the nebulae are made up of various shells of gases, rotating in different directions and each about its own axis. The axes of the various shells may not coincide. On this hypothesis we shall consider the result if the main body of the nebula were an ellipsoidal shell and that this is encircled in the plane of the major axis by a shell of material rotating in the opposite direction. If we further assume that the eccentricities of this shell are greater than those of the main ellipsoid and that it is intrinsically considerably fainter, any effect of its velocity would be masked in the region where we are viewing the main body through it. When we passed out beyond the edge of the main ellipsoid the velocity of the belt would become effective and the line would suddenly ''hook back'' in the reversed direction from the slope it had maintained up to this point. This is very similar to the effect observed in the direction of the ansae in N. G. C. 7009. In N. G. C. 6543 this effect is also evident, but to a lesser degree, and it seems to persist over a larger area in the nebula. This might be explained on the hypothesis that the principal plane of the encircling shell is inclined to the line of sight. The possible inclination is further supported by the appearance of the ansae which, instead of being sharp as in N. G. C. 7009, are rounded. The abnormal velocities in the outer portions of the nebulae proper might be explained by other bands of material with their axes shorter than those in corresponding directions in the main ellipsoidal shell. Such bands might produce the brighter spots appearing in the shorter exposure-time photographs.

The hypothesis of an ellipsoidal shell with rotating rings of material might also explain some of the irregularities in the photometric curve. Figure 7 shows the projection of an ellipsoidal shell, and of a surrounding ring which is made up of material possessing half the radiating power of the shell, both being concentric and the material of which they are constructed being non-absorbing and non-occulting of light. Even on the enlarged Crossley plates the scale is so small that the entire region of the curve from B to C would be in view in the photometer at one setting and any readings to get the true shape of the curve would be virtually impossible.

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[13] *Publ. A. S. P.*, 29, 133, 1917.

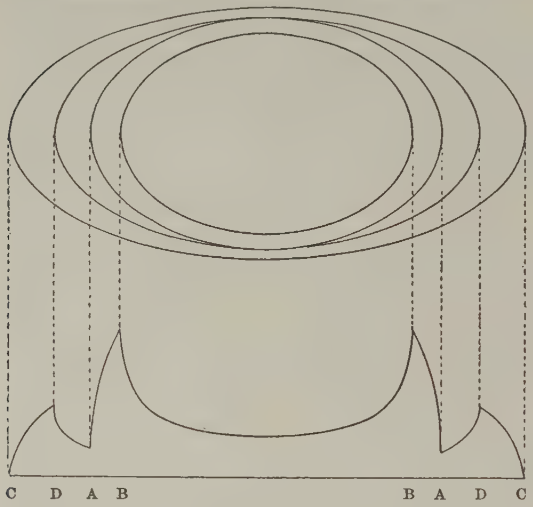

Fig. 7.—Theoretical intensity curve of ellipsoidal shell of gas, with encircling band of gas

Along the axis in the direction of the ansae in N. G. C. 7009 the intensity of the radiation from the nebula seems to be reduced (see (*a*) and (*b*) Fig. 5). This may be explained on the hypothesis that the material of which the outer ring is composed, is more absorptive of light than the material in the main shell. In that case the main characteristics of the curve (Fig. 7) would not be materially altered, but the curve would be generally flattened out.

It is difficult to understand how any such system of construction for nebulae could come into existence, or to predict its future development. It would undoubtedly be in unstable equilibrium and it is possible that the two nebulae, of different types in the Harvard Classification of Nebulae[14], represent different stages of development. There is also the possibility that the points of peculiar brightness and velocity may be produced by abnormal pressures, temperatures or other physical qualities of the gas itself. In the present state of our knowledge of the effects of such conditions on the wave-length of the produced radiation it is futile to attempt any hypothesis on these grounds.

This hypothesis for the construction of nebulae, namely: a foundation of a rotating ellipsoidal shell, together with secondary bands or shells rotating about various axes, is far from definite or conclusive. It is advanced as the only apparent explanation of the relative velocities and brightnesses existing in the planetary nebulae N. G. C. 6543 and N. G. C. 7009.

For their criticism and advice during the prosecution of this work the author is deeply grateful to Director Campbell, Dr. Moore and other members of the staff of the Lick Observatory.

W. K. Green.

Mt. Hamilton,
June 23, 1917.

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[14] *H. A.*, 76, No. 3.

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ASTRONOMY

LICK OBSERVATORY BULLETIN

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PARALLAX OF THE RING NEBULA IN LYRA

This study of the parallax and proper motion of the central star of the Ring Nebula in *Lyra* is based upon the measures of eighty-eight selected plates made at Mount Hamilton with the Crossley reflector. Of these plates fourteen constitute a special series for the investigation of the effect of atmospheric dispersion, and the other seventy-four are parallax plates. They are grouped as follows:

1906 Fall	11 plates	1913 Spring	9 plates
1907 Spring	6 plates	1915 Spring	14 plates
1910 Fall	8 plates	1916 Spring	8 plates
1911 Spring	7 plates	1916 Fall	11 plates
1911 Summer	14 plates (Dispersion series)		

The 1906 and 1907 plates were made by myself with the coöperation of Dr. Perrine, who was in charge of the Crossley telescope at that time. The remaining plates were made by Dr. Curtis after my departure from the mountain. With the exception of the 1907 series and part of the dispersion series Seed 23 plates were used with ten minute exposures. In the other cases Seed 27 plates were used with two minute exposures. All plates were made within half an hour of the meridian except the dispersion series and the 1906 and 1907 groups. The 1906 group was made at an average positive hour angle of 2 hours 51 minutes and the 1907 group at an average negative hour angle of 1 hour 28 minutes. No poor plates were included in the list to be measured and the plates have received equal weight in the solution of the parallax equations.

Nine comparison stars were used. Their coördinates measured in millimeters on the plate from an arbitrarily chosen origin are as follows. (The $+X$ and $+Y$ coördinates correspond to $+\alpha$ and $+\delta$ respectively):

TABLE II

Star	X	Y
Neb. star	—1.02	+0.04
1	+0.54	+0.07
2	—2.38	+1.26
3	—4.59	+3.11
4	—5.26	+4.46
5	+7.40	—2.37
6	+1.52	—1.46
7	+3.21	—4.26
8	—3.90	—6.02
9	+3.45	+5.22

With the exception of number one these are of about the same magnitude as the nebula star. Comparison star number one is the 12½ magnitude stars about 1 minute of arc following the nebula star, designated by Hall,[1] Barnard, and others by the letter ***a***. Of the eight other comparison stars six are somewhat brighter and two fainter than the nebula star. Arranged in order of brightness they stand as follows:

1, 7, 8, 4, [3, 9, 5, neb., 6,] 2,

those within the brackets being nearly equal.

MEASUREMENT

The plates were measured by me in the Repsold measuring apparatus of the University of Minnesota. This apparatus is built to measure plates of the size used in the astrographic programme (about 6¼ inches square) and special provision had to be made for the insertion of plates of this series (size 3¼ × 4¼ inches). A brass plate with a rectangular hole 3¾ × 4¾ inches in its center was used as a frame to hold the Crossley plates. The plates were adjusted so that all were measured with the same portion of the scale and micrometer screw. It was found possible to make this adjustment in a few minutes, and with surprising accuracy. The observed coördinates seldom differ from those upon the plate first measured by more than .01 mm.

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[1] *Astr. Nach.*, **92**, 27, 1878.

Rectangular coördinates were measured. The positive X coördinate was taken in the direction of increasing right ascension, and the positive Y coördinate in the direction of increasing declination. The plate first measured (plate 35) was oriented with the help of the position angle of the line: nebula star — star 1, as given by Professor Barnard.[2] The graduated circle of the measuring apparatus was used to measure the angle (1° 14′) between the X direction and the line joining these two stars. The other plates were oriented so that the scale and micrometer screw readings for the nebula star and an outlying comparison star coincided very closely with the readings obtained in measuring plate 35.

The X coördinates only were measured in the case of most of the plates. These were measured in the direct and reverse positions, to eliminate personal equation in setting upon images of different size and density.

Ten revolutions of the micrometer are equivalent to one millimeter of the scale. The micrometer is graduated into one hundred divisions and readings were made to one-tenth of a division. One unit of the observed reading is therefore equivalent to one ten-thousandth millimeter. This is the unit in which most of the reductions and discussions are made. The scale of the plates is such that one millimeter corresponds to thirty-nine seconds of arc, and one unit of the last place of the reduction is therefore equivalent to 0″.0039.

The intention at first was to measure X coördinates only, but in the search for the source of certain discrepancies the Y coördinates were measured on a limited number of plates. The measures of the Y coördinates of a plate are independent of the measures of the X coördinates. In most cases the plate was removed from the measuring apparatus between the measurement of the X coördinates and that of the Y coördinates. The orientation of the first plate whose Y coördinates were measured (plate 34), was obtained by turning it 90° from the X position. The other plates were oriented by comparison with this one.

Reduction

The measured coördinates were referred to a common origin and a standard orientation and scale value. The origin was taken at the centroid of the comparison stars, and the measured coördinates of each plate were first referred to this origin by subtracting from each measured X coördinate the mean of the measured X coördinates (for that plate) of the comparison stars. Corrections for orientation and scale value (including of course the linear terms of refraction and aberration) were then made by means of a least squares solution for each plate. To reduce the effect of accidental errors no single plate was taken as the standard, but the averages of the measured X coördinates of the twelve plates first measured were taken as standard X coördinates. Errors of run were observed but not corrected for, since it was found that they did not affect the measured position of the central star. Division error of the scale was eliminated by locating all plates in the measuring apparatus so that each star was always measured with reference to the same division line of the scale.

I have not used the elegant "dependency" method proposed by Professor Schlesinger in his series of parallax papers[3] which have appeared since this work was begun. This method is shorter and, aside from the "rounding off of the dependencies," entirely rigorous, but it does not yield the residuals, which I thought ought to be determined in this case. (See discussion of accidental error.) A most striking fact brought out by the dependency method is that some comparison stars may be of so little use that they should be omitted, even though the number of available stars is very limited.[4] This cannot occur, however, when the parallax star is near the centroid of the comparison star, since the dependencies all reduce to the term $\frac{1}{n}$ when X_π and Y_π are zero. This situation is very well approximated in the present case, the coördinates of the nebula star being $X_\pi = -1.02$ mm., $Y_\pi = +0.03$ mm. The dependencies of these comparison stars are as follows:

Star 1	+0.107	Star 4	+0.140	Star 7	+0.095
2	+0.126	5	+0.060	8	+0.150
3	+0.139	6	+0.102	9	+0.077

It is interesting to note also that a star which is of no value as a member of a certain group of comparison stars may become valuable by the addition of one or more comparison stars so situated as to bring the centroid of the whole group near to the parallax star.

Atmospheric Dispersion

The first two groups of plates were taken at considerable hour angles, the Spring and Fall series being made upon opposite sides of the meridian. All plates of the later parallax groups were made within approximately one half hour of the meridian. Measures of the early plates brought out discrepancies in the position of the nebula star which seemed to be due to atmospheric dispersion. A series of exposures was made for the investigation of this effect. They consist of evening and morning plates made on two nights (July 24th and 26th, 1911) at considerable hour angles. The fourteen plates selected for measurement are numbered between 2516 and 2534 inclusive. Of these, six are Seed 27 plates and the other eight are Seed 23.

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[2] *Mon. Not. R. A. S.*, 60, 248, 1900.

[3] *Ap. Jour.*, 32, 33, 1910, 1911.

[4] See *Ap. Jour.*, 33, 353, 1911, for a case in which one of the four available comparison stars is useless.

The equations of condition are derived as follows: If $\Delta\mu$ represent the difference between the refraction coefficient for the nebula star and that for the comparison stars the effect of dispersion will be to cause a relative displacement of the nebula star toward the zenith of an amount $\Delta\mu \tan Z$ where Z is the zenith distance of the nebula star, i.e., the center of the plate.

If the nebula star is at the center of the plate

$$\tan Z = \sqrt{X^2 + Y^2},$$

where X and Y are the coördinates of the projection of the zenith on the plate. The X component of the displacement of the image of the nebula star is

$$\Delta x = \Delta\mu \tan Z \cos\theta = \Delta\mu \sqrt{X^2 + Y^2} \times \frac{X}{\sqrt{X^2+Y^2}} = \Delta\mu X$$

where θ is the angle between the X axis and the line joining the center of the plate with the projection of the zenith. The values of X and Y are expressed in terms of the hour angles of the center of the plate by the following relations given by Professor Turner:[5]

$$X = \frac{\tan(S-A)\sin q}{\cos(P-q)}$$

$$Y = \tan(P-q)$$

$$\tan q = \tan\lambda \cos(S-A).$$

Where P = north polar distance of center of plate = $57^{\circ}\!.1$,

$S - A$ = hour angle of center of plate,

λ = co-latitude of Lick Observatory = $52^{\circ}\!.7$.

Similarly the Y component of the displacement due to dispersion is

$$\Delta Y = \Delta\mu Y.$$

The equations of conditions for the X coördinates are written

$$\Delta\mu X + (\xi_n)_d = \xi_n.$$

The expression $(\xi_n)_d$ represents the value of ξ_n corrected for atmospheric dispersion. The equations for the displacement in the Y direction are not used since there is little variation in the coefficients of $\Delta\mu$. For convenience in the numerical work I have added 1.0200 to each side and write

$$(\xi_n)_d + 1.0200 = \xi_d$$

$$\xi_n + 1.0200 = n.$$

The numerical equations with their solutions follow, .01 mm. = unit:

SEED 23 PLATES

Plate				v
2517	$-0.451\Delta\mu$	$+1.\ \xi_d$	$=-0.44$	-0.09
2518	$-\ .403$	$+1.$	$=-\ .55$	$+\ .14$
2522	$+\ .884$	$+1.$	$=+\ .41$	$-\ .05$
2524	$+1.017$	$+1.$	$=+\ .57$	$-\ .12$
2527	$-\ .392$	$+1.$	$=-\ .41$	$-\ .08$
2528	$-\ .366$	$+1.$	$=-\ .49$	$.00$
2532	$+\ .832$	$+1.$	$=+\ .27$	$+\ .06$
2533	$+0.858$	$+1.$	$=+0.24$	$+0.11$

$$\Delta\mu = +0.673 \pm 0.038 \text{p.e.}$$

$$\xi_d = -0.228 \pm 0.027$$

SEED 27 PLATES

Plate				v
2516	$-0.500\Delta\mu'$	$+1.\ \xi'_d$	$=-0.65$	$+0.10$
2521	$+\ .840$	$+1.$	$=+\ .34$	$+\ .08$
2523	$+\ .966$	$+1.$	$=+\ .70$	$-\ .19$
2525	$-\ .456$	$+1.$	$=-\ .60$	$+\ .08$
2526	$-\ .439$	$+1.$	$=-\ .34$	$-\ .17$
2534	$+0.955$	$+1.$	$=+0.36$	$+0.14$

$$\Delta\mu' = +0.726 \pm 0.065 \text{ p.e.}$$

$$\xi'_d = -0.197 \pm 0.047$$

Upon the basis of these solutions corrections were applied to the coördinates ξ_n found from the parallax plates. The correction to be added in units of 0001 mm. is

$-67.3\,X$ for Seed 23 plates,

$-72.6\,X$ for Seed 27 plates,

where X represents the X coördinate of the zenith at middle of the exposure time.

The difference in effective wave length corresponding to these values of $\Delta\mu$ is obtained as follows:

The ordinary formula for refraction is

$$\frac{\sin i}{\sin r} = n,$$

where n = index of refraction,

i = angle of incidence,

r = angle of refraction.

Differentiating this formula with respect to r and n we have

$$r = -\frac{\sin r}{\sin i} \times \frac{\sin r}{\cos r}\Delta_n$$

$$= -\Delta n \tan Z$$

if we allow the approximations $\frac{\sin r}{\sin i} = 1$ and $\tan r = \tan i = \tan Z$. The quantity Δ_n is therefore, except for the sign and the units in which it is expressed, identical with the quantity $\Delta\mu$ or $\Delta\mu'$ just determined. The former is expressed in radians and the latter are expressed in hundredths of a millimeter on the plate. The relation between them is approximately.

$$\Delta_n = -\frac{40}{100 \times 206000} = -1.94 \times 10^{-6}\,\Delta\mu.$$

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[5] *Mon. Not.*, 54, 19, 1893.

A value of n in terms of the wave length λ is given by Kayser and Runge[6] as follows:

$$n = 1.0002879 + \frac{0.00000132}{\lambda^2} + \frac{0.000000032}{\lambda^4}$$

(λ is expressed in units of .001 mm.).

Differentiating this with respect to n and λ we have

$$\Delta\lambda = -\frac{\lambda^5 \Delta n}{0.00000264\lambda^2 + 0.000000128}$$
$$= +\frac{1.94\lambda^5 \Delta\mu}{2.64\lambda^2 + .128}$$

As a mean value of λ for insertion in the second member we may take

$$\lambda = 0.400.$$

Burns[7] finds that most of the light of the central star lies between $\lambda = .330$ and $\lambda = .370$, and Bergstrand[8] states that the photographic refraction constant is computed for $\lambda = 0.430$. In taking $\lambda = .400$ for the computation of $\Delta\lambda$ we are therefore certainly not far wrong. This gives for Seed 23 plates

$$\Delta\lambda = 0.025.$$

The value of $\Delta\mu$ determined above is therefore conservative in view of Burns's results just quoted.

That so small a difference in effective wave length should produce so considerable a displacement (the range is over 0″.2 at very moderate zenith distances) emphasizes the importance of careful attention in the matter of atmospheric dispersion in all parallax work.

PARALLAX EQUATIONS

After the introduction of the dispersion correction the parallax equations were set up in the usual way. They are of the form

$$\xi_\pi + b\pi + c\mu = (\xi_n)_d.$$

Where ξ_π represents the X coördinate of the central star corrected for parallax and proper motion and $(\xi_n)_d$ represents the coördinate of the central star after all other corrections, including the correction for dispersion, have been applied.

For convenience we add 1.0220 to each side and set

$$\xi_\pi + 1.0220 = \xi$$
$$(\xi_n)_d + 1.0220 = n$$

and the equation becomes

$$\xi + b\pi + c\mu = n.$$

The seventy-five equations of conditions so obtained give the following values of the unknown quantities reduced to seconds of arc:

$$\pi = +0''.015 \pm 0''.007 \text{ p.e.}$$
$$\mu = +0.0055 \pm 0.0014 \text{ p.e.}$$
$$\xi = -0.018$$

The probable error of one observed position is ±0″.042.

VALUE OF THE PARALLAX

That the result obtained above represents a real parallax seems unlikely. This is suggested first by the fact that the value obtained for the parallax is only about twice the size of its probable error, and this probable error is large in view of the large number and high quality of the plates employed. A clearer light is shed upon the situation by the group means of the coördinates of the central star. These are as follows:

TABLE III

Date	No. Plates	*X* coörd.	p.e.
Oct., 1906	11	—39″.88	0″.011
April, 1907	6	39.83	.013
Sept., 1910	8	39.89	.007
May, 1911	7	39.88	.010
July, 1911	14	39.87	.009
May–June, 1913	9	39.81	.014
June, 1915	14	39.83	.012
May, 1916	8	39.81	.022
Sept., 1916	11	—39.82	0.014

Except for the April, 1907 series the measured *X* coördinates of the central star remained sensibly constant from Oct., 1906, until July, 1911, and again from May, 1913, until Sept., 1916. Between these intervals there is a change of about 0″.06. The April, 1907, series of plates is peculiar in several respects. The photographs were made on Seed 27 plates with two minute exposures; instead of the usual Seed 23 plates, with ten minute exposures. They were made at considerable hour angles and consequently required considerable correction for dispersion. Finally the exposures were made by myself practically without previous experience with the Crossley reflector. It is perhaps reasonable therefore to assume that one or more of these circumstances resulted in a changed value of the coördinate for this group, and that some change in instrument or manipulation resulted in a permanent change after July, 1911. Possibly the same cause produced both effects.

This theory was advanced as soon as the measures of the 1913 series were reduced. It had been the intention to close the parallax investigation with that series. After consultation, however, it was decided to test the matter by adding one or two further series of plates made with special precautions to secure uniform instrumental conditions. The three series of plates secured since show variations within the range indicated as accidental by the probable errors. I am therefore inclined to believe that the value of the parallax resulting from the least squares solution does not represent an actual parallactic displacement of the image. That the parallax does not exceed two hundredths of a second seems, however, highly probable.

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[6] *An. d. Phys.*, 50, 312, 1893.

[7] *Lick Obs. Bull.*, 6, 92, 1911.

[8] *Astr. Nach.*, 167, 240, 1905.

PROPER MOTION IN DECLINATION

The Y coördinate of the central star was measured on twenty-three plates in four groups. The resulting group means are as follows:

Date	No. Plates	Y coörd.	p.e.
Oct., 1906	11	+1″30	0″004
April, 1907	6	1.35	.012
May and June, 1911	4	1.29	.009
June, 1913	4	+1.35	0.025

The proper motion in declination is probably less than one second of arc per century.

ACCIDENTAL ERROR

Residuals resulting from the least squares solution for each plate furnish information regarding the accidental error. There were nine comparison stars, giving nine equations with two unknowns. After the completion of the reduction of the 1913 Spring series of plates these residuals were collected and discussed as follows:

The total number of residuals is 720. The average X residual is 10.4 units of the last decimal place. This residual is equivalent to four hundredths of a second of arc. The average Y residual is 8.5 units of the last decimal place. There are fifteen residuals larger than 30 units (=″12). A comparison of the residuals with the estimate of the plate on the basis of the appearance of the images gives the following results (for the measure in the X coördinate):

20 "fine" plates have an average residual of 9.8 = ″038.

14 "good" plates have an average residual of 9.9 = ″039.

9 "fair" plates have an average residual of 10.2 = ″040.

11 "poor" plates have an average residual of 12.3 = ″048.

For 41 Seed 23 plates the average residual in the X coördinate is 9.9 = ″039, and for 14 Seed 27 plates the average X residual is 12.1 = ″047.

Investigation of probable error has been made as follows:

The sums of the squares of the residuals have been built up for every fifth plate. This gives the following values of $.67\sqrt{\frac{[vv]}{7}}$, which is the probable error of one observation of a comparison star:

Plate			
2	6.7	=	0″026
15	7.4		.029
32	7.0		.027
43	10.7		.042
2364	12.8		.050
2480	7.9		.031
2486	5.7		.026
2521	18.0		.070
2526	6.6		.026
2534	7.2		.028
2787	10.0		.039
Average	9.2	=	0″036

For the probable error of an observation of the nebula star we have the result of the least squares solution for parallax, namely, 10.9 units (0″042). In further consideration of this point compare probable errors from the group means, Table III.

The actual discrepancies of the group means are obtained by correcting them for the parallax and proper motion determined above and comparing them with the value determined for ξ_π as follows:

Date	Coörd.	v	p.e.
Oct., 1906	—1.0228	+ 1	2.8
April, 1907	.0220	+ 8	3.4
Sept., 1910	.0222	— 7	1.7
May, 1911	.0215	—11	2.5
July, 1911	.0218	— 5	2.2
May–June, 1913	.0213	+ 5	3.5
June, 1915	.0210	— 2	3.0
May, 1916	.0208	0	5.6
Sept., 1916	.0214	— 3	3.5

This table shows evidence of systematic error. The probability of the accidental occurrence of an error $\frac{9}{2}$ times as large as the probable error is only two in a thousand.[9]

This study of residuals seems to indicate that accidental errors affect the measured position of an image on a Crossley plate by less than four hundredths of a second on the average.

SYSTEMATIC ERROR

That one or more of these groups of plates is affected by systematic error seems altogether probable. Possible sources of error have been considered as follows:

1. *The Sky.* Under this head come errors due to conditions of the atmosphere, the motion of the Earth or the motions of the comparison stars. Refraction and aberration terms of the first order are taken account of in the correction for scale value and orientation. Second order terms have been found to be negligible.

Atmospheric dispersion has been investigated and corrected for. A slight change in the value of this correction would bring the 1906 Fall and 1907 Spring series near coincidence with the mean coördinate: $=1.0217$. The other plates are made so near the meridian that their measures would not be affected by a small change in the dispersion constant. The fact should not be overlooked, however, that the dispersion constant may vary considerably with the varying transparency of the atmosphere for light of short wave length, and possibly with the condition of the reflecting surface.

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[9] *Brünnow's Lehrbuch der sphärischen Astronomie,* ed. 1881, p. 46.

Parallax or proper motion of the comparison stars would of course affect the measures of the nebula star, but the number of comparison stars is not small and any considerable variation in the position of one or more of them does not seem to be indicated by the residuals.

''Poor seeing'' and the consequent difficulty in guiding is undoubtedly responsible for large individual errors. This might combine with a fast or slow rate of the driving clock to produce an error which would be systematic for any given condition of observing. It is conceivable also that a rising or falling current of air of abnormal density might produce lateral distortion of the field photographed. The occurrence of such currents would depend upon the local topography and would undoubtedly have a tendency toward seasonal variation.

The possibility of error due to difference in magnitude between the comparison stars and the parallax star has been considered. Aside from number one the comparison stars are near the nebula star in photographic magnitude. Number six is the faintest of the comparison stars and it is not far distant on the plate from number one, which is considerably brighter than the others. The X residuals of these two stars have been compared for evidence of magnitude error. These residuals have opposite sign thirty-two times out of fifty-five. The average residual being 10.4 units, the algebraic difference of the residuals of any two comparison stars should perhaps exceed $\sqrt{(10.4)^2+(10.4)^2}=14.7$ as often as it is less than that number. In the case of stars one and two the difference of their residuals exceeds 14.7 thirty-five times out of fifty-five. These figures show very little evidence of magnitude equation even in the case of stars widely different in brightness.

2. *The Telescope.* The effect of poor focus is to cause considerable irregularity in the edges of the images. Strings or clumps of silver grains reach out from the main body of the image, and clear lanes or bays invade the image. Images of this sort frequently result in large residuals. The effect is undoubtedly produced by poor seeing, overdevelopment of the plate or fogging by moonlight. I believe that it increases the accidental error to a very considerable extent. The May, 1916, series seems to suffer especially from this difficulty. Several plates of this series were fogged by moonlight. To minimize this effect exposure should be long enough to give strong images and development should not be forced. The Seed 23 plates may have an advantage over the Seed 27 plates in this particular. One effect of having the plate too far away from the mirror is to limit the field of good definition. I have found a great range in the extent of the field of round images.

It has been found that the center of good definition is not quite at the center of the plate, and its location with reference to the center of the plate does not remain altogether constant. Observations to investigate this point were made as follows:

Plates were projected upon a white wall by means of a stereopticon carefully located so that the line of projection was normal to the wall. A discarded plate was first marked at its center and inserted in the stereopticon. The location upon the wall of the mark at the center of the plate was noted. Also, vertical and horizontal scales were laid off upon the wall, the units corresponding to one millimeter on the plate. The location of the center of good definition was then determined as accurately as possible and the location of the nebula star also observed. The indications are that the center of good definition has wandered between three and ten millimeters to the north of the center of the plate and between two millimeters following and five millimeters preceding the center. The nebula star has been generally to the south of the center of the plate, sometimes preceding and sometimes following the center. The nebula star was generally from five to ten millimeters from the center of good definition and comparison stars 5, 7, and 8 frequently showed the effect of distortion on plates that were excellent in other respects.

If the Crossley reflector is to be used for parallax work in the future, I would suggest that means be provided for observing the position of the optical axis, and for causing it to intersect the plate normally at the center, and that care be taken to have the image of the parallax star at the same point. These adjustments should be under control whenever parallax plates are made.

All photographs were made with the telescope west of the polar axis and I presume the mirror is always replaced after silvering without change of orientation.

It is believed that the photographic plate is very nearly perpendicular to the optical axis and that this factor does not vary to any considerable extent. No observations have been made to make sure of this, but the comparison stars are so near the parallax star that moderate variations in tilt of the plate would not introduce appreciable error.

3. *The Photographic Film.* Distortion of the film could scarcely be responsible for systematic error. One plate showing sagging of the gelatin plainly to the naked eye was rejected. Star images were examined by projection with the microscopic attachment of the stereopticon and the actual silver grains were found to be surprisingly small. Apparently the granular appearance at the edge of the photographic image is due to a tendency of underexposed silver grains to blacken in strings or bunches rather than to the size of the individual grains.

4. *The Measuring Apparatus.* The uniformity of treatment of the plates in measurement insures the elimination of division error of the scale. The error of run was observed but was found to have no effect upon the position of the nebula star. To make certain of this the measure of one plate was corrected completely for the largest observed error of run. No change in the coördinate of the central star resulted. Correction for run therefore was not introduced. The run was small but was constant and the residuals for the comparison stars would undoubtedly be slightly improved by making this correction. Several plates were remeasured to test the accuracy of the measurement, with the following results for the coördinate of the nebula star:

	Coörd. of Central Star	Measured	
Plate 2	—1.0207	Dec. 25, 1908	
	—1.0209	Mar. 2, 1909	
	—1.0193	Mar. 3, 1909	Measured by Prof. Leavenworth
	—1.0188	Mar. 12, 1909	
	—1.0201	Mar. 12, 1909	
Plate 32	—1.0180	Dec. 29, 1908	
	—1.0177	July 21, 1909	
Plate 2804	—1.0211	June 26, 1914	
	—1.0209	July 23, 1914	
Plate 35	—1.0223	Dec. 10, 1908	
	—1.0222	July 10, 1914	

In general the second measure does not reproduce the coördinates of the comparison stars as closely as it does the coördinate of the nebula star. This is to be expected, since the programme involves three times as many measures of the nebula star as of any comparison star.

The error of an observed position of the nebula star is clearly due in the main to an actual displacement of the image, i.e., a second measure reproduces the position within a fraction of the probable error.

CONCLUSIONS

The parallax of the central star is probably less than two hundredths of a second of arc. The proper motion is probably less than one second of arc per century.

There is pretty good evidence of the presence of systematic error amounting to four or five hundredths of a second; but four groups of plates taken since July, 1911, with special attention to instrumental adjustment give results within a *range* of two hundredths of a second.

The suitability of the Crossley reflector for parallax work is apparent. The probable error of the position obtained from one plate is 0″.042. Other evidence indicates that the accidental error of the observed position of an image is rather less than 0″.04. It should be remembered that these plates do not form part of a parallax campaign, but were taken in regular course without any special preliminary studies or arrangements.

The probable error of the parallax (0″.007) is very small, and more than ordinarily significant in this case because of the large number (75) of equations of condition. The number of plates is excessively large, considered as a series for parallax only, but this being the first stellar parallax undertaken with a reflecting telescope it was necessary to use ample observational material to guard against unsuspected sources of error. The precision corresponding to a probable error of 0″.042 for a single plate compares favorably, all things considered, with that secured in determinations made with longer focus instruments. In undertaking a parallax programme with the Crossley it would be highly important to look into the sources of the systematic discrepancies noted above. This evidence of systematic variation cannot, however, militate to the disadvantage of the Crossley as compared with other telescopes, in the absence of satisfactory evidence that they are not subject to similar variation. So far as I am aware, no one of the instruments now being used for parallax work has been subjected to a test of its ability to reproduce at successive epochs the position of a star having no sensible parallax. The chief limitation of the Crossley for parallax work is in its restricted field of good definition. Since the light of all wave lengths is brought to one focus the effect of atmospheric dispersion must be eliminated with care.

Finally I wish to acknowledge my obligation to Dr. Campbell for permission to undertake the work with the Crossley reflector and for support in its prosecution, to Dr. Perrine for coöperation at the telescope during the exposure of the 1906 and 1907 plates, and especially to Dr. Curtis for the long series of excellent plates which he made after my departure from the Observatory. Dr. Fath kindly assisted at the telescope during some of the exposures. I am also obliged to Professor F. P. Leavenworth for the use of the measuring apparatus of the University of Minnesota and for coöperation in investigating the effect of personal error and the accuracy of measurement. Mr. Hugh Wilcox made the least squares reductions for the 1916 groups of plates and the final least squares solution for parallax.

BURT L. NEWKIRK.

UNIVERSITY OF MINNESOTA,
August 8, 1917.

Issued October 16, 1917.

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ASTRONOMY

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NUMBER 300

THREE NOVAE IN SPIRAL NEBULAE

The nebula N. G. C. 4527 ($\alpha = 12^{\mathrm{h}} 29^{\mathrm{m}}.0$, $\delta = +3° 12'$; 1900), is a considerably elongated spiral about 5′ in length. In March of the present year, while comparing a Crossley negative of 1915 with one taken in 1901, a star was noticed on the 1915 image, at the edge of the inner and brighter part of this spiral, which was not on the 1901 image. It seemed advisable, before announcing it as a *nova,* to secure any evidence possible from plates taken elsewhere, in order to decide whether or not it might be a variable star instead. Director Pickering has had a search made on the Harvard plates of this region, and Professor Barnard placed at my disposal enlarged positives of the two Yerkes plates made with the Bruce telescope. Combining with the Crossley negatives the data furnished by their kind coöperation, I am enabled to give the following facts as bearing on the history of this nova:

1900 Feb. 6, H. C. O. Draper telescope; limiting magnitude 15; nova not shown.
1901 Mar. 15, L. O. Crossley; limiting magnitude about 18; nova not shown.
1901 May 8, L. O. Crossley; limiting magnitude 19.5; nova not shown.
1905 Feb. 11, H. C. O. Draper telescope; limiting magnitude 15; nova not shown.
1907 Feb. 16, Yerkes 16-inch Bruce; the nebula is 5° from the center of the plate; no trace of nova.
1915 Jan. 13, H. C. O. 1-inch Cooke lens; limiting magnitude 12.5; nova not shown.
1915 Mar. 20, L. O. Crossley. The plate was taken of N. G. C. 4536, but N. G. C. 4527 and the nova are shown near the edge of the plate. Owing to the distance from the center of the plate there is so much distortion that an estimate of the magnitude of the nova at this time is quite uncertain; estimated to be about magnitude 14.
1915 April 16, L. O. Crossley. The nova is about magnitude 15.
1915 June 3, H. C. O. 1-inch Cooke lens; limiting magnitude 12.5; an object is suspected.
1916 May 4, Yerkes 10-inch Bruce; no evidence of nova.
1917 April 18, L. O. Crossley; limiting magnitude 19; no trace of nova.

From this evidence there is almost no chance that the star was an ordinary variable. It evidently appeared shortly before March 20, 1915, and had become a magnitude fainter by April 16; there is not the faintest trace of the nova in the plates of 1901 or 1917, taken with the Crossley reflector.

The nova was situated 8″ north of and 44″ following the nucleus of the spiral. It is indicated in figure 1 by the small arrow. In this figure, the portion at the left is a reproduction, enlarged 5.4 times, of the Crossley plate of May 8, 1901, exposure time $2^{\mathrm{h}} 45^{\mathrm{m}}$; that at the right, on the same scale, was taken April 16, 1915, exposure time 2^{h}.

A comparison of older and more recent Crossley negatives has resulted in the discovery of two more novae in spirals; that both these novae should have appeared in the *same* spiral is especially worthy of note.

The spiral nebula N. G. C. 4321 (Messier 100; $\alpha = 12^{\mathrm{h}} 17^{\mathrm{m}}.9$, $\delta = +16° 23'$; 1900) is a fine, rather open, nearly round spiral about 5′ in diameter. Nova A appeared in this spiral at some time previous to March 17, 1901, 110″ west and 4″ north of the nucleus. Nova B appeared in the outer part of this spiral at some time prior to March 2, 1914, 24″ east and 111″ south of the nucleus. Though Nova B is recorded on only the one plate, there is no doubt that it is a *bona fide* stellar image; it shows the same peculiarities of structure as other stellar images on the same plate.

Data kindly furnished me by Directors Frost, Pickering, and Slipher, when combined with the Crossley negatives, give the following photographic history of these novae:

1896 June 30, H. C. O.; limiting magnitude 15.5; no trace of either nova.
1901 Jan. 16, H. C. O.; limiting magnitude 14.5; no trace of either nova.
1901 Mar. 17, L. O. Crossley; Nova A has appeared; magnitude about 13.5.
1901 April 16, L. O. Crossley; two plates show Nova A as about magnitude 14.5.
1901 April 19, L. O. Crossley; Nova A 14.5 or fainter.
1912 Feb. 12, H. C. O.; limiting magnitude 16.2; no trace of either nova.
1912 Mar. 13, H. C. O.; limiting magnitude 15.5; no trace of either nova.
1913 Feb. 10, H. C. O.; limiting magnitude 15.2; no trace of either nova.
1914 Mar. 2, L. O. Crossley; no trace of Nova A. Nova B has appeared and is about magnitude 14.
1915 April 8, Lowell Obs.; no trace of either nova.
1915 May 3, Lowell Obs.; no trace of either nova.
1917 Aug. 5, L. O. Crossley; no trace of either nova.

The nebula is recorded on plates taken for other purposes with the 10-inch Bruce telescope by Professor Barnard, on 1910 April 8, on five nights in March, 1911, and on 1917 Jan. 1. None of these photographs shows any trace of either nova, but it should be said that on some of the plates the nebular image is distorted because it is far out from the center of the field.

In the case of these two novae, also, the evidence seems reasonably clear that the objects were novae, and not variable stars.

In figure 2, the two novae which appeared in N. G. C. 4321 are indicated by small arrows. The photograph at the left is enlarged five times from a Crossley negative taken on April 19, 1901, exposure time three hours, and shows Nova A; that at the right, on the same scale, was taken on March 2, 1914, exposure time two hours, and shows Nova B.

Including Ritchey's nova in the spiral N. G. C. 6946, six novae have now been discovered in spiral nebulae; four averaged about magnitude fourteen, and two were brighter. Of the latter, one was the well-known nova which appeared in 1885 within a few seconds from the nucleus of the Great Nebula in *Andromeda;* the other, Z *Centauri,* appeared 28″ from the nucleus of the spiral N. G. C. 5253. The *Andromeda* nova and Z *Centauri* disappeared in due time and have not reappeared, though frequently looked for in the succeeding thirty-two and twenty-two years.

That one nova should appear so placed in the sky as to be directly in line with a spiral nebula, while a possibility, is highly improbable; but that six such novae should be so placed transcends the bounds of probability. There can be no doubt that these novae were actually in the spirals. Sufficient evidence has been brought forward to render invalid the hypothesis that they may be variable stars. The color index of Ritchey's nova, moreover (*H. C. O. Bulletin,* No. **642**), shows that this cannot be a long-period variable.

It appears probable that a systematic programme of photographing the spirals at short intervals, say every two years, might bring to light many other novae in spirals. The exposures would be short, only a few minutes, and such a programme could be carried through rapidly. Presumably those thus far found are the brighter examples of novae in spirals, and a larger proportion of fainter ones would be expected.

The occurrence of these novae in spiral nebulae must be regarded as a fact bearing very directly on the theories of the constitution of the spirals. It seems to me that they furnish weighty evidence in favor of the well known "island universe" theory of the spiral nebulae. The distribution of the novae thus far observed, excluding the six found in spirals, shows that they are essentially a galactic phenomenon; in fact, T *Coronae* is the only star amongst those called novae which lies outside the Milky Way, and *T Coronae* was not a typical nova. A limited number of novae—about twenty-six—have been found in our galaxy. Though reasoning by analogy frequently leads to error, the occurrence of objects of the same type in the spirals would reasonably be expected, were these spirals in fact congeries of vast numbers of stars, like our own galaxy.

The entire invisibility of many novae before and after their brief apparition periods makes it possible to assume that they may well have increased in brightness at least sixteen magnitudes. On this rough assumption the novae in spirals would possibly have been of the thirtieth magnitude before their outburst. Stars of the fifteenth magnitude, in our own galaxy, if assumed to be twenty thousand light years away at present, would have a distance of the order of twenty million light years in order to be of the thirtieth magnitude. An external galaxy at this distance, if 10′ in apparent diameter, would have an actual diameter of nearly sixty thousand light years, not an impossible dimension, so far as we may judge from our very imperfect knowledge of the size of our own galaxy. Nebulae 10′ in diameter are the giants of the class; the smaller spirals would have to be ten to one hundred times more remote, unless their dimensions were considerably smaller than our galaxy is believed to be.

There is another way of looking at the subject. The Harvard list[1] of novae contains twenty-eight entries. Adding *Nova Geminorum* No. 2, of 1912, and excluding the nova in the *Andromeda* nebula and Z *Centauri,* the arithmetical mean of the maximum observed

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[1] *H. A.,* **56** (VI), 169.

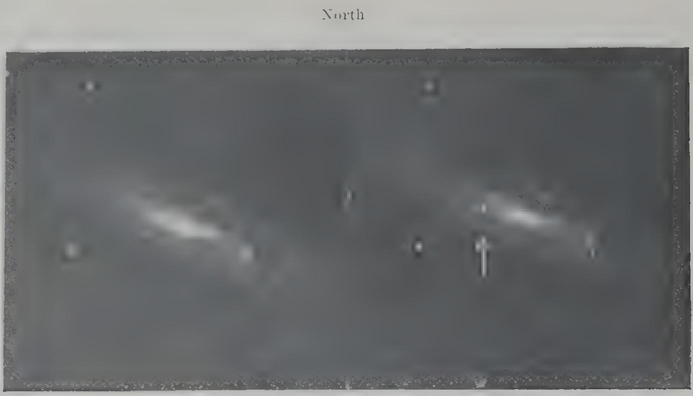

Fig. 1.—The Spiral Nebula N. G. C. 4527

Left: May 8, 1901.
Right: April 16, 1915, showing Nova.

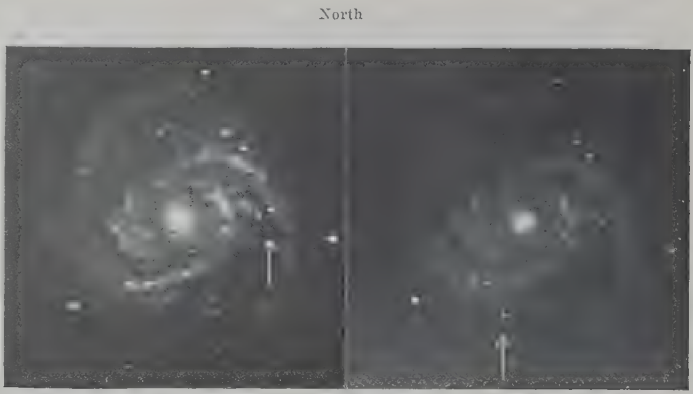

Fig. 2.—The Spiral Nebula N. G. C. 4321

Left: April 19, 1901, showing Nova A.
Right: March 2, 1914, showing Nova B.

magnitudes of the twenty-seven novae is 5½. Some of the observed maximum magnitudes were visual and some were photographic, but the spectra of novae are rich in blue and violet light, and their color equation is therefore small. In several novae the maximum brightness occurred before their discovery, and we shall not be far wrong to assume that the average (arithmetical) maximum brightness of our galactic novae is 5. Presumably the novae already discovered in spiral nebulae are the brighter ones of their class; and there seems to be safety in setting 15 as the average maximum. There is thus an average difference of ten magnitudes between galactic novae and spiral novae. Now all the evidence available assigns a great distance to the galactic novae. If we assume equality of absolute magnitude for galactic and spiral novae, then the latter, being apparently ten magnitudes the fainter, are of the order of one hundred times as far away as the former. That is, the spirals containing the novae are far outside our stellar system; and these particular spirals are undoubtedly, judging from their relatively great angular diameters, the nearer spirals. Of course the effect of any existing absorbing materials in the spirals upon the novae is to reduce their apparent brightness and thus to make them seem farther from our system than they really are.

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U. S. SHIPPING BOARD NAVIGATION SCHOOL,
SAN DIEGO, CALIFORNIA, September 8, 1917.

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ORBIT AND PERTURBATIONS OF (716) *BERKELEY*

The minor planet *Berkeley* provisionally designated as 1911 MD was discovered by Palisa at Vienna on July 30, 1911, and observed by him seventeen times from July 30th to September 23rd, 1911 (*A. N.* 4518 and *A. N.* 4565). The first provisional orbit was computed by F. Hopfner of Triest (*A. N.* 4525). In *A. N.* 4607 elements computed by Stracke are published and the planet is given number (716). These elements are given in the "Bahnelemente und Oppositions-Ephemeriden für 1917" in the *Berliner Jahrbuch* for 1919. The planet proved to be identical with the previously observed planet 1906 UN^{b} discovered by Wolf at Koenigstuhl on July 16, 1906 (*A. N.* 4607). The representation led to the following residuals, $\Delta\alpha = +2^{\mathrm{m}}.4$, $\Delta\delta = +4'$.

In 1912 Dr. Palisa named the asteroid *Berkeley* in honor of the Students' Observatory of the University of California, Berkeley, California (*A. N.* 4612).

The elements by Stracke are based on the observations of 1911 Aug. 3, 29, and Sept. 23 (*A. N.* 4607), the mean daily motion being $754''.565$. The writer computed several orbits of this asteroid by Leuschner's Short Method (*Publications of the Lick Observatory,* Vol. VII, Part VIII), and found that the observation of September 23, 1911, is slightly in error. Due to the present circumstances it was impossible to find out anything definite from the observer about this particular observation. Consequently this observation was left out of any further consideration. An orbit computed by the writer by Leuschner's Short Method based on normal places of the following dates: 1911, July 30.597722, August 22.435397, September 17.408071 B. M. T. represents the various observations more closely than the orbit in the *Jahrbuch*. The elements and constants for the equator are:

ELEMENTS

Epoch and Osculation, 1911 Aug. 22.435397 B.M.T.

$$M = 118^\circ\ 25'\ 49''.7$$

$$\left.\begin{aligned} \omega &= 48\ \ 49\ \ 14.5 \\ \Omega &= 146\ \ 54\ \ 49.3 \\ i &= 8\ \ 23\ \ 37.9 \end{aligned}\right\}\ 1911.0$$

$$\log e = 8.973135$$

$$\log a = 0.448530$$

$$\mu = 753''.72333$$

CONSTANTS FOR THE EQUATOR 1911.0

$$x = r\,[9.998571]\sin(286^\circ\ 01'\ 31''.6 + v)$$

$$y = r\,[9.982335]\sin(197\ \ 22\ \ 51.8 + v)$$

$$z = r\,[9.463909]\sin(180\ \ 31\ \ 47.5 + v)$$

The residuals of the 1906, 1912, and 1914 observations compared with the residuals of the orbit in the *Jahrbuch* are as follows:

Date	Observer, Place		Neubauer	Stracke	Reference
1906 July 16	Wolf, Koenigstuhl	$\Delta\alpha$	$+0^{\mathrm{m}}.5$	$+2^{\mathrm{m}}.4$	*A. N.* 4607
		$\Delta\delta$	$-1'$	$+4'$	
1912 Nov. 4	Palisa, Vienna	$\Delta\alpha$	$+1^{\mathrm{m}}.7$	$-2^{\mathrm{m}}.7$	*A. N.* 4612 two observations
		$\Delta\delta$	$+10'$	$-8'$	
1914 Jan. 22	Metcalf, Winchester	$\Delta\alpha$	$+0^{\mathrm{m}}.8$	$-6^{\mathrm{m}}.5$	*A. N.* 4809
		$\Delta\delta$	$-1'$	$+11'$	
1914 Jan. 25	Neujmin, Simeis	$\Delta\alpha$	$+2^{\mathrm{m}}.1$	$-7^{\mathrm{m}}.6$	*A. N.* 4714
		$\Delta\delta$	$-3'$	$+10'$	

Since the observations of 1906 and 1914 are given only to tenths of minutes of time and minutes of arc in right ascension and declination, respectively, an improvement of the foregoing elements seemed inadvisable. The two observations of 1912 by Palisa were made the same night only one hour apart, but these are the only observations during that opposition. Furthermore, the discrepancy of the two observations of 1914 shows one of them to be in error.

The mean daily motion of the planet is such that the computation of the general perturbations due to *Jupiter* by Hansen's method (*Abhandl. d. K. S. Ges. d. Wiss.* V–VII) is out of the question. The Hansen-Bohlin method as given by Professor Bohlin in his "Formeln und Tafeln zur gruppenweise Berechnung der Allgemeinen Störungen Benachbarter Planeten." Upsala, 1896, and in his "Sur le Développement des Perturbations Planétaires," Stockholm, 1902, is the method chosen here.

Professor Bohlin starts with the differential equations of Hansen and introduces a function θ defined by the equation

$$\mu_0\theta = \mu(\epsilon - e\sin\epsilon) - g' \qquad \text{I}$$

where μ_0 is the rational number of the particular group under which the planet in question comes, g' the mean anomaly of the disturbing body (*Jupiter* in this case). The quantity μ_0 differs from μ the actual ratio of n', the mean daily motion of *Jupiter*, to n, the mean daily motion of the planet, by a small amount.

From the relations

$$g' = n't + n'\delta z',$$

and

$$nz = nt + n\delta z$$

we have

$$\mu_0\theta = (\mu_0 - \mu)nz + \mu n\delta z - n'\delta z', \qquad \text{II}$$

Bohlin defines now the discrepancy between the ratio μ to μ_0 and unity by the symbol w, that is

$$1 - \frac{\mu}{\mu_0} = w$$

a quantity of the utmost importance throughout the developments. Making use of this definition we have

$$\theta = wnz + (1-w)n\delta z - \frac{n'\delta z'}{\mu_0}$$

from which

$$n\delta z = \frac{1}{1-w}\left[\theta - wnz + \frac{n'\delta z'}{\mu_0}\right] \qquad \text{III}$$

Differentiating the equation with respect to ϵ we have

$$\frac{d\theta}{d\epsilon} = [w + (1-w)\overline{W}]\ (1 - e\cos\epsilon) - \frac{dn'\delta z'}{\mu_0 d\epsilon} \qquad \text{IV}$$

Equations III and IV are the backbone of the Bohlin method. θ is now determined from IV and the perturbations are then derived from III.

In developing the perturbative function in powers of the eccentricities and inclinations, Bohlin makes use of the exponential form of the trigonometric parts. The various functions are then developed by Taylor's series in powers of the quantity w, that is, in the form

$$f(T) = f(T_0) + wf(T_1) + w^2 f(T_2) + \ldots,$$

$f(T_0)$ representing coefficients of the particular group as functions of eccentricities and inclinations, $f(T_1)$ and $f(T_2)$ are derived from $f(T_0)$. Another step is then taken by breaking the function θ into two parts, namely,

$$\theta = \theta_1 + (1-w)\zeta$$

where θ_1 is a function independent of ϵ, and ζ takes the place of θ in those terms which contain both ϵ and θ. This of course is similar to the well known expansion due to Lagrange of the type $x = y + \alpha\phi(x)$.

The quantity θ_1 can be derived only by successive approximations. The constant of integration here can be thrown into the successive approximations and since the perturbations of second order are to be neglected in most cases the quantity θ_1 can be replaced by the quantity Θ defined by Bohlin

$$\Theta = \kappa g + G$$

where $\kappa = w$ and G is the constant of integration. Tables for the group 2/5 under which the asteroid Berkeley falls have been published by Professor D. T. Wilson of Cleveland in the *Astronomiska Iakttagelser Och Undersökninger,* Band 10, No. 1, Stockholm. On the basis of these tables the writer has computed the general perturbations of *Berkeley* given below.

In applying the Bohlin method the question arises whether an extremely close approach to commensurability of the planet's mean motion to that of *Jupiter* does introduce errors in the principal terms of an order which would make the perturbations computed unreliable. In *A. N.* 3295, Professor Bohlin makes a statement to the effect that a w as small as ±0.01 would still hold. In the case of *Berkeley,* the w is less than 0.01. This fact introduces questions which are of importance on the practical side of the theory of general perturbations. As in the orbit methods, the final test of the methods consists in the representation of the observations. The same test applies in the various methods of general perturbations. One difficulty in this case is that the observational data give no aid to the theoretical investigation, since the observations are insufficient both in number and accuracy.

The entire analytical development of the Bohlin method is based on the so-called mean elements. There are three distinct ways of deriving the mean elements. One way would be to take the mean values of several sets of good osculating elements distributed over a sufficient number of oppositions. This method, however, practical as it is, is rarely used since several independent sets of elements at the various oppositions have been computed for very few of the asteroids. Another way to get the mean elements from osculating elements is the analytical method given by Hansen in his second Memoir of the *Abhandl. d. K. S. Ges. d. Wiss.* The perturbations derived from osculating elements must satisfy the following conditions at the epoch of the osculation:

$$nz = C_0\ ; \quad \nu = o\ ; \quad S = o$$

$$\frac{dz}{dt} = 1\ ; \quad \frac{d\nu}{dt} = o\ ; \quad \frac{dS}{dt} = o$$

The six arbitrary constants introduced during the development are determined by means of these six equations of condition and are introduced in the elements which thereby become mean elements. The third method makes use of the osculating elements as mean elements in the process of development. On representation of the various oppositions (the number of which must be sufficient to warrant a least square solution) the residuals are then used to correct the osculating elements in order to get mean elements.

The last method seems best adapted to the case of *Berkeley*. It is hoped that the next few oppositions will supply the data needed for a satisfactory numerical determination of the constants, which are merely indicated in the perturbations on account of their uncertainty. The writer has purposely refrained from bringing the tables below under one table at this stage. The arrangement of the arguments is similar to that used by von Zeipel in his development of the $\frac{1}{2}$ group. Wherever the mean elements appear as constants in the tables they have been kept explicit.

The perturbations $n\delta z$, in the mean anomaly, ν, in the radius vector, and s, in the third coördinate, are given by

$$\begin{aligned} n\delta z &= (n\delta z)_1 + (n\delta z)_2 + (n\delta z)_3 + (n\delta z)_4, \\ 2\nu &= 2(\nu)_1 + 2(\nu)_2 + 2(\nu)_3 + (2\nu)_4, \\ s &= (s)_1 + (s)_2 + (s)_3 \end{aligned}$$

where $(n\delta z)_1$, $2(\nu)_1$, and $(s)_1$ include the secular terms, $(n\delta z)_2$, $2(\nu)_2$, and $(s)_2$ are those terms which are functions of the quantity θ_1, $(n\delta z)_3$, $2(\nu)_3$, and $(s)_3$ those terms which are functions of both θ_1 and ϵ, the eccentric anomaly—disturbed—and the remaining terms are given by the fourth part of the right hand member. The last terms can be neglected for all practical purposes.

Preliminary Tables of Perturbations due to Jupiter

Argument	$(n\delta z)_1$	$2(\nu)_1$	$(s)_1$
ϵ^2	5.10561n		
$\epsilon \sin \epsilon$	0.02784	1.02333n	0.65787n
$\epsilon \cos \epsilon$	1.02333	0.02976n	0.64847
$\epsilon \sin 2\epsilon$	8.40083		
$\epsilon \cos 2\epsilon$	9.39440		
$\cos \epsilon$	0.02784	1.02333n	0.65787n
$\sin \epsilon$	1.02333	0.02976	0.64847n
$\cos 2\epsilon$	8.09980		
$\sin 2\epsilon$	9.09337		

$$\left.\begin{matrix} (n\delta z)_1 \\ 2(\nu)_1 \\ (s)_1 \end{matrix}\right| = \left| \Sigma \text{ Coeff.} \quad \times \text{ Argument}\right.$$

where the coefficients are given logarithmically in seconds of arc, and ϵ, where explicit, should be expressed in radians.

χ	$(n\delta z)_2$		$2(\nu)_2$		$(s)_2$	
i	$\log k$	K	$\log k$	K	$\log k$	K
0	4.20906	180°1339	2.66614	0°6367	0.91192	331°0644
1	3.08784	187.8893	3.09696	7.7465	2.01845	150.8456
2	1.47140	7.8171	9.66226	187.8171	8.58780	330.8456
3	0.14058	187.8171				
—0	9.86525	35.9506				
—1	0.95527	100.4345				
—2	9.56078	215.9506				

$$\begin{aligned} (n\delta z)_2 &= [2.29753]\,\theta_1 + [1.65273n]\,g + \Sigma\, k \sin(\chi + K) \\ 2(\nu)_2 &= [9.00256n]\,g + \Sigma\, k \cos(\chi + K) \\ (s)_2 &= [9.62246n]\,g + \Sigma\, k \sin(\chi + K) \end{aligned}$$

where the coefficients are given logarithmically in seconds of arc, θ and g are expressed in radians, and $\chi = 2\theta_1 + i\epsilon$.

unit = 1″

$(n\delta z)_3$				$2(\nu)_3$			
i	j	$\log k$	K	i	j	$\log k$	K
0	—2	0.0000n	0°	0	—2	0.0000	0°
0	—1	1.1139n	0	0	—1	0.6990	0
0	1	1.6532	0	0	1	0.7842	350.538
0	2	1.2212	327.265	0	2	0.9163	255.966
0	3	0.3495	116.568	0	3	0.4771	90
1	—1	0.0000n	45	1	—1	0.4515	45
1	0	0.8451	0	1	0	0.3010	0
1	1	2.3984	2.749	1	1	2.2127	182.810
1	2	2.9314	4.770	1	2	2.4534	184.847
1	3	2.1909	2.955	1	3	1.8988	4.344
1	4	0.3010	45	1	4	0.8621	15.945
—1	—2	0.3010	0	—1	—2	0.0000	0
—1	—1	0.4771	0	—1	—1	0.3010	0
—1	0	0.6990n	0	—1	0	0.0000	0
—1	1	0.0000	315	—1	1	0.5569	326.312
2	0	0.0000	90	2	0	0.3495	243.432
2	1	0.0000n	0	2	1	0.6990	180
2	2	2.7563	5.531	2	2	2.8149	185.536
2	3	2.6869	186.732	2	3	2.6305	6.859
2	4	2.2345	184.680	2	4	1.9751	5.469
2	6	0.4771n	0				
3	2	1.1771	183.814	3	1	0.0000	90
3	3	1.5605	187.908	3	3	1.8309	8.488
3	4	1.8871	189.707	3	4	1.9741	9.162
3	5	1.1799	7.594	3	5	0.5000	198.436
3	6	1.6243	4.086	3	6	1.3464	187.765
4	3	0.0000	0	4	3	0.0000	0
4	4	1.0852	9.462	4	4	1.2116	190.619
4	5	1.1559	12.095	4	5	1.3095	191.312
4	6	1.1846	11.310	4	6	1.2882	191.889
4	7	1.0085	11.310	4	7	1.0570	195.253
5	5	0.5000	198.436	5	5	0.7075	11.310
5	6	0.6153	194.034	5	6	0.8621	15.945
5	7	0.6153	194.034	5	7	0.7842	9.462
6	6	0.0000	0	6	6	0.3495	206.568
6	7	0.3010	0	6	7	0.5000	198.436
6	8	0.0000	0	6	8	0.3010	180

$$\begin{aligned} (n\delta z)_3 = {} & \Sigma\, a_1 \sin\, [i\epsilon + j\, 2/5(\theta_1 - \epsilon] \\ & + \Sigma\, a_2 \cos\, [i\epsilon + j\, 2/5(\theta_1 - \epsilon)] \end{aligned}$$

$$\text{where } a_1 = k \cos K$$

$$a_2 = k \sin K$$

or $(n\delta z)_3 = \Sigma\, k \sin\, (\chi + K)$

$$\text{where } \chi = [i\epsilon + j\, 2/5(\theta_1 - \epsilon].$$

Similarly $2(\nu)_3 = \Sigma\, b_1 \cos\,[i\epsilon + j\,2/5(\theta_1 - \epsilon)]$
$- \Sigma\, b_2 \sin\,[i\epsilon + j\,2/5(\theta_1 - \epsilon)]$
where $b_1 = k \cos K$
$b_2 = k \sin K$
or $2(\nu)_3 = \Sigma\, k \cos\,(\chi + K)$
where $\chi = [i\epsilon + j\,2/5(\theta_1 - \epsilon)]$

$(s)_3$ unit = 1″

i	j	log k	K
0	—3	0.1505	225°
0	—2	1.1937	39.806
0	—1	1.5846	38.660
0	1	1.6074	122.900
0	2	2.0129	143.688
0	3	1.4969	322.760
1	—1	1.3275	41.187
1	0	1.3555	41.427
1	1	1.6382	246.981
1	2	2.0011	321.075
1	3	1.7653	325.488
1	4	1.4387	326.888
—1	—2	0.4515	225
—1	—1	1.1937	219.806
—1	1	0.9685	131.187
2	0	0.8927	219.806
2	1	1.5655	225
2	2	1.7980	256.184
2	3	2.2621	138.993
2	4	1.5606	148.494
3	1	0.4515	45
3	2	0.9647	49.394
3	3	0.9934	66.033
3	4	1.5545	149.859
3	5	0.8267	153.437
3	6	1.2305	331.928
4	3	0.4515	225
4	4	0.5569	236.312
4	5	0.8267	333.437
4	6	1.0485	333.437
4	7	1.1277	333.437
5	4	0.1505	45
5	5	0.1505	45
5	6	0.3495	153.437
5	7	0.5000	161.564
6	7	0.0000	0

$(s)_3 = \Sigma\, a_1 \sin\chi + \Sigma\, a_2 \cos\chi$
where $a_1 = k \cos K$
$a_2 = k \sin K$
or, $(s)_3 = \Sigma\, k \sin\,(\chi + K)$ where $\chi = [i\epsilon + j\,2/5(\theta_1 - \epsilon)]$

$$(n\delta z)_4 = \{(8.15192n) \cos 2\theta_1 + (9.65705n) \sin 2\theta_1\} \cos \epsilon$$
$$+ (9.66564) \sin\,(187^\circ\!.8171 + 2\theta_1 + 2\epsilon)$$
$$+ (7.75912n) \cos\,(324^\circ\!.0494 - 2\theta_1 - 2\epsilon),$$
$$2(\nu)_4 = (9.66564n) \cos\,(187^\circ\!.8171 + 2\theta_1 + 2\epsilon) +$$
$$(7.75192n) \cos\,(324^\circ\!.8171 - 2\theta_1 - 2\epsilon).$$

In the terms $(n\delta z)_1$, $2(\nu)_1$, and $(s)_1$ where the angle ϵ occurs we must retain the total number of revolutions since these are the secular terms.

The next opposition will occur November, 1917. Two observations of *Berkeley* by Palisa during the year 1915 are announced in the *Vierteljahrschrift der Astronomischen Gesellschaft*, but they are not published as yet so far as the writer is aware.

The undersigned wishes to express his thanks to Miss Jessica M. Young for the kind assistance given during the progress of the work in duplicating the computation, and to Professor A. O. Leuschner for his timely and helpful advice.

FERD. J. NEUBAUER.

BERKELEY ASTRONOMICAL DEPARTMENT,
February 15, 1917.

Issued December 19, 1917.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 302

ON THE ORBIT OF (718) *ERIDA*

The minor planet *Erida* was discovered by Palisa at Vienna on Sept. 29, 1911 and given the designation 1911 MS. Eleven observations were secured, extending from Sept. 29 to Oct. 28 (*A. N.* 4565).

The orbit, I in Table I, by Dr. Fritz Cohn (*A. N.* 4607) was based upon observations of Sept. 29, Oct. 13, and Oct. 28, 1911. This held its place in the *Jahrbuch* until 1915, when a new orbit, II in Table I, by Strehlow was adopted. This orbit was obtained from the observations of Feb. 28, Mar. 18, and Mar. 20, 1914, variation of the distances being applied so as to represent the 1911 opposition also. A correction, $\Delta\mu = +1''.5$, was then found necessary in order to represent the 1904 opposition of 1904 OD on the assumption that this and *Erida* are identical.

The orbits III and IV in Table I were undertaken as a class problem in Theoretical Astronomy. This case was selected in order to test the feasibility of producing from a single opposition an orbit which would be comparable in accuracy to the usual orbits based upon several oppositions. The orbit was based upon the observations used by Cohn and was computed by the Short Method under the direction of Professor Leuschner. Satisfactory representation of the entire series of observations of the 1911 opposition was obtained.

For comparison the first and last ephemeris positions, as published in *Kleine Planeten, Jahrgang* 1917, are given below in Table II.

A test of the value of this orbit will be afforded if observations are secured in the opposition of this year (November, 1917).

The difference between orbits III and IV is purely numerical, due to range of practical solutions, and has no significance. Their agreement with the orbit II, as well as the agreement of the ephemeris based on one of them with the ephemeris based on orbit II, furnishes the desired evidence that an orbit can be produced from the first opposition alone, which is as satisfactory for observational purposes as the orbit II based on three oppositions.

TABLE I

	I	II	III	IV
Epoch	1911	1914	1911	1911
B.M.T.	Sept. 29.5	Apr. 1.5	Oct. 13.4	Oct. 13.4
Equinox	1911.0	1910.0	1911.0	1911.0
M	149° 00′ 39″.9	320° 18′ 15″.0	156° 34′ 10″.1	156° 03′ 25″.0
ω	169 56 47.2	168 08 30.2	166 36 11.9	166 55 59.5
Ω	39 22 46.8	39 44 16.3	39 42 41.2	39 43 50.8
i	7 03 55.1	6 58 13.0	6 59 04.8	6 58 49.3
ϕ	12 05 35.0	11 28 39.3	11 19 07.1	11 20 11.1
μ	664″.65	664″.412	663″.865	663″.769
log a	0.484943	0.485037	0.485285	0.485327

CONSTANTS FOR THE EQUATOR 1911.0

(Orbit III)

$$x = r\,[9.998685]\sin(296^\circ\ 06'\ 18''.2 + v)$$
$$y = r\,[9.942907]\sin(208\ \ 33\ \ 15.9 + v)$$
$$z = r\,[9.687587]\sin(198\ \ 04\ \ 20.5 + v)$$

For the purpose of comparison in method of procedure the residuals found by the writer's computation obtained by a single differential correction and arbitrary variation, and those found for orbit IV by Miss Elizabeth J. Easton by two differential corrections, are:

RESIDUALS (O–C)

	III		IV	
	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$
I	+0″.2	+0″.8	+0″.2	+0″.1
III	+0″.3	+0″.2	±0″.0	±0″.0

TABLE II—EPHEMERIS

Gr. M. T.	α(1917.0)	δ(1917.0)	
Oct. 22.5	$2^h\ 56^m.3$	+17° 09′	by III Table I
	2 55.9	17 07	by II "
Oct. 22.5	2 53.5	16 56	by I "
Dec. 1.5	2 25.3	15 39	by III "
	2 25.1	15 36	by II "
Dec. 1.5	2 22.9	+15 24	by I "

The thanks of the writer are due to Miss Jessica M. Young for checking parts of the computation.

CARLOS S. MUNDT.

BERKELEY ASTRONOMICAL DEPARTMENT,
October 26, 1917.

Issued December 19, 1917.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 303

THE ORBIT OF THE SPECTROGRAPHIC BINARY γ *PHOENICIS*

The binary character of γ *Phoenicis* ($\alpha = 1^h\ 24.0^m$, $\delta = -43^\circ\ 50'$, Mag. 3.3, Class K5) was announced by Palmer, in *Lick Observatory Bulletin,* 3, 110, 1905. The orbit has been computed from measures of thirty-one spectrograms taken with the three- and two-prism spectrographs of the D. O. Mills Expedition. The measures of the three-prism plates were made with a Toepfer engine, while the two-prism plates were compared with a single standard spectrogram by means of the Hartmann spectrocomparator. A systematic correction of —1.5 km. has been applied to each of the latter observations to bring them into accord with those taken with three-prism dispersion. The observed velocities are recorded in the fourth column of the Table of Observations, the corrected values in the fifth column. The symbols in the last two columns have the following significations: Wr, W. H. Wright; Pa, H. K. Palmer; Cs, H. D. Curtis; Pd, G. F. Paddock; Sa, R. F. Sanford; Sc, A. A. Scott; H, C. M. Huffer; and Wi, R. E. Wilson.

TABLE OF OBSERVATIONS

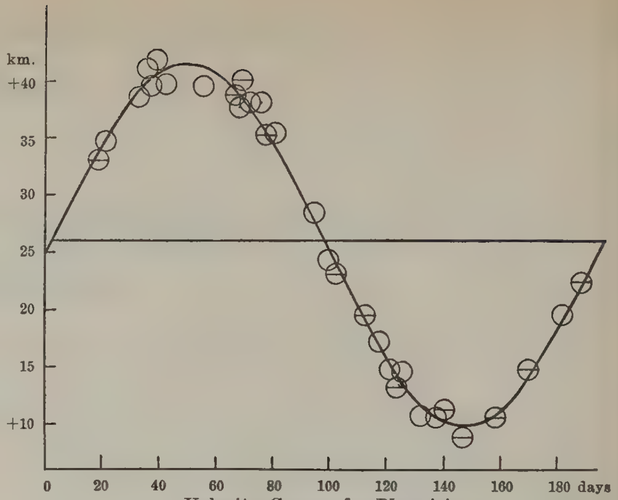
Gr. M. T.	Phase Days	Obs. km.	Adop. km.	Comp. km.	A–C km.	Disp.	Observer	Measurer
1903 Dec. 14.6	69.0	+40.0	+40.0	+38.4	+1.6	III	Pa	Pa
Dec. 22.6	77.0	35.2	35.2	35.7	—0.5	III	Wr	Pa
1904 June 22.9	66.5	38.8	38.8	39.1	—0.3	III	Pa	Pa
Oct. 3.9	169.5	14.8	14.8	13.8	+1.0	III	Pa	Pa
Nov. 15.7	18.5	33.0	33.0	33.9	—0.9	III	Pa	Pa
1906 Oct. 25.8	146.2	8.8	8.8	9.8	—1.0	III	Cs	Pd
Nov. 6.7	158.1	10.6	10.6	10.8	—0.2	III	Cs	Pd
Dec. 6.6	188.0	22.4	22.4	21.8	+0.6	III	Pd	Pd
1907 Oct. 3.8	101.6	23.1	23.1	24.1	—1.0	III	Cs, Pd	Pd
Oct. 13.8	111.6	19.5	19.5	19.2	+0.3	III	Cs, Pd	Pd
Oct. 25.7	123.6	13.4	13.4	14.2	—0.8	III	Cs	Pd
Nov. 10.7	139.6	11.3	11.3	10.3	+1.0	III	Cs, Pd	Pd
1914 June 25.0	38.5	43.3	41.8	40.4	+1.4	II	Sa	Sa, Wi
July 27.9	71.5	39.7	38.2	37.7	+0.5	II	Sc	Sa
July 31.9	75.4	39.6	38.1	36.3	+1.8	II	Sc	Sa
Aug. 5.9	80.4	36.9	35.4	34.3	+1.1	II	Wi	Wi
1915 Sept. 15.7	98.7	25.8	24.3	25.5	—1.2	II	Wi	Wi
Oct. 7.6	120.6	16.4	14.9	15.3	—0.4	II	Wi	Sc, Wi
Oct. 18.7	131.6	12.2	10.7	11.8	—1.1	II	Wi	Sc
1916 Sept. 6.8	68.2	39.1	37.6	38.7	—1.1	II	Wi	Sc, Wi
Oct. 2.6	94.0	30.0	28.5	27.9	+0.6	II	Wi	Wi
Oct. 25.6	117.0	18.7	17.2	16.8	+0.4	II	Wi	Sc, Wi
Nov. 2.6	125.0	16.0	14.5	13.7	+0.8	II	Sc	Sc
Nov. 14.7	137.1	12.1	10.6	10.6	0.0	II	Wi	Sc
1917 Jan. 31.6	21.2	36.5	35.0	35.1	—0.1	II	Wi	Wi
Feb. 12.5	33.1	40.1	38.6	39.1	—0.5	II	Sc	Sc, Wi
Feb. 16.5	37.1	41.0	39.5	40.1	—0.6	II	Sc	Wi
Feb. 21.5	42.1	41.2	39.7	40.9	—1.2	II	Wi	Wi
July 10.9	181.5	21.0	19.5	18.7	+0.8	II	Wi	Wi
Aug. 27.8	35.6	42.7	41.2	39.8	+1.4	II	Wi	Wi
Sept. 16.7	55.5	40.9	39.4	41.1	—1.7	II	H	Wi

Preliminary elements, derived by the graphical method of Lehmann-Filhés, showed the orbit to be very nearly circular. Residuals from a circular orbit were used to determine corrections to P, T, K, and V_0, by means of a least squares solution, after which an assumed eccentricity = 0.005 and $\omega = 267°$ were found to decrease the residuals. Very little advantage could be expected from further application of the method of least squares, in view of the extremely small eccentricity. The elements adopted as best satisfying the observations are:

$P = 193.79$ days
$T =$ J. D. 2417945.0
$e = 0.005$
$\omega = 267°$
$K = 15.8$ km.
$V_0 = +25.8$ km.
$\mu = 1^\circ\!.8577$
III–II = —1.5 km.
$a \sin i = 42{,}100{,}000$ km.
$$\frac{m^3_1 \sin^3 i}{(m + m_1)^2} = 0.079\odot$$

The probable error of the velocity derived from a single plate is ±0.65 km.

It should be noted that this is the only late type star at present known to be moving in an essentially circular orbit.



Velocity Curve of γ *Phoenicis*

The elements and the observations are represented in the accompanying diagram by the velocity curve and the small circles, respectively; the barred circles representing the three-prism observations and the open circles the two-prism.

I am indebted to Mr. C. M. Huffer, Assistant with the D. O. Mills Expedition, for assistance in checking some of the computations for this orbit.

Ralph E. Wilson.

The D. O. Mills Expedition,
Santiago, Chile,
October, 1917.

ON THE ORBIT OF THE SPECTROSCOPIC BINARY OF σ *PUPPIS*

The binary character of σ *Puppis* ($\alpha = 7^h\ 26.1^m$, $\delta = -43°\ 6'$, Mag. 2.99, Class K5) was announced by Palmer, in *Lick Observatory Bulletin*, **3**, 111, 1905. An approximate orbit has been published by Lunt in the *Astrophysical Journal*, **54**, 260, 1916, the elements being as follows:

$P = 258$ days
$T =$ J. D. 2420419
$\omega = 350°$
$e = 0.2$
$K = 18.0$ km.
$V_0 = +88.5$ km.

These elements were based upon thirty-three spectrograms taken at the Cape between January, 1909, and March, 1915, supplemented by four spectrograms taken by the Mills Expedition in 1904. As fourteen additional observations of this star had been secured by the Mills Expedition between 1906 and 1917 with a view to determining its orbit, and as our preliminary elements agreed well with those published by Lunt, it was deemed advisable to carry through a second solution, using all the fifty-one observations available, as listed in the following table.

Table of Observations

Gr. M. T.	Phase Days	Obs. V. km.	Adop. V. km.	Comp. V. km.	A–C km.	Spectrograph
1904 Jan. 15.8	202.2	+85.4	+85.4	+84.7	+0.7	Mills-III
Jan. 29.7	216.1	89.0	89.0	91.3	—2.3	Mills-III
Oct. 25.8	228.4	97.0	97.0	97.2	—0.2	Mills-III
Dec. 22.8	28.6	102.8	102.8	103.7	—0.9	Mills-III
1906 Dec. 21.8	242.0	106.4	106.4	104.2	+2.2	Mills-III

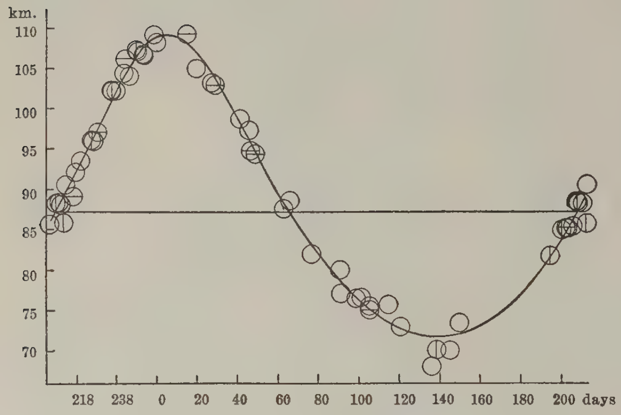
TABLE OF OBSERVATIONS—(*Continued*)

Gr. M. T.	Phase Days	Obs. V. km.	Adop. V. km.	Comp. V. km.	A–C km.	Spectrograph
1907 Jan. 21.6	15.0	109.2	109.2	108.0	+1.2	Mills-III
Feb. 24.6	49.0	94.4	94.4	94.6	—0.2	Mills-III
Nov. 7.8	46.4	94.7	94.7	95.8	—1.1	Mills-III
1908 Jan. 4.8	105.4	75.1	75.1	75.2	—0.1	Mills-III
1909 Jan. 4	212.6	91.8	90.7	89.5	+1.2	Cape
1911 Feb. 7	203.6	86.2	85.1	85.3	—0.2	Cape
Mar. 25	249.6	107.6	106.5	106.9	—0.4	Cape
Oct. 25	205.6	85.7	84.6	86.2	—1.6	Cape
1912 Jan. 5	19.6	106.0	104.9	106.8	—1.9	Cape
Feb. 20	65.6	89.8	88.7	87.2	+1.5	Cape
Mar. 26	100.6	77.8	76.7	76.1	+0.6	Cape
1913 Nov. 25.7	193.9	86.0	81.8	81.4	+0.4	Mills-II
Dec. 1.8	200.0	89.3	85.1	83.8	+1.3	Mills-II
Dec. 9.7	207.9	92.6	88.4	84.3	+1.1	Mills-II
Dec. 11	209.6	89.5	88.4	88.1	+0.3	Cape
Dec. 13.8	212.4	90.1	85.9	89.5	—3.6	Mills-II
Dec. 19	217.6	93.3	92.2	92.1	+0.1	Cape
Dec. 21	219.6	94.6	93.5	93.1	+0.4	Cape
Dec. 27.7	226.3	100.2	96.0	96.6	—0.6	Mills-II
1914 Jan. 6	235.6	103.2	102.1	101.3	+0.8	Cape
Jan. 8	237.6	103.3	102.2	102.2	0.0	Cape
Jan. 12	241.6	105.4	104.3	104.0	+0.3	Cape
Jan. 14	243.6	105.1	104.0	104.9	—0.9	Cape
Jan. 19	248.6	108.1	107.0	106.6	+0.4	Cape
Jan. 22	251.6	107.5	106.4	107.5	—1.1	Cape
Jan. 27	256.6	110.1	109.0	108.5	+0.5	Cape
Jan. 28	257.6	109.2	108.1	108.6	—0.5	Cape
Feb. 23	26.6	104.3	103.2	104.5	—1.3	Cape
Mar. 9	40.6	99.7	98.6	98.5	+0.1	Cape
Mar. 14	45.6	98.4	97.3	96.2	+1.1	Cape
Mar. 31	62.6	88.8	87.7	88.2	—0.5	Cape
Apr. 28	90.6	78.1	77.0	78.6	—1.6	Cape
Nov. 30	48.6	96.5	95.4	94.8	+0.6	Cape
Dec. 28	76.6	83.1	82.0	83.0	—1.0	Cape
1915 Jan. 11	90.6	81.3	80.2	78.6	+1.6	Cape
Jan. 19	98.6	77.6	76.5	76.6	—0.1	Cape
Jan. 26	105.6	76.7	75.6	75.2	+0.4	Cape
Feb. 4	114.6	77.0	75.9	73.6	+2.3	Cape
Feb. 10	120.6	74.2	73.1	72.9	+0.2	Cape
Feb. 25	135.6	69.2	68.1	71.9	—3.8	Cape
Mar. 6	144.6	71.3	70.2	72.0	—1.8	Cape
Mar. 11	149.6	74.8	73.7	72.2	+1.5	Cape
1916 Oct. 4.9	207.9	92.7	88.5	87.3	+1.2	Mills-II
Nov. 1.8	235.8	106.5	102.3	101.4	+0.9	Mills-II
Nov. 13.8	247.8	111.5	107.3	106.4	+0.9	Mills-II
1917 Apr. 6.5	137.7	74.4	70.2	71.9	—1.7	Mills-II

From the beginning it was quite evident that there were involved systematic differences between the velocities derived with the three- and the two-prism Mills spectrographs and those obtained at the Cape. Approximate corrections, therefore, were derived from the plotted curves which would bring the observations into accord, and in the final solution these were reduced to the basis of the III-prism Mills observations. The adopted values are given in the fifth column of the table. The two-prism results were obtained by means of the Hartmann spectrocomparator, in comparison with a single standard plate. The value assumed for the line displacements on the standard plate was evidently three or four kilometers too great.

The period was corrected by means of the early observations and a least squares solution carried through for corrections to the other five elements. The resulting elements are as follows:

$P = 257.8$ days
$T =$ J. D. 2420418.6
$e = 0.17$
$\omega = 349.^\circ 3$
$K = 18.55$ km.
$V_0 = +87.3$ km.
$\mu = 1.^\circ 3964$
III–Cape = —1.1 km.
III–II = —4.2 km.
$a \sin i = 64{,}800{,}000$ km.
$\frac{m_1^3 \sin^3 i}{(m + m_1)^2} = 0.164\odot$



Velocity Curve of σ *Puppis*

The probable error of the velocity derived from a single plate is ±0.92 km. The sums of the squares of the residuals for the thirty-seven observations used in both solutions are 71.8 from the preliminary elements and 55.6 from the secondary, corrections having been applied to the three III-prism Mills observations uncorrected by Lunt to make the solutions comparable.

The elements and observations are represented in the accompanying diagram by the velocity curve and small circles, respectively; the open circles representing the Cape observations; those horizontally barred, the Mills III-prism observations; and those vertically barred, the Mills II-prism observations.

Mr. C. M. Huffer assisted in making the least squares solution for this orbit.

RALPH E. WILSON.

THE D. O. MILLS EXPEDITION,
SANTIAGO, CHILE,
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A SPECTROGRAPHIC STUDY OF κ *PEGASI*

The star κ *Pegasi* is a visual binary (discovered by Burnham in 1880) having one of the shortest known periods (11.37 years). The two components are described as yellowish and of magnitudes 4.8 and 5.3. Dr. Aitken, however, as well as other visual double-star observers, estimates them to be of equal magnitude. One of the two components (that whose spectrum is stronger in the $H\gamma$ region) was discovered in 1900 by Dr. Campbell to be a spectroscopic binary.[1] κ *Pegasi* is therefore a triple system of great interest. According to Dr. Campbell, changes in the appearance of the spectrum occur.

Since the discovery of the binary character of that visual component, a considerable number of spectrograms of it have been taken at the Lick Observatory, mostly in 1900, 1912, and recently in 1917. The spectrograms of 1917 have been taken on Seed 23 plates, with exposures averaging 2½ hours, by the author, helped greatly by Dr. Paddock, Mr. Thiele, and Mr. Shane. The series of spectrograms of 1900 had been measured by Dr. H. D. Curtis, but for the uniformity of the measurements it seemed preferable to the author to measure all the plates himself. The measurements have been made by the use of a Hartmann spectrocomparator, using a spectrogram of α *Canis Minoris* as a standard plate. The first three columns of the following table give the Greenwich mean time of each exposure, the corresponding Julian day and the observed radial velocities.

Gr. M. T.	J. D. 241	Observed velocity km.	Computed velocity km.	Residuals O–C km.	Residuals Eq. of Cond. km.
1896 Aug. 31	3803.785	—36.35	....	......	......
1897 July 17	4123.948	—40.32	....	......	......
1900 Aug. 6	5238.822	+35.74	+35.41	+0.33	......
Aug. 7	5239.817	+27.92	+28.59	—0.67	......
Aug. 8	5240.828	—11.09	—11.08	—0.01	......
Aug. 12	5244.832	+37.71	+36.05	+1.65	......
Aug. 21	5253.789	—40.78	—39.26	—1.52	......
Aug. 22	5254.795	—30.49	—32.06	+1.57	......
Sept. 24	5287.783	+21.81	+22.21	—0.40	......
Sept. 25	5288.696	—13.83	—14.86	+1.03	......
Oct. 16	5309.782	+13.17	+14.05	—0.88	......
Oct. 23	5316.734	+38.50	+38.47	+0.03	......
Oct. 24	5317.746	+17.58	+18.28	—0.70	......
1904 July 6	6668.906	—51.44	....	......	......
Aug. 23	6716.939	—56.16	....	......	......
Sept. 27	6751.781	—26.55	....	......	......
Oct. 17	6771.796	—31.99	....	......	......
1905 July 24	7051.000	—51.57	....	......	......
1912 July 6	9590.967	— 2.66	— 1.54	—1.12	......
July 7	9591.905	+33.74	+30.74	+3.00	......
July 18	9602.898	+ 0.23	— 2.03	+2.24	......
Aug. 3	9618.939	—41.71	—41.73	+0.02	......
Aug. 4	9619.811	—35.63	—36.44	+0.81	......
Aug. 17	9632.952	+ 4.49	+ 5.95	—1.46	......
Aug. 20	9635.950	—16.41	—16.33	—0.08	......
Sept. 15	9661.828	—32.13	—30.76	—1.37	......
Sept. 20	9666.811	—42.67	—42.67	0	......
Sept. 23	9669.831	+36.27	+36.17	+0.10	......
Sept. 29	9675.824	+35.67	+36.38	—0.71	......
Oct. 9	9685.811	—28.25	—27.81	—0.44	......
Oct. 10	9686.631	+ 3.81	+ 3.33	+0.48	......
Oct. 11	9687.778	+34.99	+36.47	—1.47	......
Oct. 13	9689.776	—19.44	—19.65	+0.21	......
Nov. 14	9721.709	—26.01	—25.58	—0.43	......
	242				
1917 July 27	1437.990	+20.69	+21.42	—0.73	—0.77
July 29	1439.987	—51.61	—49.80	—1.81	—1.88
July 30	1440.979	—52.57	—52.55	+0.02	—0.04
Aug. 1	1442.988	+19.88	+20.22	—0.34	—0.42
Aug. 3	1444.750	— 2.68	— 2.41	—0.27	—0.19
Aug. 5	1446.728	—56.62	—56.30	—0.32	—0.31
Aug. 6	1447.775	—22.03	—22.86	+0.83	+0.78
Aug. 6	1447.986	—13.87	—13.90	+0.03	—0.09
Aug. 8	1449.780	+25.00	+23.69	+1.31	+1.27
Aug. 9	1450.960	—12.10	—12.31	+0.21	+0.18
Aug. 10	1451.715	—41.52	—43.21	+1.69	+1.62
Aug. 12	1453.958	—15.15	—13.48	—1.67	—1.74
Aug. 13	1454.690	+14.81	+14.29	+0.52	+0.41
Aug. 14	1455.972	+20.07	+20.13	—0.06	—0.09
Aug. 15	1456.968	—13.87	—13.53	—0.34	—0.40
Aug. 27	1468.675	— 4.34	— 3.98	—0.36	—0.36
Aug. 28	1469.985	—52.45	—53.13	+0.68	+0.63
Aug. 30	1471.692	—20.02	—21.50	+1.48	+1.42

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[1] *Ap. Jour.*, **12**, 257, 1900.

After a careful examination of the plates, in which Dr. Moore kindly assisted, it was found that the spectrograms record not only the spectrum of the spectroscopic binary but also, very faintly, the spectrum of the other visual component. The latter shows for a few days and even for a few months a velocity which is practically constant. This was to be expected since the two visual components are so close together (about $0\overset{\prime\prime}{.}25$ is their angular distance) that they cannot be separated on the slit of the spectrograph. During the period of one revolution of the spectroscopic binary around its center of mass the spectrum of the other visual component does not show any shifting of the lines, while the lines of the bright component of the spectroscopic binary do shift. The result is that sometimes lines of the two visible spectra coincide and reinforce one another, and sometimes they are separated. This produces very curious variations of intensity of the lines, which variations will be described later.

THE ORBIT OF THE SPECTROSCOPIC BINARY

Period.—The velocity curves having been determined fairly accurately for the three epochs of 1900, 1912, and 1917, the very long interval of time allowed us to determine the period of the spectroscopic binary very accurately. In this determination account was taken of the different times necessary for the light to cross the different chords of the visual binary orbit. The spectroscopic binary having been relatively farther away at one epoch than at the other (1900, 1912, and 1917), this introduces a difference in length of path traveled by the velocity vibrations in reaching the Earth. However, the correction due to this was very small.

The period of the spectroscopic binary was found to be $5\overset{d}{.}9715$.

SPECTROSCOPIC ORBIT IN 1917

Preliminary elements of the orbit were deduced by the author's second method,[2] as follows:

$K = 41.12$ km.
$e = 0.03$
$\omega = 199\overset{\circ}{.}8$
$P = 5\overset{d}{.}9715$
$T = 2421440.84$ Julian Days
$\gamma = -14.65$ km.

In which K is the semi-amplitude of the velocity variation, e the eccentricity, ω the angular distance of periastron from the ascending node, T the time of periastron passage, and γ the velocity of the center of mass.

A least squares adjustment was applied to these elements, using Schlesinger's method, assuming the period P to be constant, and the following corrections to the elements were obtained:

$\delta K = +0.41$ km.
$\delta e = +0.0042$
$\delta\omega = -10\overset{\circ}{.}41$
$\delta T = -0\overset{d}{.}155$
$\delta\gamma = +0.03$ km.

The probable errors of the principal elements having been computed, the definitive elements for the orbit in 1917 are:

$K = 41.53 \pm 0.34$ km.
$e = 0.034 \pm 0.007$
$\omega = 189\overset{\circ}{.}4 \pm 13\overset{\circ}{.}9$
$P = 5\overset{d}{.}9715$
$T = 2421440.685$ J. D. $\pm 0\overset{d}{.}23$
$\gamma = -14.62$ km.
$a \sin i = 3{,}408{,}000 \pm 29{,}000$ km.

The residuals computed from the equations of conditions and the residuals computed directly from the new elements will be found in columns five and four of the table on the first page of this bulletin. Their close agreement indicates that the least squares adjustment has been correctly made.

SPECTROSCOPIC ORBIT IN 1900

The preliminary elements furnished by the same method as for 1917 were:

$K = 40.0$ km.
$e = 0.05$
$\omega = 6\overset{\circ}{.}7$
$P = 5\overset{d}{.}9715$
$T = 2415239.30$ J. D.
$\gamma = -3.04$ km.

The method of least squares was applied as before. It was found that the coefficients in the third and fifth normal equations were almost proportional throughout, which indicates practical indetermination. No further attempt was made to apply the method of least squares to the 1900 orbit, but the preliminary elements were bettered as far as practicable by successive approximations, which reduced the sum of the squares of the residuals to a very small amount. We arrive, then, at the definitive elements:

$K = 39.81$ km.
$e = 0.04$
$\omega = 4\overset{\circ}{.}7$
$P = 5\overset{d}{.}9715$
$T = 2415239.25$ J. D.
$\gamma = -2.80$ km.
$a \sin i = 3{,}266{,}000$ km.

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[2] *Publ. A. S. P.*, **29**, 195, 1917.

SPECTROSCOPIC ORBIT IN 1912

The preliminary elements furnished by the same method as for 1917 were:

$K = 40.6$ km.
$e = 0.05$
$\omega = 56°63$
$P = 5^{\mathrm{d}}9715$
$T = 2419593.32$ J. D.
$\gamma = -4.22$ km.

The observations for 1912, having been made rather at random, since the observers did not know the period at that time, are not distributed uniformly along the velocity curve. They happen to cluster at five different and very limited regions of the curve. It was deemed in that case better to combine the observations into five normal places and compute the corrections to the preliminary elements. Under these circumstances this method is fully as satisfactory as the method of least squares. We shall have five equations of condition which must be satisfied. From them the following corrections to the elements were obtained:

$\delta K = -0.25$ km.
$\delta\omega = -9°11$
$\delta e = -0.023$
$\delta T = -0^{\mathrm{d}}146$
$\delta\gamma = +0.63$ km.

The definitive elements for the orbit in 1912 become:

$K = 40.35$ km.
$e = 0.03$
$\omega = 47°5$
$P = 5^{\mathrm{d}}9715$
$T = 2419593.174$ J. D.
$\gamma = -3.79$ km.
$a \sin i = 3{,}312{,}000$ km.

The residuals furnished by these elements for the five normal places are —0.02 km., +0.11 km., —0.20 km., —0.03 km., and +0.15 km., respectively.

COMPARISON OF THE THREE ORBITS

The elements of the three spectroscopic orbits are brought together in the following table:

Element	1900.7	1912.7	1917.6	Probable errors for 1917
K	39.81 km.	40.35 km.	41.53 km.	±0.34 km.
e	0.04	0.03	0.034	±0.007
ω	4°7	47°5	189°4	±13°9
P	$5^{\mathrm{d}}9715$	$5^{\mathrm{d}}9715$	$5^{\mathrm{d}}9715$	....
T	2415239.25 J. D.	2419593.17 J. D.	2421440.69 J. D.	±0.23 J. D.
γ	—2.80 km.	—3.79 km.	—14.62 km.	....
$a \sin i$	3,266,000 km.	3,312,000 km.	3,408,000 km.	±29,000 km.

The change of γ is at once evident, but this had to be expected, since γ indicates the velocity of the center of mass of the spectroscopic system, and this center of mass revolves around the center of mass of the visual system. These three values of γ will be used to determine the real size of the visual orbit.

A very important fact is the change of ω, which indicates a rotation of the line of apsides. It seems very probable that the line of apsides rotates 360° in the same time that the spectroscopic body revolves around the center of mass of the visual system; that is, in 11.35 years. The mean yearly motion of the line of apsides from 1912.7 to 1917.6 appears to be 28°7±3°; from 1900.7 to 1912.7, $\frac{360° + 47°5 - 4°7}{12} = 33°6 \pm 2°$; and from 1900.7 to 1917.6, 32°2 ± 1°; the probable errors being deduced from the probable error for ω in 1917, divided by the number of years that separate the two sets of observations. The mean yearly motion of the spectroscopic body is 31°7. These four mean motions are equal within the limits of their probable errors, which seems to show that the above law of rotation is very probable. This rotation, although possibly accomplished according to the preceding supposed law, is not likely to be uniform, however; in the assumption that it would be such, we could adopt the three values of ω, for 1900, 13°7, for 1912, 34°1, and for 1917, 189°4, that would give an exact mean yearly motion of 31°7. The error for 1917 being assumed zero, the error for 1900 would be +9° and that for 1912, —13°4. All of these errors are certainly within the limit of the probable errors for each determination, which, as we have seen, are certainly greater than ±13°9.

It is not likely that the eccentricity changes much, but a small change in the value of $a \sin i$ could be possible. This may be due to a small change in the major axis or a small variation in the inclination of the orbit.

The rotation of the line of apsides is strikingly shown if we put the velocity curves for 1900, 1912, and 1917 (figs. 1, 2, 3) upon each other so that their center-of-mass lines coincide (fig. 4). A big change due to the variation of ω is at once apparent. This rotation is probably due to perturbations occurring in the spectroscopic binary orbit under the influence of the third body. This system presents, indeed, very great similarities to the system Earth-Moon-Sun, in

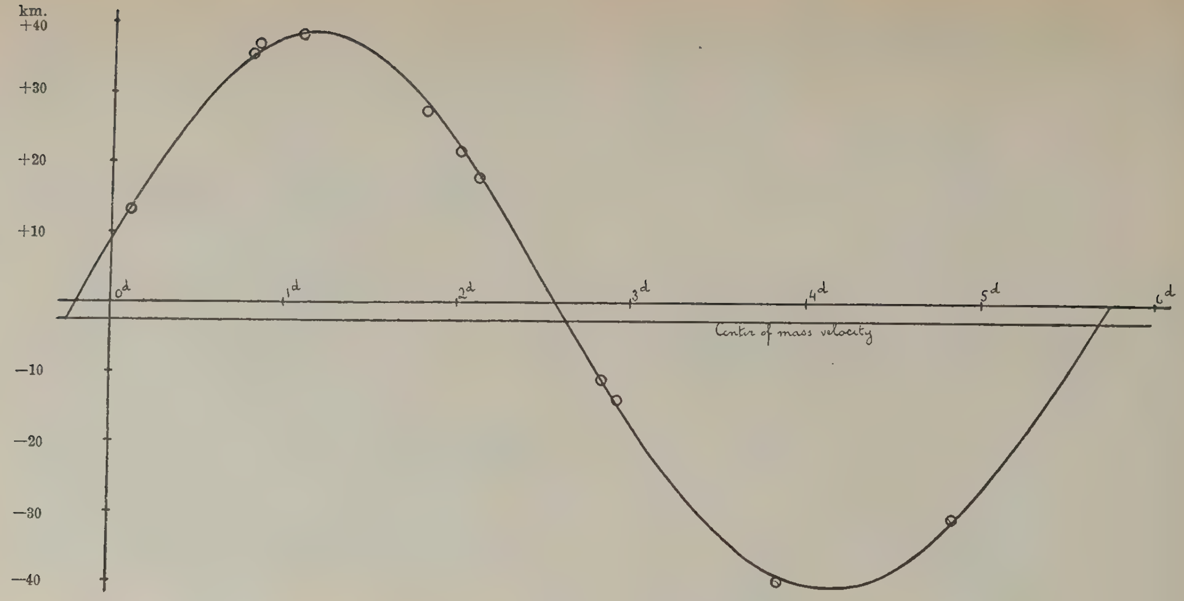

Fig. 1. Velocity curve for 1900

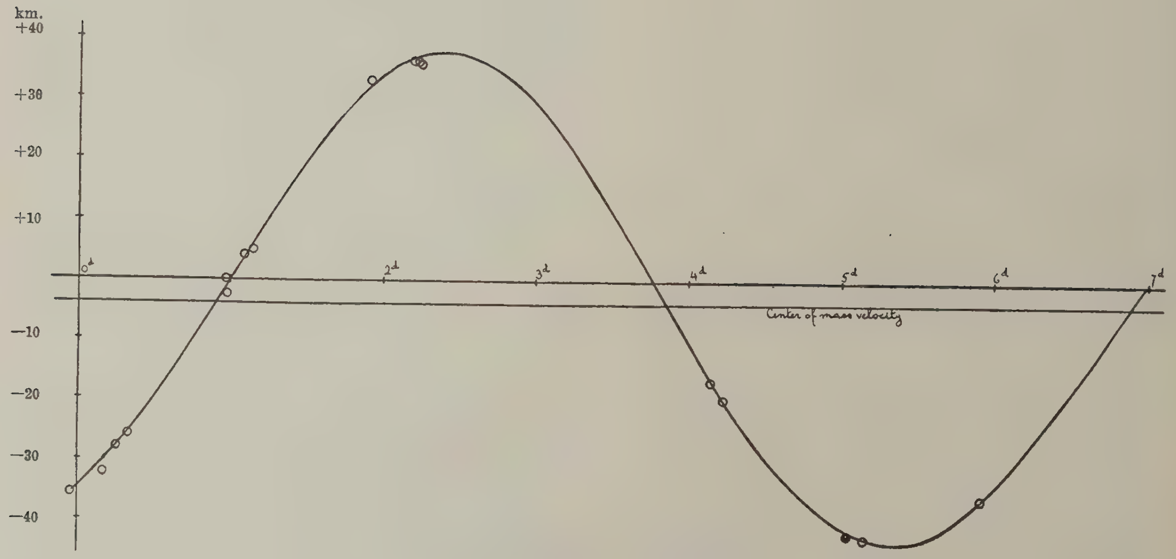

Fig. 2. Velocity curve for 1912

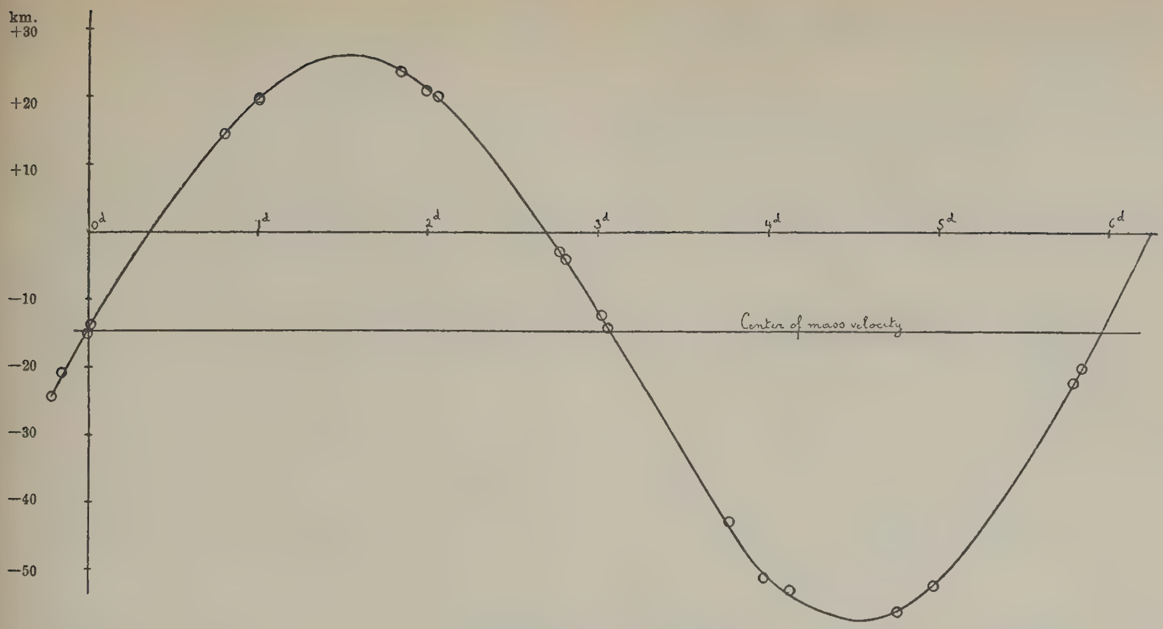

Fig. 3. Velocity curve for 1917

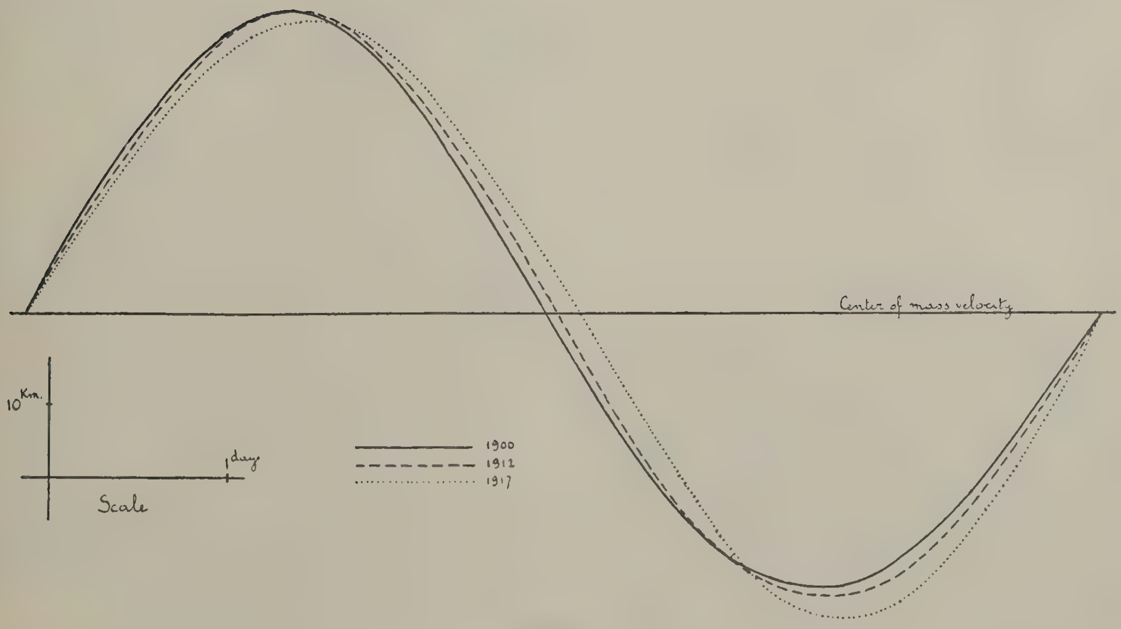

Fig. 4

which we see that the line of apsides of the orbit of the Moon completes a direct revolution, once in about nine years (8.855 years).

REAL SIZE OF THE ORBIT OF THE SPECTROSCOPIC BODY AROUND THE CENTER OF MASS OF THE VISUAL SYSTEM

The very good elements of the visual orbit derived by Lewis are:

Time of periastron	1897.8
Period	11.35 years
Eccentricity	0.49
Nodal point	109°2
Inclination	77°5
Angle from node to periastron	106°1
Semi-major axis	0″29

The only published parallax of the system is that determined by Flint, +0″028, from meridian circle observations at the Washburn Observatory. The probable errors of such determinations are probably great, and it is impossible to rely on such a parallax to determine the real size of the visual orbit of κ *Pegasi.*

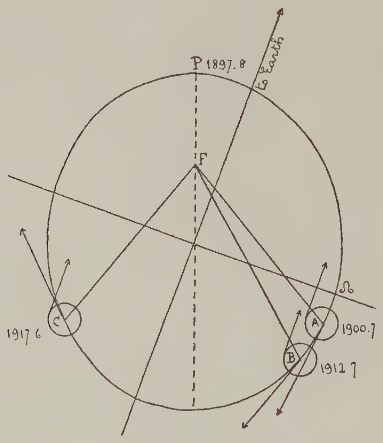

Fig. 5

Having the elements of the visual orbit, let us represent that orbit in its plane on an arbitrary scale of measurement (fig. 5). We can also represent the line of nodes and the positions of the body in 1900.7, 1912.7, and 1917.6. Let us now suppose that this orbit represents that of the spectroscopic body around the center of mass of the visual system. The Earth, from the spectroscopic data, is as indicated by the arrow, and not in the opposite direction, as will be shown below.

From the visual data and according to Kepler's laws, we can compute three numbers proportional to the velocities at the three epochs projected on the line of sight. These three numbers are:

For 1900.7	+30
For 1912.7	+27
For 1917.6	—22

These are the radial velocities for the three epochs, but expressed in a certain arbitrary unit that we do not yet know. Let us now call x the projection of the proper motion of the visual system on the line of sight expressed in the same arbitrary unit. Since we know the absolute radial velocities at the three epochs we have the equations:

$$\frac{x - 22}{x + 30} = \frac{14.62}{2.80}$$

$$\frac{x - 22}{x + 27} = \frac{14.62}{3.79}$$

These equations become:

$$11.82x = -500.20$$

$$10.83x = -478.12$$

In order that the data furnished by the visual observation of the double star and that furnished by spectroscopy shall agree, the two equations ought to have their coefficients nearly proportional:

$$\frac{11.82}{10.83} = 1.09; \qquad \frac{500.20}{478.12} = 1.04.$$

They are seen to be very nearly proportional. A good value of x will be obtained by taking the mean of the two solutions. They give us the value $x = -43$; and, taking account of the difference 0.05 of the two above ratios, we find that the probable error for x is not greater than ±1. The lengths (expressed in the arbitrary unit) proportional to the observed radial velocities will then be:

$$-43 - 22 = -65 \text{ for } 1917$$

$$-43 + 27 = -15 \text{ for } 1912$$

$$-43 + 30 = -13 \text{ for } 1900$$

The negative signs obtained for these values show that the direction of the Earth from the orbit has been properly chosen. The radial velocity of the system will then be, in kilometers per second:

$$\frac{-14.62 \times 43}{65} \pm \frac{14.62 \times 1}{65} = -9.67 \pm 0.22 \text{ km.}$$

We know that the radial velocity at any point of the orbit is expressed by the formula:

$$V = \gamma + Ke \cos \omega + K \cos (v + \omega)$$

Where γ is the radial velocity of the center of mass, K the semiamplitude of the velocity curve, ω the angular distance of the periastron from the ascending node, and v the true anomaly. Since we shall now determine K, the above equation becomes:

$$K = \frac{V - \gamma}{e \cos \omega + \cos (v + \omega)}$$

But from the elements of the orbit we have:

$$e = 0.49$$
$$\omega = 360^{\circ} - 106^{\circ}.1 = 253^{\circ}.9$$

Computing K separately from the three values of γ, we obtain:

$$K_{1900} = 10.04 \text{ km.}$$
$$K_{1912} = 10.27 \text{ km.}$$
$$K_{1917} = 9.81 \text{ km.}$$

These values of K agree pretty well. We may then accept their mean value:

$$K = 10.04 \pm 0.20 \text{ km.}$$

Now, having K, we shall obtain the semimajor axis a by the formula:

$$a = \frac{f\ KU\ \sqrt{1-e^2}}{\sin i}$$

Where f is the constant of attraction which, expressed by its logarithm, is [4.13833], K is the semi-amplitude of the velocity curve as found above, U is the period expressed in days. We obtain:

$$a = 511{,}100{,}000 \text{ km.}$$

From the measurements of two or three lines of the spectrum of the other visual component on three plates where the separation was great, the following velocities were obtained (reduced to the Sun):

Gr. M. T.	Velocity
1917 Aug. 5 $17^h\ 29^m$	+3.20 km.
1917 Aug. 10 $17^h\ 10^m$	+2.33 km.
1917 Aug. 28 $23^h\ 28^m$	+3.87 km.

But we have seen before that the velocity of the center of mass of the spectroscopic binary in 1917 is —14.62 km. Since the radial velocity of the visual double system is —9.67 km., the absolute radial velocities of the two visual components in the system will be:

Center of mass of spectroscopic component, $-14.62 + 9.67 = -4.95$;
Second visual component, $+3.14 + 9.67 = +12.81$.

Since the motions of the two components in their respective orbits are always parallel and in opposite directions, the ratio of the absolute values of these two preceding velocities will be the ratio of the sizes of the two orbits. Since the value of the semi-major axis of the first orbit is 511,100,000 km., the value of the semi-major axis of the apparent orbit of one component around the other will be:

$$\frac{511{,}100{,}000 \times (4.95 + 12.81)}{4.95} = 1{,}826{,}000{,}000 \text{ km.}$$

The apparent semi-major axis of the ellipse is $0''.29$, from which the parallax of *κ Pegasi* would be:

$$\frac{0''.29 \times 150{,}000{,}000}{1{,}826{,}000{,}000} = 0''.025$$

For which a probable error of $\pm 0''.002$ can be computed. It is interesting to compare this value with that obtained by Flint, $0''.028$.

It is important to have an idea of the masses of the three bodies constituting the triple system. Let us call m and m_1 the masses of the spectroscopic body and its companion. Let $m + m_1 = M$, and let M_1 be the mass of the other visual component.

In the visual orbit we have:

$$M \sin^3 i = [3.01642 - 10]\ (1 - e^2)^{\ \frac{3}{2}}\ (K + K_1)^2 K_1 P \odot$$

Where i is the inclination of the orbit, e the eccentricity, K the semi-amplitude of the radial velocity curve of the visual orbit described by the center of mass of the spectroscopic binary, K_1 of that described by the other visual component $\left(K_1 = \frac{K \times 12.81}{4.95} = 25.98\right)$, and P the period of the visual orbit expressed in days.

$$M \sin^3 i = 9.62 \odot$$

But $i = 77^{\circ}.5$, from which $M = 10.33 \odot$ and $M_1 = \frac{K}{K_1} \times M = 4.00 \odot$.

Now in the formula for the spectroscopic binary, $\frac{m_1{}^3 \sin^3 i}{(m + m_1)^2}$, we know $m + m_1 = M$.

We have:

$$\frac{m_1{}^3 \sin^3 i}{(m + m_1)^2}\ [3.01642 - 10]\ (1 - e^2)^{\frac{3}{2}} K^3 P \odot$$
$$= 0.443 \odot$$
$$m_1{}^3 \sin^3 i = 106.7 \times 0.443 \odot$$
$$m_1 \sin i = 3.62 \odot$$

If with Dr. T. J. J. See[3] we suppose that the planes of the two orbits (visual and spectroscopic) coincide, we have $i = 77^{\circ}.5$ from which:

$$m_1 = 3.71 \odot \text{ and } m = 6.62 \odot$$

This last result is logical, but we may as well suppose that the inclination of the planes of the two orbits is different.

VARIATIONS OF THE SPECTRUM OF *κ Pegasi*

At first sight it seems that the spectral class of *κ Pegasi* shows a variation in the same period as the orbital revolution (about 6 days). As in *δ Cephei* and in other Cepheid variables,[4] the enhanced lines appear sometimes stronger, sometimes weaker. An examination of the plates of 1917, especially at maximum and minimum velocity, was made, and the following interesting results were obtained for a great many lines of the region from λ4500 to λ4575, which was the best region on the plates.

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[3] *Researches on the Evolution of the Stellar Systems*, Vol. II, p. 583.

[4] *The Spectrum of δ Cephei*, by W. S. Adams and H. Shapley. *Proc. Nat. Acad. Sc.*, 2, 136, 1916.

Element	Wave length	Maximum velocity	Minimum velocity	Character of the line
Ti	4501.46	Sharp	More diffuse	Enhanced
Mn	4502.4	Faint	Stronger	Arc
Fe	4508.45	Broad	Fainter	Enhanced
Fe	4515.50	Fairly strong	A little weaker	Enhanced
Fe	4520.39	Fairly strong	A little weaker	Enhanced
Fe	4522.80	Strong	Weaker	Enhanced
Mn	4523.57	Weak	Stronger	Arc
Ca	4527.2	Same strength	Same strength	
Ti	4534.1	Strong	Weaker	Enhanced
Ti Fe	4598.8	Very strong	Much weaker	Enhanced
Fe	4550.9	Weak	Much stronger	Arc
Ti-Fe	4552.5	Same strength	Same strength	Arc
Ba	4554.2	Same strength	Same strength	
Ti	4555.6	Weak	Stronger	Arc
Fe	4556.1	Rather strong	Weaker	Enhanced
Ti	4572.1	Rather strong	Weaker	Enhanced
Fe	4574.6	Weak	Stronger	Arc

Besides these lines, however, some arc lines seem to show a variation of the same sense as the enhanced lines.

In 1917 the star seems to have an earlier class of spectrum near maximum than near minimum velocity; but, which is interesting, *this is reversed in 1900 and 1912.* In these two years the star seems to be of an earlier type of spectrum near minimum than near maximum velocity: the enhanced lines in general seem to be stronger at minimum than at maximum velocity.

This apparently strange behavior of the spectrum of κ *Pegasi,* and especially the reversal of the phenomenon from one epoch to the other, is readily explained by the superposition on the first spectrum of the spectrum of the second visual component. This last component is possibly of the same type. When these enhanced lines of the second spectrum coincide with those of the first, they are reinforced, while the other lines do not change in intensity. Some dissymmetry appears on one side of enhanced lines when we are near, but not at the exact coincidence. When the difference in the velocity of the two components is great the strong lines of the second spectrum appear sufficiently distinct for measurement on three of the fine-grained plates of 1917. In other words, the phenomena occurring are produced by the superposition of an F class spectrum oscillating very quickly (six days) upon a spectrum of possibly the same class oscillating by a smaller amount and very slowly (period 11.35 years).

It is with pleasure that I acknowledge my indebtedness to Director Campbell for his valuable advice and supervision in the present study. I also want to express to him my gratitude for having so generously assigned me the Martin Kellogg Fellowship, and for his deep sympathy with the unfortunate citizens of my stricken country, Belgium. I also wish to thank Dr. Aitken, Dr. Paddock, and especially Dr. Moore, for the help they have given me in the present work.

F. HENROTEAU.

MT. HAMILTON,
December 15, 1917.
Issued January 15, 1918.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 305

ON FINITE VELOCITY OF GRAVITATION AS A POSSIBLE FACTOR IN STELLAR EVOLUTION

The subject of a finite velocity for gravitation has been treated by a number of authors with the uniform conclusion, except on the part of the relativitists, that the velocity is either infinite or at least many times greater than that of light. Tisserand in the fourth volume of his celestial mechanics gives a bibliography and summary of the work done by previous authors. A careful search through various indices of scientific publications since that date has shown no new material, except that based on the postulates of relativity.

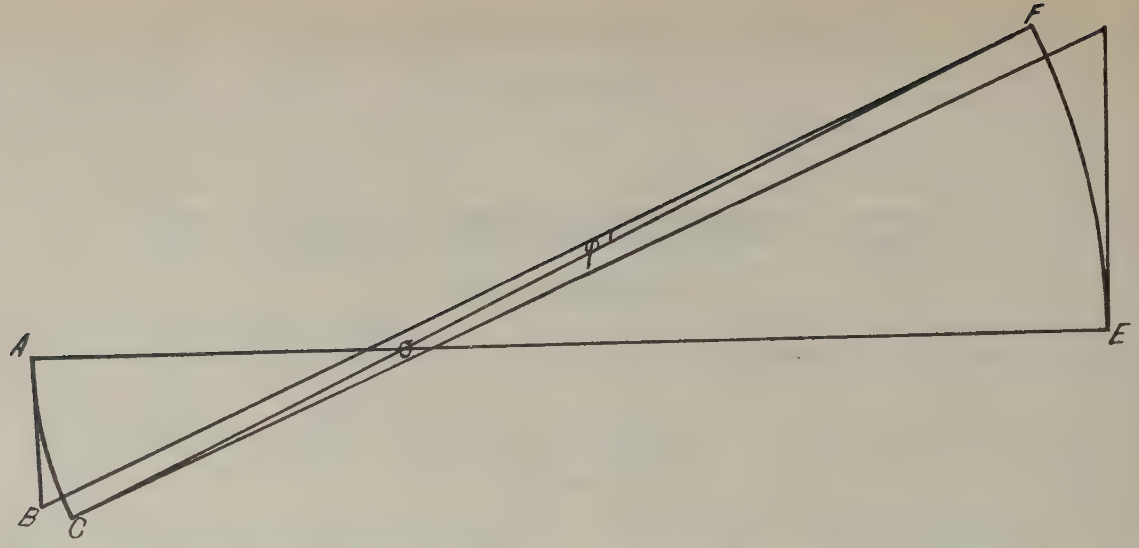
Perhaps the most thorough treatment of this problem is that by Hepperger in *Sitzungsberichte der Akademie der Wissenschaften in Wien,* 1888, Seite 337–362. In this paper he discusses the effects of such a velocity on the orbits of the planets, separating those due to the velocity of the Sun in space, due to the motion of the Sun because of the attraction of the planets, and due to the rotation of the Sun. His conclusion is that the velocity of gravitation must be at least a thousand times as great as that of light. The relativitists, on the other hand, making certain assumptions regarding mass, variation of dimensions, etc., find no difficulty in assuming a velocity as small as that of light.

In all the papers which I have been able to find on the subject the authors have made one common assumption, apparently without realizing that it is a mere assumption and that another may be fully as logical. They assume that the gravitational field generated at any instant radiates for all time from the point occupied by the center of mass of the creating body at the instant of creation of the field. Also that the velocity of the field away from this point is the same in all directions, irrespective of the velocity of the body. In this paper I shall attempt to show that in view of certain facts of stellar evolution and in view of the Michelson-Morley experiment it is simpler to assume that the centers of gravitational and light disturbances move through space with the velocities which the bodies have at the instants of generating the fields. By this assumption we gain many of the advantages of the relativitists, without the use of varying masses and dimensions, non-Euclidean geometries, a maximum velocity, etc. The assumption approaches relativity at several points, yet in the final analysis is not relativity, for it does not necessarily imply relativity's primary postulates.

Following are the assumptions made in this paper:

1. Gravitation has a finite velocity. In calculations the velocity of light will be used.
2. The center of any gravitational field is not subject to accelerations, but moves with the velocity possessed by the body at the instant of generating the field. (If we were to assume the center of the field accelerated the same as the body, it could be shown that the conditions of infinite velocity would be almost perfectly reproduced, irrespective of any assumed high velocity of gravitation, which might even be less than the velocity of light.)
3. All assumptions of the ordinary Newtonian mechanics, except as stated above.

Let us consider two attracting bodies of which the primary is the larger. At the instant t_0 we know the velocities of each and the distance between them. We also know the masses m' and m''. Consequently we know perfectly all their future movements under the assumptions of the ordinary two-body problem. Here we shall be concerned with the divergencies from these movements, produced by the foregoing postulates. At our starting instant the primary is at A and the secondary at E. After the interval Δt which it takes a gravitational impulse to travel from the primary to the secondary, let the secondary be at F and the primary at C. Had the primary moved in a straight



line with constant initial speed it would have been at B. According to our assumptions, the center of the gravitational field created at the instant the primary was at A is now at B. If it had moved with the body to C, instead of to B, the effect would be that of infinite velocity to a much higher order of agreement than under our hypothesis, no matter how slow the velocity of the disturbance. The effect on the secondary is that which we would have if the velocity of the gravitational field were infinite and the primary were now actually at B, for in both these cases the line of force would be along FB. If the secondary were infinitesimal, so that the primary remained at A, or traveled to B instead of C, we would have our standard two-body problem, to an almost perfect agreement. This is the important fact that we must remember before taking up a study of the effects of our hypothesis on the solar system and on double stars, which are of comparable magnitude. The distance CB is very small even in the most rapidly moving double stars, and in the solar system is practically zero. The larger the value of CB, or rather of the angle CFB, the greater is the effect due to a finite velocity of the field.

The following causes tend to make this angle large. The reasons are evident in each case.

1. Equality of masses. If one mass is infinitesimal, there is no measurable difference from the normal two-body problem. The nearer the masses to equality the greater is the effect produced by the secondary on the primary. As explained above, the only perturbation or divergence from the normal path of the secondary large enough for us to consider is that due to this second order action.

2. The angular velocity of the masses. The normal acceleration in the case of circular motion is $\omega^2\rho$. This factor entering in the second power is very important.

Let us designate the angle CFB by ϕ and the radial force which we would have under the assumption of infinite velocity of gravitation by r. This force would act in the direction of FC. We can neglect any difference in the lengths CF and BF. This difference is always smaller than the length of CB. We have, then, a force r acting in the direction FB. We will resolve it into its components in the direction FC and perpendicular to FC. The first of these is $r\cos\phi$ and the second $r\sin\phi$. For convenience we will write $R = r - r\cos\phi$ and $S = r\sin\phi$. When we make calculations we will find that R is very much smaller than S, and may be set equal to zero. Our error from assuming $FB = FC$ is seen at once to be even of a higher order than this quantity that we shall usually neglect. There is no component perpendicular to the orbit plane.

The following variations of the elements are from Watson's *Theoretical Astronomy*, pp. 518 *et seq.* All terms containing Z as a factor have been set equal to zero.

$$\frac{da}{dt} = \frac{2a^2}{k\sqrt{p(1+m)}}\left(e\sin g\,R + \frac{p}{r}S\right)$$

For the circle this becomes $\dfrac{da}{dt} = \dfrac{2a^{\frac{3}{2}}}{k\sqrt{1+m}}S$

$$\frac{dp}{dt} = \frac{2\sqrt{pr}}{k\sqrt{1+m}}S$$

$$\frac{de}{dt} = \frac{1}{k\sqrt{p(1+m)}}\left[p\sin v\,R + p(\cos v + \cos E)\,S\right]$$

$$\frac{d\Omega}{dt} = \frac{di}{dt} = 0$$

$$\frac{d\pi}{dt} = \frac{1}{k\sqrt{p(1+m)}}\,\frac{1}{e}\left[-p\cos g\,R + (p+r)\sin g\,S\right]$$

The problem may now be treated in three ways:

1. The equations may be integrated if possible.
2. The equations may be expanded in series and integrated term by term.
3. The perturbations may be obtained by a method of special perturbations.

The last of these methods is very easy and the calculation is very short. It will, therefore, be adopted for this preliminary paper. Let us divide the path of the primary into n parts, enough being chosen that the motion during the interval P/n is sensibly linear. The derivative of each element with respect to the time may be computed for each part of the orbit. The sum of these derivatives multipled by P/n is the total change in the element during one complete revolution. Since the changes in the perturbations for successive revolutions depend merely upon the changes produced in the elements, we may multiply the changes produced for one revolution by the total number of revolutions as long as the elements are not seriously changed. When this occurs we will compute the derivatives again as before, but on the basis of the new elements and repeat the operation. Even in the most rapidly moving double stars this will be unnecessary for several centuries.

Attention is called to the fact that da/dt is always positive and that, although de/dt may be negative, the greater value of S when positive will cause it to integrate to a positive variation in e when a complete revolution is taken. This means that there is a secular perturbation which continually forces the two bodies farther apart and makes the orbit more eccentric. It is a well known fact that early type spectroscopic binaries have the shortest periods and the nearest to circular orbits. Tidal action seems entirely inadequate to account for this. If it can be shown that the changes in the orbits of double stars are of considerable magnitude and that the changes in the solar system are smaller than can be detected by our best observations, the hypothesis at once becomes worthy of attention. If in addition we can explain the Michelson-Morley and electro-magnetic experiments as well as is done by the postulates of relativity without running counter to the results of any experiments, a further development of the hypothesis with respect to electro-magnetic laws will be warranted.

I have computed the perturbations in the case of σ *Aquilae.* The elements of the orbit of this binary are published by Professor Jordan in the *Publications of the Allegheny Observatory,* Vol. III, No. 22. They follow:

$P = 1.9502$ days ± 0.0001
$e = 0.0$
$a_p = 4{,}380{,}000$ km. (assuming $\sin i = 1$)
$a_s = 5{,}340{,}000$ km.

As stated previously, I have assumed the velocity of gravitation to be that of light for the purposes of calculation. Gravitation will then require 32^{s}4 to travel the distance between the bodies. Since the orbit is circular, the bodies move with constant speed. The arc CA is, therefore, equal in length to the straight line AB. The tangent of the angle BOA is equal to the angle COA, which is equal to 32.4 (assumed exact), multiplied by the number of seconds in a circumference and divided by the number of seconds of time in P. This operation gives $COA = 249''.20520$. We then find $BOA = 249''.20514$. These values are based on an eight-place calculation and are accurate to one in the last place. Since the masses are nearly equal, the angle CFB is half COB or $0''.00003$. CFB is ϕ. Let us choose, as the unit of distance, that from the center of mass of the system to the secondary, i.e., OF. We will choose the mass of the secondary as unity and the unit of time as the mean solar day. Therefore the radial force, under the assumption of infinite velocity of gravitation is ω^2. $S = \omega^2 \sin 0''.00003$. $R = -\omega^2 (1 - \cos 0''.00003)$. As stated above, R may be neglected in a first approximation. $\text{Log}\, S = 1.178898 - 10$. Since a is in this case taken as unity, $da/dt = 2S$. $\text{Log}\, da/dt = 1.479928 - 10$. Multiplying this by the number of days in a century, we get $7.042518 - 10$ as the logarithm of the total change in a. We will add this increment to the present value of a, raise to the $3/2$ power and multiply by the present value of P. We get, therefore, the value of P for a hundred years hence as 1^{d}9534, an increase of 0^{d}0032 over the present value. Since the probable error in the period is only 0^{d}0001, the computed change may probably be tested within the next quarter of a century. It is unfortunate that there is a third body in the system of Algol, for otherwise we could make this demonstration at once. However, there is, even in the case of Algol, some supporting evidence, for the Berlin Jahrbuch of 1787 gives a period shorter than that which Algol has at present, although there is now a tendency to ascribe a periodic character to this variation.

It is now necessary for us to show that the effect in the solar system is smaller than it is possible to measure. An attempt was made to calculate the effect directly in the case of *Jupiter.* However, the computation required more than eight places of logarithms and was abandoned. It is possible from the double-star data given here to get a quite approximate value of the perturbation, and this indirect method was used. In this case the angle ϕ has about $\frac{1}{12500}$ the value we found for σ *Aquilae.* ω is about $(\frac{1}{2200})$ its value in that case. Therefore, taking *Jupiter's* distance from the Sun as unity, we get $da/dt = \frac{1}{12500} (\frac{1}{2200})^2$ times the value for the binary. We find from this that the value of a a hundred years from now will be 1.00000000000002. μ will have changed by about 0″.000000000009. The displacement of the planet will be less than 0″.0001.

Every one is familiar with the Michelson-Morley experiment. The negative results obtained are those to be expected from this hypothesis if applied to the velocity of light. It is therefore worth while to note that it may be possible to test our hypothesis by a modification of the experiment. If we will admit light to the apparatus directly from the Sun when on the horizon, instead of from a source of light which moves with the Earth, and if we then change to an artificial source, a movement of the interference bands should be produced. Using the artificial source, the source moves through space with the mirror A and the time according to our hypothesis, from A to B and back is exactly equal to that from A to C and back, no matter what the orientation. If, however, we admit sunlight directly, the time of passage from A to C is changed, lengthened if the apparatus is receding from the Sun and shortened if approaching.

An almost exactly analogous hypothesis has been made in the case of light by Ritz and by Comstock. The arguments for and against such an hypothesis for light have been discussed by Comstock in the *Physical Review,* Vol. 30, No. 2, under the title, "A Neglected Type of Relativity," and by de Sitter in *Physikalische Zeitschrift,* Vol. 14. The arguments against such an hypothesis for light have considerable weight, but do not seem to be absolutely conclusive. It has been pointed out by Tolman, about 1912, that with a repetition of the Michelson-Morley experiment modified so as first to admit light to the apparatus directly from the Sun when on the horizon, instead of from a source of light which moves with the Earth and later from such an artificial source, a movement of the interference bands should be found if the hypothesis is true for light. Such an experiment would be of great value and could be performed with some modifications of the original apparatus. I am indebted to Professor Tolman for the interest he has taken in this paper and for pointing out the negative evidence regarding emission theories of light. A velocity of propagation which depends on the velocity of the source need not necessarily demand an emission hypothesis. Certain results observed in the firing of big guns may be published later. Preliminary evidence tends to indicate that the velocity of sound waves is affected by the velocity of the source. This speculation is not necessarily intended to associate the velocity of light with that of gravitation, the use of the velocity of light in the numerical applications being purely arbitrary and a matter of convenience.

A speculation such as this will, by analogy, be carried at once to the study of the composition of atoms, in view of our present theories regarding them as composed of positive and negative electrons. Atoms in which the masses of positive and negative electrons were at all comparable would be much more unstable than others. It seems that a complete investigation of all electro-magnetic and radio-active effects might well be worth the labor of an immediate study.

We find that our hypothesis requires a rather rapid increase in the distances of binaries like σ *Aquilae.* Its effects on the solar system are much too small to measure for centuries. In sufficient time they would force the larger planets farther from the Sun than the smaller ones, a condition which is approximately fulfilled. We need consider only the action due to the planet's own mass in this problem, because all other perturbations produced must be periodic. If true, we would have a rough relative measure of the age of the planets. The effect in separating compact star clusters is probably quite important. Astronomers have felt the insufficiency of tidal action to account for phenomena similar to these. May it not be that here we have the required explanation?

I desire to thank Professor Leuschner for the encouragement and suggestions he has given me and for reading the paper before the Philadelphia meeting of the National Academy of Sciences, November, 1918.

DINSMORE ALTER.

BERKELEY ASTRONOMICAL DEPARTMENT,
May, 1917.

Issued February 4, 1918.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 306

ONE HUNDRED NEW DOUBLE STARS

TWENTY-FOURTH LIST

The systematic survey of the sky initiated in April, 1899, was completed early in 1915 for the area from the North Pole to —14° declination. Between 13^h0 and 1^h0 right ascension it was completed to —22° declination. The majority of the double stars given in the present list, like the ones in my preceding lists, were discovered in that survey.

In examining many thousands of stars it was inevitable that a considerable number should be found which did not look like single stars and yet could not with certainty be seen double. All such were specially marked on my charts and a number of them have recently been reëxamined. The result has been the discovery of several close or difficult pairs, among them A 2901, A 2906, A 2984, and A 2985. The angular separation in at least the last two of these has probably increased since the date of the first examination.

Twelve pairs in this list were discovered in the course of the examination of a few of the charts in the small part of the zone —14° to —18° which was not completed in 1915.

A list containing 45 unpublished new double stars was sent to M. Robert Jonckheere in the summer of 1915, and these pairs are included in his *Catalogue and Measures of Double Stars Discovered Visually from 1905 to 1916* (*Memoirs, Royal Astronomical Society,* LXI, 1917) with the designation A 29—. Two of these pairs, nos. 1650 and 1962, have as yet been measured on one night only and are therefore not included in the present list. Their designation should be changed from A 29— to A 30—.

Robert G. Aitken.

January, 1918.

			R.A. 1900.0			Decl. 1900.0		Date	Angle	Dist.	Mag.	B.D. Mag.	n	
A 2901	A.G. Christiania	190	0^h	54^m	55^s	+68°	49′	1917.85	9°9	0″18	7.6– 7.6	7.0	2	
2902	A.G. Strassburg	634	2	31	58	— 4	59	15.77	339.2	1.46	9.0–12.5	9.0	2	
2903	A.G. Washington	704		34	51	—16	37	18.02	190.6	3.41	9.0–13.0	9.0	2	
2904	A.G. Washington	716		36	46	—17	13	18.02	181.0	2.99	8.0–13.8	8.0	2	
2905	A.G. Washington	737		42	15	—14	23	16.84	185.2	3.39	9.0–12.0	9.0	2	
2906[1]	A.G. Harvard	1310		45	46	+52	35	17.86	115.5	0.23	7.9– 9.2	......	2	A and B
								17.86	303.2	1.42	7.6– 7.8	8.0	2	AB and C
2907[2]	A.G. Washington	767	2	49	02	—16	28	16.84	131.0	1.72	8.9–13.0	8.9	2	
2908	A.G. Washington	897	3	19	26	—17	39	17.86	255.6	4.26	8.9–12.2	8.8	2	
2909	A.G. Washington	901		19	45	—16	00	18.02	162.5	0.13	7.4– 7.4	6.7	2	A and B
								18.00	252.9	16.69	13.5	......	1	AB and C
2910	A.G. Washington	911		22	04	—17	38	18.02	279.4	1.98	8.6–11.0	8.6	2	

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[1] A 2906. The new pair forms the brighter component of Σ 314. Elongation was suspected in 1906, but no measures were made until October, 1917. The Struve pair shows very slow, direct angular motion.

[2] A 2907. This is the preceding and brighter star of a very wide pair.

			R.A. 1900.0			Decl. 1900.0		Date	Angle	Dist.	Mag.	B.D. Mag.	n	
A 2911	A.G. Washington	915	0h	23m	01s	—14°	43′	1918.02	115°0	1″96	6.8–13.7	6.8	2	
2912	A.G. Washington	977		37	38	—16	17	18.05	24.8	1.28	7.3–11.5	7.3	2	
2913	A.G. Washington	1076	3	59	08	—16	02	15.59	339.6	0.45	9.0– 9.0	8.4	2	
2914	A.G. Washington	1110	4	05	08	—17	09	15.03	39.3	1.66	9.6– 9.6	9.0	2	
2915	A.G. Washington	1265	4	32	22	—17	45	17.18	341.6	0.26	9.6– 9.6	9.0	3	
2916	A.G. Strassburg	1654	5	30	12	— 1	58	15.81	61.8	0.79	9.0–13.0	9.0	2	
2917	A.G. Nicolajew	1402		30	30	— 1	55	15.81	263.2	0.60	8.9–11.2	8.9	2	
2918	A.G. Strassburg	1755		39	03	— 2	32	15.81	180.4	0.40	9.7–10.4	9.2	2	B and C
								15.80	315.6	95.68	9.0– 9.2	......	2	A and BC
2919	A.G. Strassburg	1808		48	23	— 3	34	15.81	334.6	0.37	8.8– 9.6	8.5	2	A and B
								15.80	324.0	14.50	8.5–13.5	......	1	AB and C
2920	A.G. Strassburg	1854	5	53	28	— 5	03	15.43	223.9	4.66	9.0–13.0	9.0	2	
2921	A.G. Strassburg	1904	6	00	38	— 4	47	15.97	176.5	1.70	8.8–10.5	8.8	2	
2922	A.G. Vienna	1790		03	01	— 7	20	15.99	344.8	0.90	9.0–12.2	9.0	2	
2923	A.G. Vienna	1870		10	44	— 9	23	15.99	125.5	1.05	9.6– 9.6	9.0	2	
2924	A.G. Strassburg	2037		15	12	— 5	33	15.98	104.1	1.68	9.0–14.0	9.0	2	
2925	A.G. Cambridge (U. S.)	1886		15	16	—10	36	15.11	141.6	0.30	9.2– 9.2	8.5	3	
2926	A.G. Strassburg	2142		26	01	— 5	49	16.18	278.2	3.80	9.0–13.0	9.0	2	
2927[3]	A.G. Leipzig II	2988		26	37	+ 5	01	14.65	284.6	3.14	7.4–13.1	7.3	2	AB
								14.84	318.6	6.74	14.0	......	1	AC
								14.65	288.9	12.30	13.2	......	2	AD
								14.65	197.3	13.32	13.5	......	2	AE
2928	A.G. Vienna	2014		27	16	— 7	58	15.97	96.0	1.95	9.6– 9.6	9.0	2	
2929	A.G. Albany	2275		30	09	+ 4	33	14.65	23.8	0.92	9.0–13.0	9.0	3	
2930	A.G. Strassburg	2203		32	43	— 3	36	17.43	1.2	0.26	9.6– 9.6	9.0	2	
2931	A.G. Strassburg	2214		33	29	— 2	19	17.43	121.2	0.26	9.6– 9.6	9.0	2	A and B
								17.43	53.8	14.26	11.7	......	2	AB and C
2932	A.G. Strassburg	2224		34	47	— 5	18	17.43	79.4	1.51	9.6– 9.6	9.0	2	
2933	A.G. Albany	2352		36	18	+ 4	19	14.55	186.3	1.47	9.0–12.1	9.0	2	
2934	A.G. Strassburg	2246		37	06	— 4	40	17.43	144.4	3.18	9.4–10.0	8.9	2	
2935	A.G. Cambridge (U. S.)	2172		43	52	—12	20	15.14	304.6	0.27	9.6– 9.6	9.0	2	
2936	A.G. Vienna	2280	6	51	54	— 8	40	15.98	303.9	0.29	9.2– 9.4	8.6	2	
2937	A.G. Strassburg	2518	7	04	07	— 5	27	15.81	86.7	3.36	9.0–14.7	9.0	2	
2938	A.G. Strassburg	2519		04	13	— 4	46	16.00	77.4	0.28	9.0–11.0	9.0	2	
2939	A.G. Leipzig II	3738		15	21	+ 9	51	14.94	242.8	3.81	7.6–13.8	7.6	2	
2940	A.G. Leipzig II	3744		15	58	+ 9	19	14.94	359.8	1.21	9.0–11.0	9.0	2	
2941	A.G. Leipzig I	2859		15	59	+10	44	14.94	78.9	3.66	9.0–12.5	9.0	2	
2942	A.G. Albany	2793		17	27	+ 1	26	15.21	285.0	1.30	8.8–12.8	8.8	2	
2943	A.G. Nicolajew	2157		19	18	+ 0	49	15.25	88.4	0.58	9.0–12.0	9.0	2	AB
								15.28	280.4	18.90	13.0	......	1	AC
2944	A.G. Leipzig II	3816		20	40	+ 7	59	14.94	172.0	0.56	9.2–10.9	9.0	2	
2945	A.G. Leipzig I	2951		25	19	+10	44	14.94	31.9	3.64	9.0–14.0	9.0	2	
2946	A.G. Leipzig II	3932		28	29	+ 9	26	15.00	267.1	4.59	9.0–13.8	9.0	2	
2947	A.G. Leipzig I	2977		28	36	+11	12	15.00	355.0	4.04	9.0–13.5	9.0	2	
2948	A.G. Leipzig II	3984		32	44	+ 9	44	15.00	299.8	0.56	9.5– 9.5	8.9	2	
2949	A.G. Cambridge (U. S.)	2790		37	02	—11	01	14.51	228.0	4.00	8.8–13.8	8.8	2	
2950	A.G. Vienna	2843		40	26	— 8	46	15.99	124.0	2.90	9.5– 9.7	9.0	2	
2951	A.G. Vienna	2854		41	06	— 9	35	15.99	71.0	1.02	9.5– 9.7	9.0	2	
2952	A.G. Leipzig II	4197		47	48	+ 8	26	15.05	49.8	0.30	9.2–10.4	9.0	2	A and B
								14.89	230.8	9.47	14.0	......	1	AB and C
2953[4]	A.G. Leipzig I	3153		49	35	+10	39	14.94	162.0	1.93	9.2–13.1	9.2	2	
2954	A.G. Leipzig I	3201		55	00	+10	11	15.14	355.8	0.44	9.1– 9.1	8.5	2	A and B
								15.14	42.0	3.46	8.5–12.5	......	2	AB and C
2955	A.G. Leipzig I	3205		55	18	+10	59	15.24	7.2	0.65	8.5–11.8	8.5	3	

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[3] A 2927. The principal star is possibly an exceedingly close pair, equal magnitudes. Many other stars in the field of view.

[4] A 2953. A star of 8.9 magnitude is n. fol. in the 520-power field.

			R.A. 1900.0			Decl. 1900.0		Date	Angle	Dist.	Mag.	B.D. Mag.	*n*	
A 2956	A. G. Leipzig I	3223	7h	57m	47s	+10°	13′	1915.14	170°7	4″61	8.0–13.8	8.0	2	
2957	A.G. Leipzig II	4360	8	01	34	+ 9	10	15.19	58.6	0.54	9.3–10.1	8.9	3	
2958	A.G. Leipzig II	4458		10	09	+ 7	57	15.25	286.8	0.56	8.3–11.5	8.3	2	
2859	A.G. Leipzig I	3352		12	08	+11	09	15.20	266.2	0.76	9.0–10.8	8.8	2	
2960	A.G. Leipzig II	4555		18	23	+ 7	52	15.06	48.3	0.22	9.3– 9.6	8.8	2	
2961[5]	A.G. Leipzig I	3389		18	27	+10	57	15.25	259.3	0.98	6.5–11.5	6.5	2	
2962[6]	A.G. Leipzig II	4703		32	38	+ 6	45	14.87	253.0	1.94	9.0–13.2	9.3	2	
2963	A.G. Leipzig II	4732		36	07	+ 7	11	14.86	260.2	3.24	9.2–10.1	9.0	2	
2964	A.G. Leipzig I	3576		48	03	+11	32	15.06	32.0	1.13	8.7–12.5	8.7	2	
2965	A.G. Leipzig I	3586		49	07	+11	11	15.06	148.2	4.14	8.8–13.4	8.8	2	
2966	A.G. Strassburg	3454		49	19	— 5	31	17.14	319.4	0.68	9.4–10.0	9.0	2	
2967	A.G. Leipzig I	3590		50	01	+11	02	15.08	0.7	4.86	9.0–12.8	9.0	2	
2968	A.G. Leipzig I	3598		51	41	+11	08	15.14	315.9	1.12	9.1– 9.1	8.5	2	
2969	A.G. Leipzig I	3613		54	40	+11	01	15.14	353.2	2.86	9.0–13.0	9.0	2	
2970	A.G. Strassburg	3497		56	20	— 2	25	17.14	318.1	2.90	9.0–12.8	9.0	2	
2971	A.G. Leipzig II	4939	8	58	55	+ 6	10	14.82	341.7	0.33	9.4– 9.4	8.8	2	
2972[7]	A.G. Vienna	3581	9	07	15	— 7	04	16.15	338.9	3.80	9.7–10.1	9.0	2	
2973	A.G. Vienna	3585		07	29	— 6	31	16.15	9.0	2.58	7.7–12.5	7.7	2	
2974	A.G. Leipzig I	3686		07	32	+11	05	15.06	42.8	1.33	8.0–11.0	8.0	2	
2975	A.G. Leipzig I	3706		13	35	+11	22	15.13	356.8	0.66	8.0–11.2	8.0	2	
2976	A.G. Leipzig II	5066		15	22	+ 8	35	15.18	139.9	0.74	10.8–13.0	......	2	B and C
								15.06	236.8	29.05	9.0–10.8	9.0	1	A and B
2977	A.G. Leipzig II	5084		17	14	+ 7	51	15.14	265.5	1.40	8.8–13.2	8.8	2	
2978	A.G. Leipzig II	5127		22	36	+ 8	44	15.19	99.2	1.37	8.5–12.2	8.5	3	
2978	A.G. Strassburg	3824	9	44	23	— 4	25	18.02	241.6	1.89	8.6–13.0	8.6	2	
2980	A.G. Strassburg	3935	10	04	58	— 6	08	17.12	203.0	0.40	9.6– 9.6	9.0	2	
2981	A.G. Vienna	3947	10	11	51	— 9	16	16.16	177.5	2.05	9.0–11.5	9.0	2	
2982[8]	Anon		12	02	04	— 5	34	17.09	76.4	0.46	11.0–11.3	......	2	
2983[9]	A.G. Leipzig I	5218	14	44	24	+10	38	16.40	7.4	0.19	9.1– 9.1	8.5	3	
2984[10]	41 *Ophiuchi*		17	11	29	— 0	20	15.54	298.3	0.52	4.6– 7.6	4.3	3	
2985[11]	A.G. Albany	5841		30	43	+ 4	13	16.54	333.7	0.32	9.1– 9.1	8.5	2	A and B
								16.54	176.2	3.21	10.0	......	1	AB and C
2986	A.G. Nicolajew	4372		33	24	— 1	18	15.71	63.9	0.81	9.5–10.4	9.3	2	
2987	A.G. Lund	7384	17	51	46	+35	33	17.65	60.9	1.17	9.0–13.2	9.0	2	
2988	A.G. Berlin B	6602	18	36	53	+24	44	16.43	12.4	0.19	8.5– 8.5	7.9	3	
2989	A.G. Cambridge	9332		49	21	+25	46	16.42	227.4	0.41	8.7– 9.7	8.4	2	
2990[12]	A.G. Cambridge	9428		54	25	+25	07	16.46	139.4	2.89	7.9–14.5	7.9	3	A and B
								16.46	43.1	14.46	14.5	......	2	A and C
2991	A.G. Berlin B	6812		59	17	+25	03	16.44	82.2	0.63	9.4–10.1	9.0	2	
2992	A.G. Cambridge	9538	18	59	48	+26	33	16.44	59.7	0.23	9.4– 9.4	8.8	3	
2993	A.G. Strassburg	6810	19	41	13	— 2	03	15.67	159.7	0.24	9.3– 9.5	8.7	2	
2994[13]	A.G. Leipzig II	9585	19	45	24	+ 6	58	16.55	198.0	0.39	10.2–10.2	......	2	B and C
								16.55	273.8	5.54	9.6– 9.6	9.0	2	A and BC
2995[14]	A.G. Berlin B	7484	20	02	28	+24	09	16.44	59.0	1.64	8.4–12.5	8.4	2	

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[5] A 2961. The bright star is 21 *Cancri*. The magnitude in *H. A.* 50 is 6.29, the spectral class, K 5 p. According to Boss, the proper motion is 0″021 in 159°8.

[6] A 2962. The Leipzig catalogue gives the magnitude 8.7. It is certainly as bright as 9.0.

[7] A 2972. The star is certainly fainter than 9.0; it is marked 9.6 in the A.G. catalogue.

[8] A 2982. This pair is too faint to be in the B.D. It is 141″85 in 194°9 from the 7.8 magnitude star B.D. —5°3423 = A.G. Strassburg 4498.

[9] A 2983. The angle measures of this pair are unusually discordant, the extreme range being 12°2.

[10] A 2984. *H. A.* 50 gives the magnitude of 41 *Ophiuchi* as 4.82 and its spectrum as G 5. According to Boss's P.G. catalogue, the proper motion is 0″065 in 204°6. The pair was "suspected" in 1905, but was then too difficult to measure.

[11] A 2985. The wide pair is Σ 2187 and has shown no change since 1830. The fact that the brighter component was double was suspected in 1910, but could not be verified. The angular separation has probably increased in the interval to 1916.

[12] A 2990. The bright star is strong orange color, the faint stars dull and apparently blue.

[13] A 2994. The wide pair is called A.G. 238 in Burnham's G.C. and was noted as "duplex" by the Leipzig A.G. observers.

[14] A 2995. There are six or seven faint stars in the field, near the double.

			R.A. 1900.0	Decl. 1900.0	Date	Angle	Dist.	Mag.	B.D. Mag.	n
A 2996	A.G. Cambridge	10858	0h 03m 03s	+25° 54′	1916.41	241°.1	2″.64	8.2–13.4	8.2	2
2997	A.G. Vienna	7114	06 30	— 7 48	15.66	72.8	1.75	9.0–13.6	9.0	2
2998	A.G. Vienna	7237	19 35	— 7 07	15.71	9.9	0.47	9.1–10.5	9.0	2
2999	A.G. Vienna	7412	39 19	— 7 43	15.74	228.6	1.95	9.5–10.0	9.0	2
3000	A.G. Vienna	7422	20 40 11	— 7 37	1915.75	61.3	0.55	9.0–11.2	9.0	2

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THE MAGNITUDE EQUATION AND THE VISUAL SCALE

Observations made with a screen over the objective of the meridian instrument, for the purpose of determining the magnitude equation, can be used to test the visual magnitude scale, and to indicate the variation in the unit light ratio of this scale for different brightness of stars. The reduction in light, due to the interposition of a screen, when measured by the difference of magnitude between a bright star and its screened image, is the so-called screen value, and this should be constant if the various classes of stars have a uniform light ratio. Diaphragms can also be used to reduce the brightness, and in this case the reduction in light can be computed. For screens the amount of the reduction can not be calculated, but must be determined by observation.

In the usual photometric methods the brightness of a star is reduced by a varying amount, subject to measurement, to bring the reduced image into equality either with a star adopted as standard, or with an artificial light of constant brightness. The employment of screens, or diaphragms, may be considered as a photometric process, in which the amount of the reduction in light is constant for any particular screen or diaphragm. In my experience with screens, the observed decrease in visual magnitude has been found to exhibit a consistent variation, within the range of the scale between the brightest classes of stars and the faintest that can be observed with a dense screen. Observations with screens of less density, and with diaphragms, have shown similar variations. The variation in screen value can be best explained by a difference of unit light ratio, between the bright and faint stars of the visual scale. Such a difference has been recognized as existing, and the screen results give a definite indication of the amount.

The adopted visual scale has been built up progressively, and in the earlier stages was not subject to any exact measures of light ratio. For the bright stars, it dates back to the earliest classification by naked eye estimates. Six grades of brilliancy were first adopted, and there was wide variety in the intrinsic brightness of stars assigned to any one of the grades. The successive uranometries were designed to classify the same stars more precisely, in fractions of a unit of magnitude. Even in the Southern Uranometry of Dr. Gould, the most elaborate of these catalogues of naked eye stars, no attempt was made to grade the stars by any empirical light ratio, as has been stated in the Introduction.[1] The design was to carry forward, into the southern sky, the same scale that had been adopted in the northern uranometries. When photometers were employed, the design was plainly to reproduce the adopted visual scale as a general system, with such improvement in grading individual stars, and in the classes, as could be introduced by more exact interpolation. In the successive approximations to a nearly uniform light ratio, the sixth magnitude has come to represent a reasonably definite place in the visual scale. It may be taken to be the dividing point between naked eye and telescopic stars, and as the starting point for the magnitude scale of the latter.

The visual scale for the fainter stars has been established, down to the ninth magnitude, by the estimates of Argelander's *Durchmusterung* of the northern sky. His scale was in practical conformity with those of the best earlier observers with the meridian circle, and it has been adopted by modern observers with this instrument. The magnitudes of the international scheme of zone catalogues, now virtually complete from 80° north to 23° south, agree quite precisely with Argelander's scale. Indirect means of comparison show that Gould carried forward the same scale, in his southern zones, at Cordoba. The ninth magnitude has become a well established grade, in the course of meridian circle and durchmusterung observations, and the estimation of brighter stars, grading back to the naked eye classes, is a matter of interpolation.

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[1] *Uranometria Argentina*, page 13.

The extension of the visual durchmusterung to the southern sky, first by Dr. Schönfeld to —23°, and later at Cordoba to —62°, has included fainter grades of stars than Argelander's limit of 9.0 for the complete enumeration. With his definite limit of completeness, Argelander observed many stars of 9½, and fainter. Schönfeld, with an enumeration designed to be complete to 9.5, includes many stars of tenth magnitude. Cordoba, with an enumeration designed to be complete to the tenth magnitude, has purposely included many stars fainter than the limiting grade. Schönfeld's scale is in good conformity with that of Argelander, to his lowest grade. The ninth magnitude stars appear to have been estimated nearly a quarter of a magnitude brighter than Argelander's scale, in the first two volumes of the Cordoba *Durchmusterung.*

The visual scale is then defined by the uranometries and the several parts of the durchmusterung. It is reproduced in our ephemerides and in catalogues of fainter stars. These catalogues include nearly all of the ninth magnitude stars in the sky. For stars fainter than that grade, the scale of estimates has to be extended below that of the usual practice of observers with meridian instruments; and such an extension is subject to possible variation in the unit light ratio, such as has undoubtedly developed, to some extent, in the faintest classes of stars in the separate parts of the durchmusterung.

Photometric magnitudes do not differ from the visual scale by large amounts, down to the ninth magnitude; but, beyond that point, the recent photometer measures indicate that large systematic differences may be expected. The purpose of the following comparison is not to adjust the visual scale, nor to correct the system of estimates, but rather to show what divergence already exists in the unit light ratios of the adopted visual magnitudes.

Two screens have been employed with the meridian circle of the Lick Observatory, for the observations of the magnitude equation. One screen, of four thicknesses of wire netting, reduces the apparent brightness of a star by an average of three and eighth-tenths magnitudes, and was used in all the observations up to 1914 inclusive, except for the series of 1911. An assistant held this screen in front of the object glass. The screen employed in 1911, and in 1917, is in the form of a cap for the objective, and no assistance is required in its use. It weighs eight ounces and has two thicknesses of accurately spaced wire gauze. This screen reduces the apparent brightness of a star by an average of four and six-tenths magnitudes. The weight of the cap might presumably change the flexure of the telescope, as ordinarily measured, but it is not used in zenith distance observations. Variations from the observed average values of these screens follow, in tabulated form. And the recent tests of two corresponding screens of less density, one for hand manipulation and one of the cap form, are also tabulated below, with some tests of the reduction by diaphragms.

The magnitude equation resulting from the first series below was printed in Volume IV of our *Publications.* It was determined for stars south of the zenith, only. Each of the following tables gives the number of observations used for screen value at the Lick Observatory (an equal number having been made full brightness), the limits of magnitude included in each group of results, the mean of the magnitudes of the groups, and the mean of the observed screen values. In general, the screen values will be found to diminish as the average brightness of the group diminishes. The rate of decrease in screen values is very closely one quarter of a unit for each full unit of the magnitude scale. The effect is that of a relative shortening of the scale for the faint stars, each faint grade being estimated too bright, in comparison with the stars of brighter grades. For the faint end of the visual scale, three-quarters of a unit of magnitude would apear to correspond, in light ratio, to a full unit of magnitude at the bright end of the scale included. This corresponds to a total change in unit light ratio of twenty-five per cent. Indications of this variation had been noted in some of my earlier computations of magnitude equation, and reference had been made to the apparent differences of screen values.[2]

A. 1896

No. Obs.	Limits	Mean	Screen Value
24	1.1 to 2.8 Mag.	2.5 Mag.	4.3 Mag.
28	3.0 3.9	3.4	4.0
16	4.0 5.2	4.3	4.0

The magnitude equation for south stars from the series next following was printed in Volume VI of our *Publications.*

B. 1898

27	1.2 to 3.0 Mag.	2.5 Mag.	3.5 Mag.
33	3.2 4.0	3.5	3.3

The results for magnitude equation from the two next following sets were printed in Volume X of our *Publications.* In 1907 stars were observed on both sides of the zenith, to give the respective equations for the two positions of the observer, facing north and south. In the later sets, observations have been made in both positions.

C. 1902

12	1.3 to 2.6 Mag.	2.1 Mag.	4.3 Mag.
26	3.0 3.6	3.3	3.8
10	4.0 6.0	4.8	3.6

In the series for 1907, a large proportion of the faintest stars that could be observed with the screen was included. The arrangement of the groups is that originally printed in Volume X.

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[2] *Publ. Lick Obs.*, 10, 12, 1907. *Publ. Astron. Soc. Pac.*, 26, 188, 1914.

D. 1907

26	South stars	2.0 to 5.0 Mag.	3.7 Mag.	3.7 Mag.
42	North	1.0 6.0	5.1	3.5
23	South	5.1 6.5	5.7	3.4
49	North	6.0 6.5	6.3	3.2

In the series of 1911, the results were reduced to the photometric scale of magnitudes. For this purpose faint stars were selected from the *Harvard Photometric Catalogue* in Volume 54, and the estimates of their magnitudes on the visual scale were later compared with the photometric magnitudes. The photometric magnitudes were also adopted for the bright stars, observed with screen. The difference of the two scales is as follows:

PHOTOMETRIC SCALE, 1911

No. Stars	Limits	Photom. Mean	Obs.–Photom.
4	6.5 to 6.9 Mag.	6.75 Mag.	+0.25 Mag.
40	7.0 8.0	7.50	—0.12
23	8.0 9.0	8.52	—0.41
16	9.0 9.9	9.37	—0.47

The last column in the table following has been corrected for the observed difference of scales for the faint classes, from the preceding tabulation.

E. 1911

No. Obs.	Limits	Mean	Screen Value	Corrected
13	1.9 to 2.9 Mag.	2.6 Mag.	4.9 Mag.	4.8 Mag.
24	3.0 .39	3.6	4.5	4.7
60	4.0 5.2	4.6	4.3	4.8

The observed variation in screen value by the visual scale does not appear in the above results when reduced to the photometric scale.

The two following sets, the latest made here, one with the hand screen and one with the cap screen, include observations made on both sides of the zenith, but the results for the screen values have been combined without distinction of position. The average visual magnitudes, and the average photometric magnitudes have been tabulated separately. The two tables show that the variation of screen values is not dependent upon the scale of magnitudes adopted for the bright stars.

F. 1914

No. Obs.	Limits	Visual	Screen	Photom.	Screen
16	2.1 to 2.8 Mag.	2.6 Mag.	4.1 Mag.	2.7 Mag.	4.0 Mag.
32	3.0 3.9	3.4	4.3	3.6	4.1
17	4.0 4.9	4.4	3.9	4.5	3.8
13	5.1 6.4	5.7	3.4	5.6	3.5

In the 1917 series, separate combinations have been made for the stars by the visual and photometric scales.

G. 1917. VISUAL SCALE

No. Obs.	Limits	Mean	Screen Value
14	2.3 to 2.8 Mag.	2.6 Mag.	5.0 Mag.
40	3.0 3.9	3.4	4.8
24	4.0 4.9	4.4	4.4
16	5.0 5.8	5.2	4.4

G. 1917. PHOTOMETRIC SCALE

16	2.4 to 3.0 Mag.	2.8 Mag.	4.8 Mag.
34	3.2 3.9	3.6	4.6
32	4.0 4.9	4.5	4.4
12	5.0 5.6	5.1	4.5

The next series following includes some results of screen determinations made at the San Luis Observatory of the Carnegie Institution, in the years 1910 and 1911. The notes which I have on hand are probably not complete for my own work, and do not include any observations of the two assistants who made similar determinations. But the results may be taken as representative, for one observer. The eight-inch telescope of the Dudley Observatory, which we used at San Luis, was provided with both screen and diaphragm, each operated by the transit observer, from the eye end. Besides being used for the determination of magnitude equation, both screen and diaphragm were used constantly in current work, to reduce the brightness of the standard stars, in order that the final corrections to transits, for magnitude equation, should be reduced as much as possible. The diaphragm, of four inches aperture, could be used alone; or both screen and diaphragm could be applied conjointly; but the screen could not be applied without the diaphragm. This gave a choice between reducing the apparent magnitude by about 1½ units, from diaphragm alone, and approximately 5½ units, from screen and diaphragm combined. The measurement of the screen value depends upon estimates of magnitude of the images, when reduced by both screen and diaphragm, from which the effect of the diaphragm alone is to be deducted. Several hundred observations were made for deriving magnitude equation, by each of the five telescope observers. The determinations of screen value were made by the three fundamental observers, only.

The observations tabulated below were made on six nights. The range of magnitudes is small for the screen observations, since a fourth magnitude star was about the faintest that could be satisfactorily observed in transit, with both screen and diaphragm. Stars for observation with the diaphragm alone were selected from the *Harvard Photometry*, and were mostly between seventh and eighth magnitude. The complete plan was to correct our estimates of the diaphragmed stars, to reduce them to the photometric scale of faint stars. This would include a constant of reduction for the diaphragm, to represent the images on the assumed photometric scale. In the two following tabulations, no corrections to the visual scale have been applied, since I have not the complete record for the individual stars employed in the various combinations. The epoch of tables H and J is between those of D and E, since the work at San Luis was performed during my three years' leave of absence from the Lick Observatory. Both screen and diaphragm results are similar to those found here, for observations with a separate instrument. The diaphragm results are first tabulated.

H. San Luis. Diaphragm

No. Obs.	Limits	Mean	Diaphragm Value
25	6.0 to 7.0 Mag.	6.65 Mag.	1.84 Mag.
90	7.0 7.5	7.19	1.66
99	7.5 8.0	7.56	1.47
9	8.0 9.0	8.57	1.49

In the following table, the last column includes the correction for the reduction by the diaphragm, derived with the application of a rate of 0.2 per unit of magnitude, from table H.

J. San Luis

No. Obs.	Mean	Screen and Diaphragm	Screen Value
26	2.7 Mag.	5.9 Mag.	4.2 Mag.
31	3.5	5.6	4.0
32	3.9	5.4	3.8

The mean value of the screen, 4.0 magnitudes, differs slightly from the value, 3.8,[3] finally adopted in the computations of our work, which are made at Albany. The difference may be due to the application of corrections to reduce the visual estimates to the photometric scale. And also a larger number of observations were used in the final computations. The much smaller value, 3.2, for the same screen and instrument, derived from observations at the Dudley Observatory, may be the result of using stars of an average magnitude fainter than the average magnitude of those observed at San Luis. Possibly the Albany value had been derived from the observations of stars without the use of the diaphragm, when naturally much fainter stars could have been observed for magnitude equation.

A variation of screen values, similar to that under discussion here, appears in the results of the observations of magnitude equation made by the late Professor Lewis Boss, in the years 1879 to 1882, during the progress of work on the Albany A. G. zone, in which I was assistant observer. Variations of the same size, approximately 0.2 per unit of magnitude, are evident in his tabulations of the screen values,[4] for which he expressed himself as unable to definitely account. One of his surmises, however, was that the adopted system of magnitudes might not be based upon a true geometric ratio of light. In this connection, he found the quality of the images by the use of diaphragms to be unsatisfactory, though we constantly employed the four-inch reduction of aperture on the same eight-inch telescope, at San Luis, thirty years later, for reducing the less brilliant standard stars nearly to uniformity with the faint stars on our observing list.

Supplementing the screen values above, which were determined in observing for magnitude equation, the following values of two extra screens were determined in January of this year. The hand screen has two thicknesses of wire netting, one being placed diagonally on the frame, and its observed effect shows that it cuts off one-quarter as much light as the similar dense screen of four thicknesses of the same wire netting. The cap screen has one thickness of the wire gauze with which the similar dense screen is covered, and its observed effect shows that it cuts off one-eighth as much light as the similar screen of two thicknesses.

K. Hand Screen, 1918

No. Obs.	Limits	Mean	Screen Value
18	5.5 to 6.8 Mag.	6.4 Mag.	1.8 Mag.
12	7.0 7.8	7.4	1.6
10	8.0 8.5	8.3	1.4

L. Cap Screen, 1918

No. Obs.	Limits	Mean	Screen Value
18	5.5 to 6.8 Mag.	6.4 Mag.	1.8 Mag.
11	7.0 7.8	7.4	1.7
6	8.0 8.5	8.2	1.4

The magnitudes adopted for the stars in the two tables preceding were taken from the *Bonner Durchmusterung.*

The computation of the magnitude equation from the series of 1917 furnishes a test of the precision of transits of screened stars, as compared with transits at full brightness, using identically the same material for both tests. Since four nights were utilized, as many individual values of the magnitude equation can be derived as there are transits, though the weight of the mean result will be no greater than if the usual method of combining the results in two pairs of nights were followed. Thus, the combination of nights 1 and 2, 2 and 3, 3 and 4, and 1 and 4 all give values of the magnitude equation; and all these combinations give the probable error of the difference of two transits, for full brightness minus screen. The alternate combinations of nights 1 and 3, and 2 and 4, give the probable errors of transit for full brightness and for screened images. And the same individual accidental errors are employed in the two comparisons, to derive the computed probable errors.

Average Residuals, 1917

	South stars	North stars
Full–Full,	$\pm 0^{s}019$	$\pm 0^{s}023$
Screen–Screen,	± 0.027	± 0.031
Full–Screen,	± 0.023	± 0.027

The probable errors of the difference of transits between screened stars and stars observed full brightness fall exactly half way between the probable errors of the difference of two transits with screen and two transits observed full brightness, both for stars south of the zenith and for stars north. The observations of transits full brightness are more precise than the screened observations, in the proportion of weight two to weight one. For the usual range of stars observed here, no sensible decrease in the precision can be noticed, down to the ninth magnitude. Observations of screened images fainter than that limit evidently suffer some decrease in precision. The computed probable errors for one determination of the screen effect upon transits would be: $\pm 0^{s}020$ for south stars, and $\pm 0^{s}022 = 0^{s}013$ sec. δ for north stars. The mean values of the magnitude equation, from the series, would be subject to computed probable errors of less than $0^{s}001$, in both south and north groups.

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[3] *Year Book of the Carnegie Institution,* 188, 1913.

[4] Introduction *Albany A. G. Zone,* page 16.

Tests of the effect of diaphragms in reducing the brightness of the star images were made here in October last. Four diaphragms were employed, of apertures: half an inch, one inch, two inches, and three inches. In calculating the light ratios of the full aperture of the meridian circle telescope to the diaphragms, allowance has been made for the opaque area covered by the half-inch mirror, attached to the inner surface of the object glass, which serves to reflect direct illumination to the field of view. While this mirror cuts down the light received by the full objective by only $^{1}/_{155}$, it cuts down the light of a one-inch diaphragm by one-quarter. The half-inch diaphragm was placed eccentrically, since the opening would be exactly covered by the mirror, if placed opposite the center of the objective.

The smallest diaphragm gives star images that are entirely unsatisfactory, as they are too large and diffuse to estimate the magnitudes by the usual methods. With the one-inch diaphragm, the images are not sharp, but permit of approximate estimates. Images through the two-inch diaphragm are good, and those through the three inch are quite perfect. Reduction to one-quarter of the full aperture is as much as the method allows for satisfactory estimates. With screens, the images are sharp and like those of faint unscreened stars for the densest screens hitherto used; and no diffraction effects are noticed for stars of second magnitude or fainter. Estimates of magnitude depend to some degree upon the size of the star image, and for this reason the diffuse images of the small diaphragms do not permit of precise estimates. The image of a ninth magnitude star is about 1″ in diameter, under the usual observing conditions with this telescope. In full daylight, the image of a fourth magnitude star is of about the same size as that of a ninth magnitude star at night. Stars fainter than third magnitude are generally very difficult to observe here, under the conditions of atmospheric unsteadiness that usually prevail in the middle of the day. Full daylight is at least one hundred times as bright as the field illumination ordinarily employed at night. The mean results of the diaphragm observations are here tabulated. The variations depending upon magnitude follow, in another table.

M. DIAPHRAGMS, 1917

Aperture	Stars	Limits		Mean	Obs. Diff.
½ inch	11	2.9 to	4.1 Mag.	3.6 Mag.	5.2 Mag.
1	52	2.0	6.0	4.2	4.6
2	32	4.2	7.3	5.9	2.0
3	28	5.1	7.2	6.2	1.4
3	15	8.2	8.8	8.5	0.9

The magnitudes adopted in in reducing the diaphragm observations were taken from Boss's *Preliminary General Catalogue* for all stars brighter than eighth magnitude, this catalogue having been chosen because it gives a large range of magnitudes. The catalogue magnitudes of the brighter stars average one-tenth fainter than the visual magnitudes commonly adopted in our standard ephemerides. The photometric magnitudes[5] are fainter than those of Boss for the brightest classes, and brighter than those of Boss for the fainter classes. This difference in grades is represented as follows:

Mean Magnitude	2.3,	Visual–Photometric	−0.24
	3.7,		−0.05
	4.5,		−0.01
	5.4,		+0.11
	6.3,		+0.20

In a range of four units of magnitude, the relation of the two scales changes by more than 0.4. The magnitudes of the faintest stars in table M, above, were taken from the *Harvard A. G. Zone Catalogue*, and they differ from those of the *Durchmusterung* by an average of only 0.03. The last two groups of table M, for the three-inch diaphragm, show very distinctly the variation in light ratios, which has been presumed to account for the preceding tabulated variations in screen values. Omitting the few and unsatisfactory observations with the half-inch diaphragm, the remaining results are tabulated below, according to grades of magnitude.

N. DIAPHRAGMS, 1917

Aperture	Stars	Limits		Mean	Difference
1 inch	18	2.0 to	3.8 Mag.	3.3 Mag.	4.4 Mag.
	18	4.0	4.6	4.4	4.8
	16	4.7	6.0	5.1	4.5
2 inch	9	4.2	5.6	5.1	2.0
	16	5.7	6.3	6.0	2.0
	7	6.4	7.3	6.8	1.9
3 inch	6	5.1	5.7	5.4	1.5
	14	6.0	6.5	6.3	1.4
	8	6.6	7.2	6.8	1.1

No sensible progressive variation appears in the tabulation of the bright stars, for the one-inch and two-inch diaphragms. But the stars observed with the three-inch diaphragm show the usual form of the variation, quite apart from the indication given in table M for the faintest stars, combined there in a separate group. The estimates made in the three-inch, only, were entirely satisfactory.

The variations in screen and diaphragm values, from the preceding tables, are combined below in one table, with the computed rate of variation. One value of the rate is derived from each table that contains either two or three groups of bright stars. When there are four groups, two values have been derived, by comparing the first and third and the second and fourth. In tables M and N, only the results with the three-inch diaphragm have been tabulated, below.

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[5] *Annals H. C. O.*, 45 and 50.

P. Computed Rates

Table	Δ Mag.	Mean	Var.	Rate	v	Weight
A	1.8	3.4	0.3	0.17	—0.08	2
B	1.0	3.0	0.2	0.20	—0.05	1
C	2.7	3.4	0.7	0.26	+0.01	3
D	2.0	4.7	0.3	0.15	—0.10	2
	1.2	5.7	0.3	0.25	0.00	1
E	2.2	3.5	0.6	0.27	+0.02	3
F	1.8	3.5	0.2	0.11	—0.14	2
	2.3	4.5	0.9	0.39	+0.14	3
G	1.8	3.5	0.6	0.33	+0.08	2
	1.8	4.3	0.4	0.22	—0.03	2
H	0.9	7.1	0.4	0.44	+0.19	1
	1.4	7.9	0.2	0.14	—0.11	1
J	1.2	3.3	0.4	0.33	+0.08	1
K	1.9	7.3	0.4	0.21	—0.04	2
L	1.8	7.3	0.4	0.22	—0.03	2
M	2.3	7.3	0.5	0.22	—0.03	3
N	1.4	6.1	0.4	0.29	+0.04	1
Average,	1.7	5.0	0.42	0.25	±0.07	
Weighted mean,				0.25	v, weight 1 = ±0.065	

The individual results have been weighted by assigning weight one to differences of group magnitudes less than 1.5, and weight three to differences above 2.0. Neither the average result nor the average residual has been sensibly changed by the use of weights. This variation of screen values could be attributed to a difference in the unit light ratios, between the various groups of bright stars. Or, it could be similarly charged entirely to a difference of unit light ratios among the faint stars, corresponding to the screened images. But the difference is probably progressive, throughout the scale, with possibly a distinct break at the point of division between naked eye stars and telescopic stars, near the sixth magnitude. It appears more consistent to consider the total variation as due to the difference in unit light ratios between the average of the bright stars and the average of the faint stars. The amount of variation is quite precisely one-quarter of a magnitude, per unit of the scale for the bright classes. The variation is the same for the various classes, as shown by the following classification of table P.

7	values, brighter than magnitude 4, Rate =	0.24
4	between 4 and 6,	0.25
6	fainter than 6,	0.25

The tabulation of the light ratios, according to theory, will be convenient here, in making comparisons to illustrate the significance of the screen variations. For convenience the letter L will be used to represent the light ratios of two stars, or of two classes of stars. Then L_1 will be the light ratio for two stars, or classes, that differ by one unit of magnitude. The most commonly accepted light ratio for a difference of one magnitude has been that of the number whose logarithm is 0.40, each grade in the magnitude scale being very closely 2.5 times as bright as the next fainter grade. This will be referred to here as the normal scale. The naked eye stars have been generally conceded to have been graded more nearly according to a light ratio of which the logarithm is 0.36, each grade being about 2.3 times as bright as the next fainter grade. This has been known as the ''historic'' or ''natural'' scale, and will be referred to here as the historic scale. For the purposes of the comparison, two additional computations of light ratios have been tabulated; one with light ratio [0.32], and one with [0.44]. These extra scales will be called the lower and upper test scales, respectively. The light ratio of the lower is about 2.1, and that of the upper about 2.7. The differential values of L are tabulated below, for tenths of a unit of magnitude for one full unit and for the even units up to five.

Light Ratios

Mag. Diff.	Upper Test	Normal	Historic	Lower Test
0.1	1.11	1.10	1.09	1.08
0.2	1.22	1.20	1.18	1.16
0.3	1.35	1.32	1.28	1.25
0.4	1.50	1.45	1.39	1.34
0.5	1.66	1.58	1.51	1.45
0.6	1.84	1.74	1.65	1.56
0.7	2.03	1.91	1.79	1.68
0.8	2.25	2.09	1.94	1.80
0.9	2.49	2.29	2.11	1.94
1.0	2.75	2.51	2.29	2.09
1.5	4.6	4.0	3.5	3.0
2.0	7.6	6.3	5.2	4.4
3.0	20.9	15.8	12.0	9.1
4.0	58	40	27	19
5.0	158	100	63	40

The table of light ratios may be useful in deriving the effective magnitude of a close double star, the components of which can not be separated in a small telescope. Some discussion has arisen lately among observers of double stars in regard to the propriety of assigning the B.D. magnitude to the principal component, as the combined effect, only, is given in the B.D. estimates. Thus, two stars of equal magnitude would have a combined effect, $L = 2.0$, which would correspond to a magnitude 0.75 brighter than either component, on the normal scale. If the two components differ by a full magnitude, the combined light is 1.4, and the effective magnitude is between 0.3 and 0.4 brighter than the bright component, on the same scale. Two stars differing by 0.3 would have combined light of 1.74, and the effective magnitude of the pair would be 0.6 brighter than the bright component, on the normal scale. If only the difference of magnitudes has been estimated by the double star observer, the respective magnitudes can also be assigned from the B.D. estimate of the combined effect. Thus, a B.D. 8.0, which has components that differ 0.3, would be composed of $8.6 = 0.57l$, and $8.9 = 0.43l$, on the normal scale, where l is the light of the combined effect.

The variation of screen values can be translated into the equivalent change of light ratios, from the table above. If the average unit light ratio of the range of magnitudes in the tabulated screen values lies between those of the normal and historic scales, the rate of 0.25 per unit of magnitude corresponds to the difference between the upper and lower test scales. A difference of three-quarters of a magnitude, in the first column, corresponds closely to a full unit of magnitude in the last column. The inference would be that the bright stars have been graded with a unit light ratio as low as 2.1, while the faint stars, corresponding to the screened images, have been graded with a unit light ratio as high as 2.7. This change amounts to twenty-five per cent of the average unit light ratio. Or, taking the five sets of results for the dense hand screen employed here, the respective screen values for the limiting groups of magnitudes, 2.5 and 6.3, would be 4.1 and 3.2, which correspond to light ratios of 21 and 25, in the last column and first column, respectively. The light ratios derived in this manner are mean values, for a point midway between the bright stars and the screened images. If the unit light ratio at sixth magnitude is between those of the normal and historic scales, then the fourth magnitude stars would be graded with a unit light ratio of 2.1, and the eighth magnitude stars with a unit light ratio of 2.7, as general inferences from these screen results.

The effect of the diaphragms in testing the light ratio is illustrated in the table following. It gives the average magnitude of the stars observed, the computed light ratio of each aperture, the observed average reduction in magnitude, and the unit light ratio resulting from this reduction, corresponding to a point midway between full brightness and reduced magnitude.

UNIT LIGHT RATIOS FROM DIAPHRAGMS

Aperture	Mag.	L	Obs Δ	L_1	
½ inch	3.6	155	5.2 Mag.	2.6 at Mag.	6.2
1	4.2	51.7	4.6	2.4	6.5
2	5.9	10.3	2.0	3.2	6.9
3	6.2	4.4	1.4	2.9	6.9
3	8.5	4.4	0.9	5.2	9.0

The combined result of the first four reductions is a mean unit light ratio of 2.8 at magnitude 6.6. The value for the faint stars, from the three-inch diaphragm, is notably larger.

A diaphragm of exactly half the aperture of the telescope should reduce the apparent magnitude by 1.5 units of the normal scale and by 1.7 units of the historic scale. The reduction would be 1.4 and 1.9, by the upper and lower test scales, respectively. These figures should be compared with the diaphragm results at San Luis (table H), where the mean reductions apply to mean grades of magnitude between 7.5 and 9.3.

The comparison of photometric measures of the naked eye stars shows that there are differences of sensible amount in the photometric determinations of magnitudes. Some of the early measures were discussed with much detail by the late Dr. S. C. Chandler, and the results of his discussion[6] have been condensed in the following tables. They include the photometric measures of the light ratios of the various grades of stars in Argelander's *Uranometry,* as derived by the authorities quoted. Omitting the erratic values for stars brighter than the second magnitude, the following mean values of L_1 and the range of the values for each authority have been computed from the original discussion.

PHOTOMETRIC MEASURES OF VISUAL LIGHT RATIOS

Authority	Limits		L_1	Range	
Seidel	2 to	4 Mag.	2.8	2.3 to	3.1
Zöllner	2	6	2.2	1.6	2.6
Pierce	3	6	2.2	1.9	2.8
Wolff	2	6	1.9	1.4	2.5
Pickering	2	6	2.3	1.9	2.9
Pritchard	2	6	2.4	1.9	2.9
Mean,			2.3		
Average residual,			±0.2		
Average range				1.0	
Total range,				1.4 to	3.1

The photometric measures of the unit light ratio for the grades between second and sixth magnitude have a range among the various authorities of 0.9, or 40 per cent of the average value. Omitting the first authority, the range would be 0.5, above 20 per cent of the average values.

Tabulating separately the ten grades of magnitude in the *Uranometry,* of which the unit is one-third of a magnitude, the following table gives the comparison of the measures of L_1 for each grade by each authority, computed from the same material as the table preceding.

PHOTOMETRIC MEASURES OF ARGELANDER'S GRADES

Mag.	Lowest	Mean	Highest	Range
2.0–3.0	2.3	2.6	3.1	0.8
2.3–3.4	2.3	2.6	2.9	0.6
3.2–4.3	2.0	2.5	3.0	1.0
3.0–4.0	1.9	2.3	2.6	0.7
3.4–4.5	1.8	2.0	2.3	0.5
4.3–5.4	1.7	2.2	2.8	1.1
4.0–5.0	1.8	2.1	2.3	0.5
4.5–5.6	1.6	1.9	2.2	0.6
5.4–6.5	1.5	1.9	2.6	1.1
5.0–6.0	1.4	1.8	2.4	1.0
Means,	1.8	2.2	2.6	0.8

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[6] *Astr. Nach.,* 115, 145, 1886.

There is an evident progressive decrease in the values of the unit light ratio, with the decrease in the brightness of the stars. This is contrary to the change observed here, between the bright naked eye stars and the faint stars, by means of the screen reductions. The average range of measures between various authorities, for the same grade, is 0.8, or 36 per cent of the quantity measured. The difference rises to 50 per cent for some grades, and the extreme values do not occur systematically in any one of the authorities.

In the same discussion a relatively small number of measures of the magnitudes of faint stars, by photometric methods, gives an average unit light ratio, 2.5, for the grades from sixth to ninth.

The usefulness of screens as a photometric test of the magnitude scale has probably not been fully recognized. With precise estimates of the screened images, in the part of the visual scale most commonly and most accurately estimated by meridian observers, from seventh to ninth magnitudes, the magnitudes of the bright stars could be consistently tested by the light ratio already adopted for these grades of faint stars. Without regard to the amount of light cut off by the screen, a difference of one-tenth of a magnitude in the estimate would produce the same difference in unit of magnitude for any bright star. Considering the difficulty of assigning exact magnitudes to the bright stars by other photometric methods, this process presents some advantages. Several screens would be necessary, in order to have all the reduced images fall within the uniform range of magnitude best adapted to precise estimates.

A difference of one-tenth of a magnitude in the estimate of a screened image corresponds to 10 per cent of the light ratio of the screen. For a screen that reduces the light by four magnitudes, this difference gives a variation of 0.05 in the unit light ratio, fixed by its effect, or 2 per cent. For a screen that reduces the magnitude by one and a half units, the same difference of one-tenth of a magnitude in the estimate give a variation of 0.14 in the unit light ratio, or 6 per cent.

The variation of screen values, which has been under discussion here, might result in a computed variation of the magnitude equation, for different classes of stars. But the basis for variation in the magnitude equation should be a progressive change in the total correction to transits, depending upon magnitude. With a constant value of the effect of the screen upon transits, a constant magnitude equation is the most reliable correction. In the greater part of the observations tabulated here, the screen effect has been the same for all classes of stars, within the limits of probable error. Evidence of a variable magnitude equation, for stars north of the zenith, was found in the 1907 series. A small variation for my observation of stars north of the zenith, at San Luis, was also derived in the computations made at Albany.[7]

The tabulation of successive determinations of my magnitude equation follows. One determination with our instrument consists of the results of two transits, one observed full brightness, and one with screen. The results at San Luis are quoted from the *Year Book of the Carnegie Institution,* as cited above, but diminished one place of decimals. A value of magnitude equation at San Luis is derived from a single transit of a star, since the instrument was provided with three sets of wires, nine in each set, and any one set or two sets could be observed with the screen. More than one hundred observations with screen are included, for stars south of that zenith, and more than three hundred observations for stars north. Since the stars north of that zenith were the more rapidly moving ones at transit, contrary to the condition here, the relative difference in the amount of the screen effect is evidently due to the direction of motion, rather than to the velocity. If the screen effect at the Lick Observatory were to be considered alone, the relative difference might be attributed to the velocities, since my average magnitude equation at this station is approximately $0^{s}01$ sec. δ in both positions, facing north and south.

MAGNITUDE EQUATION

LICK OBSERVATORY

Epoch	South Stars		North Stars	
1896	75 Det.	—$0^{s}009$		
1898	60	—0.006		
1902	48	—0.006		
1907	49	—0.010	91 Det.	—$0^{s}025$
1911	45	—0.010	50	—0.016
1914	60	—0.007	30	—0.008
1917	48	—0.012	40	—0.020
Sum	385		211	
Mean		—0.009		—0.017
		SAN LUIS		
1910–1911		—0.010		—0.015

The computed probable error in any one series has not exceeded one unit in the third place of decimals, in the results tabulated above. For the observation of stars south of the zenith, such a probable error is consistent with the mean result. The individual observed values for north stars do not conform so well to the mean result, and some slight changes in the personal habit of observation of such transits may possibly be indicated, during the past ten years.

R. H. TUCKER.

March 8, 1918.
Issued April 11, 1918.

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[7] *Year Book of the Carnegie Institution* for 1911, page 160.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 308

MISCELLANEOUS MERIDIAN CIRCLE RESULTS

OBSERVED CORRECTIONS TO THE RIGHT ASCENSION OF THE MOON

The following observations of the Moon at meridian transit have been employed to derive the corresponding corrections to the Moon's right ascensions in the *American Ephemeris.*

1918	Limb	Obs'd R.A.	Correction
Jan. 30	Following	$11^h\ 46^m\ 58^s.55$	$+0^s.73$
Jan. 31		12 31 58.38	+0.59
Feb. 26		11 31 36.56	+0.79
Mar. 1		13 49 54.66	+0.44
Mean			+0.64
Mar. 20	Preceding	7 17 43.03	+1.00
21		8 9 47.70	+0.92
22		8 58 58.88	+0.92
Mar. 23		9 45 53.09	+1.00
Mean			+0.96

There is probably a systematic personal error in the observations of the transits of one or both limbs of the Moon. The average residual from the mean correction for one limb is $\pm 0^s.08$, while the average residual from the mean correction for both limbs is $\pm 0^s.16$. The mean of the corrections, from observations of both limbs, is $+0^s.80$. The application of this correction to the tabular right ascension of the Moon would bring the times of contact for the solar eclipse of June 8, 1918, earlier by twenty seconds than the times predicted from the *Ephemeris.* The observations have been reduced with the right ascensions of stars, four or five each night, from the *American Ephemeris.* The adopted right ascensions differ from those of the *Berliner Jahrbuch* by $\pm 0^s.01$, for a group of stars observed on one night.

OBSERVED CORRECTION TO THE RIGHT ASCENSION OF *VENUS*

The right ascension of the eastern limb of *Venus* was observed at meridian transit, following the occultation of 7 *Aquarii,* on March 1, 1918. The observed right ascension, $20^h\ 52^m\ 31^s.44$, has been reduced to the center of the planet by applying the correction for the semi-diameter, $25''.7$, from the *American Ephemeris.* The resulting correction to the *American Ephemeris* right ascension is $+0^s.09$. Irradiation may be responsible, to some extent, for the plus correction; but, as the planet was observed in full daylight, less than two hours before noon, the effect of irradiation at the limb must have been comparatively slight.

MERIDIAN CIRCLE OBSERVATIONS OF *NOVA MONOCEROTIS*

The magnitude of this star has been estimated at from 9½ to 9¾, on March 1, 15, and 20. The observed positons include the right ascensions corrected for magnitude equation, in a separate column.

Star	Epoch	Mag.	R.A. 1918.0	Corr'd	Decl. 1918.0	No. Obs.
Nova	1918.19	9.6	$7^h\ 22^m\ 46^s.989$	$46^s.951$	$-6^\circ\ 30'\ 35''.86$	3
Nova follows B.D.	—6° 2103		by $1^m\ 22^s.67$			2
	—6° 2107		0 58.49			2
	—6° 2109		0 17.36			2
—6° 2109	1918.16	9.8	7 22 29.76	29.72	—6 31 34.83	1

COMPARISON STARS FOR ASTEROID (21)

These stars were observed at the request of Professor E. E. Barnard, for his micrometer observations.

Star	Epoch	Mag.	R.A. 1918.0	Corr'd	Decl. 1918.0	No. Obs.
B.D. 24° 933	1918.16	9.5	$5^{h}\ 38^{m}\ 32^{s}088$	$32^{s}034$	+24° 53′ 10″84	2
24° 940	1918.16	8.	5 39 35.171	35.125	+24 52 52.86	2
24° 940	Correction to Berlin A. G.		—0.002	—0.048	—0.40	
24° 974	1918.08	9.2	5 44 26.326	26.272	+24 48 28.11	2
24° 1040	1918.08	8.3	5 52 36.319	36.273	+24 45 14.26	1
24° 1040	Correction to Berlin A. G.		—0.006	—0.052	+0.89	
24° 1045	1918.08	7.5	5 53 34.082	34.041	+24 47 26.55	1
24° 1045	Correction to Berlin A. G.		+0.079	+0.038	—1.70	

OBSERVATIONS OF *X Z CYGNI*

This star was observed last year, at the request of Dr. H. Shapley, of Mount Wilson Solar Observatory. It is a variable, having a large proper motion, and a large radial velocity.

Epoch	Mag.	R.A. 1917.0	Corr'd	Decl. 1917.0	No. Obs.
1917.57	9.3	$19^{h}\ 30^{m}\ 46^{s}593$	$46^{s}485$	+56° 12′ 31″30	4

Proper motions from comparisons of my observations with

Hels. Gotha A. G.	$+0^{s}0117$	$+0^{s}0095$	—0″022	2
O. A. Zone	+0.0147	+0.0133	—0.082	1
Weighted P.M.	+0.0127	+0.0108	—0.042	

Since the earlier observations may have been subject to corrections for magnitude equation similar to my own correction, the proper motion in right ascension has been derived above from the differences of right ascension, both with the correction for magnitude equation, and without the use of the correction.

THE DIURNAL VARIATION IN THE ATMOSPHERIC REFRACTION

The diurnal changes in the observed effect of the atmospheric refraction, discussed in *Lick Observatory Bulletins*, **7**, 130, 1913, and **9**, 56, 1917, have been confirmed very precisely by the results of the meridian circle observations at the San Luis Observatory of the Carnegie Institution, recently reduced at Albany.[1] The southern results have been tabulated by the computors at Albany, in seven groups, for each of the three fundamental observers. Weights were assigned to each group, depending upon the number of observations made in each period of the day and night. The number was ample in each group, for the purpose of the investigation. The period close to noon was not represented, as observations usually began at three or four in the afternoon, and were generally continued up to eight or nine in the forenoon. There are no bright close circumpolar stars in the southern sky for observations in full daylight. In the northern sky, *Polaris* and *β Ursae Minoris* make an excellent pair of stars, that can be observed here at either culmination, at any time of the day.

Without distinction of weights, the results of both investigations are given in the following table, in a form that admits of direct comparison. The table contains the observed percentage of increase or decrease in the corrections derived from the Poulkova refraction tables. The total amount of the change is quite precisely one per cent of the correction, at each station. My earlier discussion of the Lick Observatory results gave a small progressive rate during the night hours, such as has been derived for San Luis. In the later discussion, from which the results below have been taken, the correction for the greater part of the night was tabulated as constant.

In the comparison of the variation, the time of dawn has been taken at one hour preceding sunrise. The first and last periods of the computation for San Luis have been adopted as corresponding to the tabular values below, for Mount Hamilton. The tables give the results for observer Tucker, at San Luis, and then the mean results for the three San Luis observers, Tucker, Roy, and Varnum. The last column gives the differences in the observed corrections at the two stations. The averages of the corrections to the Poulkova refractions, at the two observatories, are not the same, as might be anticipated from the difference of altitude above sea level and from other circumstances. But the last column shows how consistently the variation is represented, in character and in amount, by these independent investigations.

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[1] *Year Book of the Carnegie Institution for 1916*, page 220.

PERCENTAGE CORRECTIONS TO THE POULKOVA REFRACTIONS

SAN LUIS	R. H. T.	Mean (3)	LICK OBSERVATORY		Diff.
Afternoon	—0.001	—0.001	Sunset —3^h	+0.004	—0.005
Early night	—0.008	—0.007	Sunset	—0.002	—0.005
Late night	—0.009	—0.009	Night	—0.004	—0.005
Pre-dawn	—0.011	—0.010	Night	—0.004	—0.006
Dawn	—0.011	—0.007	Sunrise —1^h	—0.004	—0.003
Early sunrise	—0.008	—0.007	Sunrise	—0.002	—0.005
Late sunrise	—0.002	+0.001	Sunrise +4^h	+0.006	—0.005
Extreme change	0.010	0.011		0.010	
Mean difference					—0.005

The daytime values of the refraction correction at San Luis are nearly in agreement with those of Poulkova, while the night values are smaller by nearly one per cent. The fall is sharp near sunset, and the rise is sharp near sunrise.

At Mount Hamilton the refraction corrections are larger than those of Poulkova in the day time, and the night corrections are smaller than those of Poulkova, with similar rapid changes near sunset and sunrise. The Poulkova refraction corrections are smaller than those of an earlier date, derived by Bessel, and the present results indicate as necessary a further diminution in the corrections for the night hours, when the greater part of our star positions are observed.

An abstract[2] of the results of the declination observations made at the U. S. Naval Observatory at Washington, in the years 1903 to 1911, was presented at the 1917 meeting of the American Astronomical Society, at Albany, by Professors Eichelberger and Littell. The comparison of zenith distances of equatorial stars, observed both by day and night in the period 1909 to 1911, shows a variation which could possibly be attributed to variation in atmospheric refraction. It would have the opposite sign, apparently, to the variation recorded above for San Luis and Mount Hamilton. The extreme difference at Washington, which amounts to 0″.8 in one period, averages 1″ tan Z for the change between day and night, which is nearly two per cent of the total refraction at sea level. The epochs of the extreme daylight groups are but one and a quarter hours from sunset and sunrise; and the variation depends upon observations south of the zenith, only. In the Lick Observatory discussion, observations both north and south of the zenith, over a meridian arc between 150° and 160° in length, have been combined, and any variation in the determination of the zenith point has been eliminated. The zenith distances at Washington show no sensible variation during the greater part of the night hours. The mean correction to the Poulkova refractions is 0″.14 tan Z, during the night. All three stations agree in deriving corrections of the same sign, for the night hours, the value adopted at Lick Observatory being —0″.20 tan Z.

OBSERVED ERRORS IN THE HARVARD A. G. ZONE

The following corrections to the places of two stars in the *Harvard A. G. Zone Catalogue* have been observed.

Star	Epoch	Mag.	R.A. 1917.0	Corr'd	Decl. 1917.0	No. Obs.
6722	1917.76	9.3	20^h 40^m $48^s.552$	$48^s.446$	+50° 44′ 57″.57	2
Correction to Harvard A. G.			+12^s		+10″	

This error has been explained by Director E. C. Pickering as due to a misidentification. Also, the star No. 3196 has been found to have right ascension approximately 5^s too great, in the Harvard Catalogue. Two observations in January of this year give the position:

1918.08 Mag. 8.6, R.A. 8^h 33^m $8^s.694$ Corr'd $8^s.668$, Decl. +51° 45′ 57″.53

R. H. TUCKER.

March 27, 1918.

Issued April 11, 1918.

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[2] *Pop. Ast.*, **25**, 598, 1917.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 309

ELEMENTS AND EPHEMERIS OF WOLF'S OBJECT, 1918 DB

The following elements were derived from observations by Barnard, February 12, 21, and March 7, numbered 9, 26, and 30 in the list given below.

ELEMENTS

Epoch = 1918 Feb. 21.8242 Gr.M.T.
$M_0 = 12° \; 00' \; 59''$
$\left.\begin{array}{l} \omega = 347 \; 45 \; 05 \\ ☊ = 110 \; 53 \; 50 \\ i = 8 \; 58 \; 10 \end{array}\right\} 1918.0$
$\phi = 32 \; 15 \; 42$
$\log a = 0.404070$
$\mu = 878.''824$
Period = 4.0374 years
T = 1918 Jan. 3.6005 Gr.M.T.

CONSTANTS FOR THE EQUATOR 1918.0

$x = [9.995344] \, r \sin (188° \; 53' \; 04'' + v)$
$y = [9.972978] \, r \sin (101 \; 57 \; 24 + v)$
$z = [9.570289] \, r \sin (77 \; 19 \; 00 + v)$

Since these elements represent the observations satisfactorily, the computation of new elements will be deferred until a longer arc shall have become available.

An ephemeris for the month of April was telegraphed to the *Harvard College Observatory*, and published in *Bulletin* **656**. The ephemeris given here is a continuation.

EPHEMERIS FOR GREENWICH MEAN MIDNIGHT

1918		True α			True δ		log Δ	Mag.
April	24.5	10ʰ	11ᵐ	44ˢ	+25°	51.′2	0.0138	14.9
	26.5		15	22	25	21.2		
	28.5		18	58	24	51.1		
April	30.5		22	33	24	21.0		
May	2.5		26	06	23	50.9	0.0580	15.2
	4.5		29	37	23	20.8		
	6.5		33	07	22	50.8		
	8.5		36	35	22	20.7		
	10.5		40	02	21	50.7	0.1000	15.4
	12.5		43	27	21	20.8		
	14.5		46	51	20	50.9		
	16.5		50	14	20	21.1		
	18.5		53	36	19	51.4	0.1398	15.7
	20.5	10	56	56	19	21.8		
	22.5	11	00	16	18	52.2		
	24.5		03	35	18	22.7		
	26.5		06	53	17	53.4	0.1775	16.0
	28.5		10	10	17	24.2		
May	30.5		13	26	16	55.1		
June	1.5		16	41	16	26.1		
	3.5		19	55	15	57.3	0.2131	16.2
	5.5		23	09	15	28.6		
	7.5		26	22	15	00.0		
	9.5		29	34	14	31.5		
June	11.5	11	32	46	14	03.1	0.2468	16.5

Recent observations by Barnard are represented as follows:

1918 April 4.8, (O–C), $\Delta\alpha \cos \delta = +2^s$, $\Delta\delta = -0.'4$
13.7, $= +2$ $= -0.6$

The following is a list of available observations and their representation:

	Date Gr.M.T.		Apparent α			Apparent δ			(O–C) cos δΔα	(O–C) Δδ	Observer
1	Feb.	5.3594	6ʰ	56ᵐ	09.ˢ5	+34°	19′	19″	+0.ˢ7	+ 7″	Wolf, Konigstuhl.
2		8.7342	7	07	37.6	35	16	21	[+2.9]	+11	Wilson, Northfield.
3		10.28009	7	12	46.53	35	37	26	0.0	+ 5	Palisa, Vienna.
4		10.6917	7	14	08.6	35	42	37	+0.6	+ 3	Barnard, Yerkes.
5		11.40832	7	16	31.3	35	51	01.9	+0.4	+ 3	Birck, Potsdam.
6		11.43294	7	16	36.1	35	51	14.9	+0.5	— 1	Birck.
7		11.7695	7	17	43.2	35	55	21	+0.3	+12	Curtis, Lick.
8		12.6206	7	20	32.0	36	04	08	—0.2	+ 5	Wilson.
9	Feb.	12.747916	7	20	56.45	36	05	23.2	+0.3	+ 5	Barnard.

	Date Gr.M.T.		Apparent α			Apparent δ			(O–C) cos $\delta\Delta\alpha$	(O–C) $\Delta\delta$	Observer
10	Feb.	14.26792	7^h	27^m	$57^s.13$	36°	19′	01″	$+0^s.3$	+ 2″	Mundler, Konigstuhl.
11		15.43015	7	29	42.20	36	27	53.0	+0.5	+ 3	Hoffmeister, Bamberg.
12		15.6616	7	30	28.4	36	29	35	+0.6	+ 6	Curtis.
13		16.34510	7	32	39.23	36	33	47.7	+0.1	0	Hoffmeister.
14		16.56668	7	33	22.29	36	35	13.5	+0.1	+ 5	Barnard.
15		16.58030	7	33	24.85	36	35	19.2	+0.1	+ 5	Barnard.
16		16.83254	7	34	11.77	36	36	38.5	+0.5	+ 2	Van Biesbroeck, Yerkes.
17		17.37325	7	35	55.90	36	39	39.1	0.0	+ 7	Hoffmeister.
18		17.58626	7	36	37.30	36	40	39.9	+0.2	+ 3	Van Biesbroeck.
19		17.64612	7	36	47.15	36	40	45.0	+0.5	+ 4	Hoffmeister.
20		18.36892	7	39	0.44	36	44	05.9	[—4.2]	+ 1	Hoffmeister.
21		19.65834	7	43	5.85	36	48	31.5	—0.6	+ 6	Hoffmeister.
22		19.73091	7	43	20.62	36	49	05.8	0.0	+14	Barnard.
23		19.81549	7	43	35.63	36	49	16.3	—0.2	+14	Van Biesbroeck.
24		[19.82017]	7	43	35.63	36	49	16.3	[—1.0]	+13	Van Biesbroeck.
25		20.71338	7	46	23.43	36	51	30.6	—0.1	+13	Van Biesbroeck.
26		21.82653	7	49	48.01	36	52	58.3	+0.2	0	Barnard.
27		23.80143	7	55	46.56	36	53	38.7	+0.4	+ 1	Van Biesbroeck.
28	Feb.	28.75972	8	10	13.22	36	42	58.7	0.0	+ 1	Barnard.
29	Mar.	2.55578	8	15	17.45	36	35	17.0	+0.2	+ 1	Barnard.
30		7.726145	8	29	14.12	36	03	23.7	+0.1	— 1	Barnard.
31		14.766852	8	47	03.22	35	01	17.1	+0.4	— 2	Barnard.
32	Mar.	16.615393	8	51	32.15	34	42	02.7	+0.3	— 8	Barnard.

The residual in the observation by Barnard on February 12, which was used for the first place of the orbit, is due to a correction which he made on the basis of a better place for the comparison star. The corrected observation is given in the list.

The two observations by Van Biesbroeck on February 19 are identical, except in date. The first was given in *H. C. O. Bulletin* **654**, the second was sent in a letter. Apparently the first of the two dates is the correct one.

The following ephemeris extends backwards from the date of discovery. It is hoped that if any photographs have been taken in the region, they will be examined for possible pre-discovery positions. If any are found, it will be greatly appreciated if they are sent to the Student's Observatory.

FOR GREENWICH MEAN MIDNIGHT

Date			α 1918.0		δ 1918.0		Mag.
1917	Dec.	3.5	4^h	$37^m.0$	— 5°	29′	11.4
		7.5		39.3	4	28	
		11.5		42.5	2	58	11.1
		15.5		46.5	— 1	03	
		19.5		51.2	+ 1	14	10.9
		23.5	4	56.7	3	53	
		27.5	5	3.1	6	51	10.8
	Dec.	31.5		10.5	10	02	
1918	Jan.	4.5		19.1	13	24	10.8
		8.5		28.5	16	49	
		12.5		39.1	20	11	10.8
		16.5	5	50.5	23	22	
		20.5	6	2.9	26	17	11.0
		24.5		15.7	28	52	
	Jan.	28.5	6	29.1	+31	05	11.2

An observation by Wolf indicates a correction to this ephemeris on January 3, 1918, of $0^m.0$ in α, + 1′ in δ.

Since the computations involved in the determination of these elements form a good example of the complications caused by inaccurate observations, it seems worth while to give an account of the successive steps in the solution of the orbit.

When the first discovery positions by Wolf arrived by telegraph, it was of course seen that the object might be of unusual interest, since its right ascension was increasing.

When for several days no further positions arrived, it was decided to determine the orbit on the basis of the first three available observations, to furnish a finding ephemeris. The observational material consisted merely in the observations by Wolf of February 3, 4, and 5. The first two were only approximate. Professor Crawford and I undertook the computation. It was readily found that the observations would not yield a solution. In the meantime, an observation was received from Dr. Curtis, at the Lick Observatory, made with the Crossley Reflector. The four observations so far available, with the velocities per day, are given below.

1918		α app.				δ app.				Observer
Feb. 3.3654	6^h	49^m	08^s		+33°	40′				Wolf.
				$+228^s$					+20′	
4.3342	6	52	48		+33	59				Wolf.
				+197					+20	
5.3594	6	56	09.5		+34	19	19″			Wolf.
				+202					+15	
Feb. 11.7695	7	17	43.2		+35	55	21			Curtis.

Professor Leuschner noticed immediately, upon making the computation of the daily velocities as given above, that the discrepancy was probably in the right ascension of February 3. Upon his suggestion, I undertook the solution of an orbit assuming the following velocities and accelerations in α and δ, the unit of time being one day:

$$\begin{array}{ll} \alpha' = +200^{s} & \delta' = +19' \\ \alpha'' = \quad 0 & \delta'' = -1 \end{array}$$

The observation of February 4 was still retained for the middle place. The resulting orbit was strongly hyperbolic. This being very improbable, a parabolic solution was undertaken. The resulting elements represented the available four observations excepting only the first right ascension* within the accuracy of the approximate numerical computation. The parabolic elements are:

$$\begin{aligned} T &= 1918 \text{ Jan. } 14.551 \text{ Gr.M.T.} \\ \omega &= 0°57' \\ \Omega &= 105\ 53 \\ i &= 11\ 06 \\ \log q &= 0.1272 \end{aligned}$$

	O–C	
Feb. 3	$\Delta\alpha = -8'$	$\Delta\delta = -1'$
4	0	0
5	−2	0
8	−1	+3
11	−3	+2

These are the elements from which the ephemeris given in *Harvard Bulletin* **652** was computed.

The observation of the 8th is by Wilson, number (2) in the foregoing list, received in the mean time. The probably high eccentricity of the orbit and the fact that Wilson suspected the object to be surrounded by faint nebulosity suggested at this time that the object might be a comet.

The next orbit was based on the positions by Wolf on the 5th, Wilson on the 8th, and Curtis on the 11th. It was a differential correction from the approximate elements given above. With the parabolic condition imposed, the best representation that could be obtained, with the outstanding discrepancy forced into the declination of the first place was $\Delta\delta_1 = -70''$.

An error of observation was again suspected, and a new differential correction was attempted, to represent Wilson's observation of the 8th, Barnard's of the 12th, and Curtis's of the 15th, the last two having in the mean time become available. Neither a parabolic, nor a general, nor conditioned solutions with assumed periods yielded a satisfactory representations. The general solution gave a period of 2.9 years. Arbitrary variations, which resulted in a four-year period, gave:

$$\begin{array}{lll} \text{(O–C)} & \Delta\alpha = +\ 8'' & -37'' \\ & \Delta\delta = -10 & -26 \end{array}$$

These residuals also showed, by comparison with those previously obtained, that there was an error in the right ascension of the observation of the 8th. This was not entirely unexpected, since Wilson, in sending his observation, remarked that it was uncertain owing to the faintness of the trail.

The last orbit served as the basis for the next solution. This was based on Wolf's observation of the 5th, Barnard's of the 12th and 19th and yielded a satisfactory result, as the erroneous observations of Feb. 8 was not involved. The resulting elements are:

$$\begin{aligned} T &= \text{Jan. } 3.9600 \text{ Gr.M.T.} \\ &\left.\begin{aligned} \omega &= 348°\ 12.'8 \\ \Omega &= 110\ \ 41.0 \\ i &= 9\ \ 06.9 \end{aligned}\right\} 1918.0 \\ \phi &= 33\ \ 32.7 \\ \log a &= 0.42435 \\ \mu &= 819.''36 \\ \text{Period} &= 4.331 \text{ years.} \end{aligned}$$

These elements, together with an ephemeris, were sent to Harvard College, and were published in *Bulletin* **655**.

Succeeding observations showed that this orbit could be improved. The new elements, based on Barnard's observations of February 12, 21, and March 7, are the result of one differential correction from the last elements given above. They are those given at the beginning of this paper, from which the ephemerides have been computed.

It is interesting to note that only one direct solution for the orbit was made, the very approximate one at the beginning. All succeeding results were derived therefrom. The method of arbitrary variation was, however, twice employed. In one of the earlier orbits the $\Delta\rho$ was changed in such a way as to diminish the residuals, and the z' of the last intermediate orbit was changed so as to remove some residual from the declination.

Wolf, on some of his plates, has detected what appears to be a satellite of the object. No American observer has so far reported the existence of such a companion, nor, apparently, has any other European observer. There is slight evidence of periodicity in the residuals, but no definite conclusions are warranted until the residuals shall have been computed more accurately. There is a further difficulty in the fact that the mass of an object which could hold a satellite at the distance given by Wolf, and with such a rapid change in position angle, would have to be considerable, a thing which is not in harmony with the observed faintness. It appears impossible to make any decision at the present time, as to whether Wolf's satellite is physically connected to the principal object or not.

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* In *A. N.* 4926 a more accurate value of this right ascension is published based on a new reduction of the plate of Feb. 3.

The majority of the observations by Professor Barnard were kindly sent to the Student's Observatory by Director Frost and Professor Barnard.

I wish to express my appreciation for the constant encouragement and advice given by Professor Leuschner during the progress of the work.

I am indebted to Mr. Carlos Mundt, who duplicated a considerable amount of the earlier parts of the computation, and to Miss Mary Heger, and Miss Dorothy Schwan, for representing all the observations from the ephemeris.

H. M. Jeffers.

Berkeley Astronomical Department,
April 19, 1918.

Issued April 24, 1918.

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ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 310

ON VARIATION OF VELOCITY OF WAVES DUE TO MOTION OF THE SOURCE*

In the former paper was the following statement:

"A velocity of propagation which depends on the velocity of the source need not necessarily demand an emission hypothesis. Certain results observed in the firing of big guns may be published later. Preliminary evidence tends to indicate that the velocity of sound waves is affected by the velocity of the source."

At that time it was impossible to make a statement concerning a variation of the velocity of sound due to the velocity of the source. This can now be done with some definiteness. The velocity of a wave depends not only on the medium in which it moves, but also on the velocity of the source from which it emanates. The evidence, theoretical and experimental, now seems fairly conclusive. The theoretical proof of the variable velocity of sound waves originating in a moving source is very simple. In the *Encyclopaedia Britannica,* Vol. 25, page 440, Poynting develops the equation for propagation of a longitudinal wave in an elastic medium by introducing an external pressure "X." He gives, after development of his equation to this point:

$$X = (E - \rho_0 U^2)\, dy/dx. \qquad (5)$$

"If then we apply a pressure X given by (5) at every point, and move the medium with any uniform velocity U, the disturbance remains fixed in space. Or if we now keep the undisturbed parts of the medium fixed, the disturbance travels on with velocity U if we apply the pressure X at every point of the disturbance.

"If the velocity U is so chosen that $E - \rho_0 U^2 = 0$, then $X = 0$, or the wave travels on through the action of the internal forces only, unchanged in form and with velocity

$$U = \sqrt{(E/\rho)}. \qquad (6)$$

The pressure X is introduced in order to show that a wave can be propagated unchanged in form. If we omitted it we should have to assume this, and equation (6) would give us the velocity of propagation if the assumption were justified. But *a priori* we are hardly justified in assuming that waves can be propagated at all, and certainly not justified in assuming that they go on unchanged by the action of the internal forces alone. If, however, we put on external forces of the required type X it is obvious that any wave can be propagated with any velocity, and our investigation shows that when U has the value in (6) then and only then, X is zero everywhere, and the wave will be propagated with that velocity when once set going."

From this point all that is necessary for us to do is to show that such an external pressure exists. Imagine the source as a vibrating tongue and the air particles as attached to it at certain intervals by a light elastic string. Increase in the intensity of sound makes the tongue vibrate through a greater arc, but it moves the same distance in each direction backward or forward with the same velocities at like phases so long as the source is fixed. Therefore, there is no total pressure X exerted by it, the sum of the positive pressures exactly equaling the sum of the negative. However, let us now consider the tongue itself as moving forward with a velocity U. Its forward velocity of vibration is, therefore, $2U$ greater than its backward velocity, at corresponding phases of the motion. These motions exist through equal intervals of time. The tongue, therefore, transmits to the neighboring air particles a motion depending upon its own. The energy received by the first particles is transmitted by each of them in turn to the particles beyond. Because of the fact that this energy is not added to except by the source and is distributed over a surface, whose area increases as the square of the distance from the source, the external energy received by any particle must vary inversely as the square of its distance. The additional velocity which it therefore receives varies inversely as the distance from the source. In explosive sounds the unit of distance, according to which the excess velocity decreases, is the radius of a sphere centered at the point of explosion and within which all the energy of the explosion is developed. In the case of a moving sphere the unit is the radius of the sphere.

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* Continuation of a paper "On Finite Velocity of Gravitation as a Possible Factor in Stellar Evolution," *Lick. Obs. Bull.* 305. Read before Berkeley meeting, American Physical Society, March 30, 1918.

Regnault's differences in determining the velocity of sound are, therefore, expected. Using a pistol as the source at the distance of 1280 meters and 2445 meters, he obtained respective velocities of 331.37 and 330.71 meters per second. These results are in line with all the data, which will be quoted later.

During March I was able, accompanied by Lieut. George McFarland, C.A.C., and Private G. H. Bardsley, formerly Whiting Fellow in Physics at the University of California, to listen for the whistling of shells coming directly toward us at about twice the normal velocity of sound. Of seven shots we heard the whistling of four before the splash occurred. Usually the whistle preceded the splash by about two seconds. The intensity of the whistling varies greatly with different shots, and seems to depend on several circumstances, the main one of which is probably the tearing of the copper "rotating band" during the passage from the gun. Such a tearing will produce a tongue which acts as an excellent reed. Of the four shots which we heard whistle, only one was loud enough to be heard by all three of us. In this case there was a higher note than in the case of any of the other three, and the rotating band flew off during the richochet, indicating it had been badly torn. The chief engineer of the tug which was towing the target, told me that usually only a small percentage of such shells whistle loudly enough to be heard, but related some amusing incidents in which the whistling was very loud, and in which men on the tug ducked for safety. I questioned him very carefully concerning his observations and personally would be convinced by them alone, even though I had not myself heard the sound on the day I accompanied the target. More than half a dozen coast artillery officers have also told me of hearing the whistling from the tug before the shell struck the water. The diagram, figure 1, shows the relative position of the tug, target, and battery; also the point of splash for each of the seven shots. Shots 1, 3, 6, and one other were heard. Number 6 is the one from which the rotating band was torn. I have talked with a number of officers, and have been at the batteries myself, but have found no one who has heard the whistling of a shell from high velocity direct fire guns, when he was at the battery. In this connection I must warn against confusion between the whistling of a shell and the noise due to the inrush of the air to the vacuum left by the shell after it has passed. In the case of mortars and howitzers, which are firing at velocities less and often not more than half the velocity of sound, the whistling is sometimes very plainly heard from the battery.

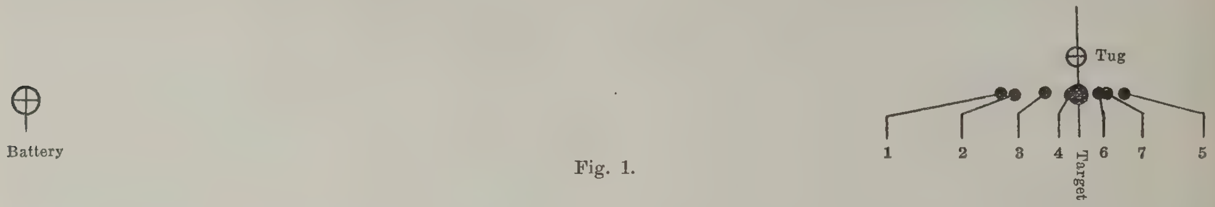

Fig. 1.

Winkelmann in his *Handbuch der Physik,* Band II, Seite 523–528, gives data obtained by Mach, Berthelot, and Wolf on the velocity of explosive sounds at various distances from the point of explosion. Unfortunately, weather conditions and temperature are not stated in any case, and, therefore, it has been impossible to obtain the normal velocity of sound, except in the case of Wolf's experiments, where the data are complete enough that a curve shows this velocity to have been about 341 meters per second. This curve is given as figure 2, and the data by Wolf in Table 1 as columns

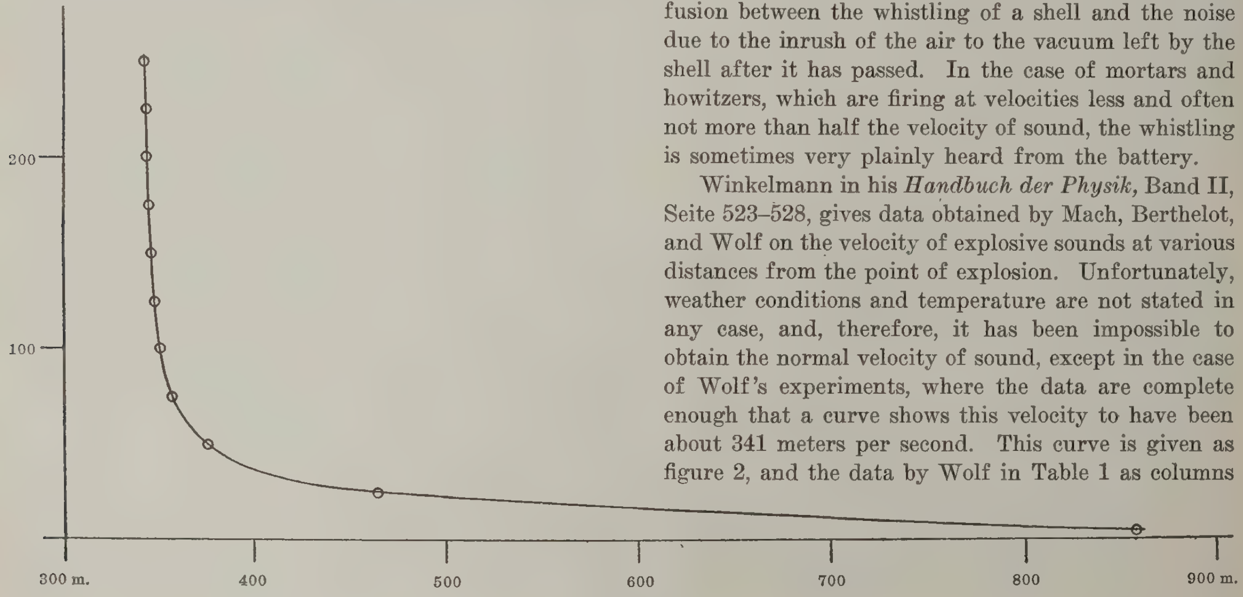

Fig. 2.

I, distance of observer from source, and II, observed velocities of sound. An examination of Wolf's velocities shows that the uncertainty of observation is about one meter per second. Subtracting the normal velocities from the observed velocities gives the excess velocities of column III. The first two of these observations seem to have been made within the explosion as defined in an earlier paragraph of this paper. I therefore took the excess velocities, observed at the distance of 50 and 100 meters from the center of the explosion, and, arbitrarily applying the ratio of their excess velocities, obtained 30 meters as the radius of the explosive sphere. Column IV of this table gives the distances of each of the points of observation from this surface. Column V gives the excess velocities required by our theory, to the nearest meter at the various distances observed by Wolf. The first and third of these must agree exactly, having been used in the determination of the unit of distance, according to which the excess velocity decreases. The other seven are not used in any way and are, therefore, entirely independent. Their agreement with the observed excess velocities depends only on the truth of the theory. It is to be noticed that the greatest difference between observed and theoretical excess velocities is one meter. Since Wolf's observations are uncertain to this amount and since the normal velocity of the day is uncertain to nearly a meter, such an agreement is perfect.

TABLE 1

I	II	III	IV	V
10	858	516	......	....
25	465	123	......	....
50	376	35	20	35
75	357	16	45	16
100	351	10	70	10
125	348	7	105	7
150	346	5	130	5
175	345	4	155	4
200	344	3	180	4
225	344	3	205	3
250	343	2	230	3

We have, therefore, two sets of experimental data entirely different from each other, as well as our mathematical theory to assert that the velocity of sound depends upon the velocity of the source as well as upon the kind of medium.

Since our equations have been built from energy relations it can be seen at once that the same reasoning will apply to all waves in any similar medium. If, therefore, we assume a medium similar to air for light, the same reasoning as for sound must apply and the velocity of light will depend upon the velocity of the source. It is interesting to say, in this connection, that the Michelson-Morley experiment, which has been assumed by many to prove that the ether does not exist, becomes, from the point of view of this theory, one of the strongest proofs of its existence.

The arguments against an emission hypothesis for light as obtained from observations of binary stars and quoted by Tolman, Comstock, and others do not apply to the variable velocity of light demanded by this work because at a distance of a few light hours from the source the velocity of light will have become sensibly normal.

At short distances, observed results, according to either the emission hypothesis, or to this, would be nearly the same in this regard. At the very great astronomical distances, varying all the way from four to thousands of light years, at which binaries are observed, there is a great divergence. The primary postulate of relativity seems to be that it is impossible from observations made in any system to tell which bodies of the system are moving. At astronomical distances this can be done under our hypothesis, and, therefore, it is not relativity.

By analogy we have somewhat strengthened the hypothesis for gravitation stated in the earlier paper. The astronomical arguments advanced there seemed fairly strong, even before there was this substantiation of what seems a rather analogous case. I very much doubt that the gravitational case will prove to be exactly the same as regards the rapidity with which the excess velocity dies out. The perturbations due to this hypothesis should be computed for every possible binary in order to get a certain test as soon as possible.

Professor H. C. Plummer has called my attention to three articles in the *Monthly Notices of the Royal Astronomical Society*, Vol. 74. The first two by Mr. R. S. Capon, on pages 507 and 658; the third by himself on page 660. Mr. Capon shows that any theory of variable velocity of light, similar to the Ritz theory, demands a grouping of the periastra of spectroscopic binaries around the position $\omega = \frac{\pi}{2}$. He shows from a discussion of a large number of binaries that such a grouping actually does exist but that the effect is very much smaller than is demanded by the Ritz theory. In his work he has employed a quantity "p" which according to the Ritz theory must have a value of one. He shows in his second article that the maximum value of this quantity is less than $4 \cdot 10^{-6}$. He suggests that the evidence favors to some extent the Stokes ether-drag theory although he calls attention to the fact that there are serious theoretical difficulties in accepting the Stokes theory. The quantity "p" in Mr. Capon's work is the factor by which the velocity of the star must be multiplied in order to get the excess velocity of light from it over the normal velocity of light. In our theory the average value of this quantity for any star is $p=\dfrac{\int_1^r \frac{dn}{n}}{n-1}$. It decreases from one at the surface of the star to zero at an infinite distance. For a star of five hundred thousand miles radius and thirty light years distance from us the mean value of "p" becomes $2.2 \cdot 10^{-7}$. For closer or larger stars the value of "p" will be greater. It must be remembered that the unit of distance according to which the excess velocity decreases is the radius of the source. Professor Plummer in his article discusses the modifications of the Doppler effect due to the Ritz theory and shows that if the Ritz theory were true they would be such as almost to preclude the possibility of any actual spectroscopic binaries. However, according to our theory, the rapidly decreasing excess velocity gives a very slight modification of the Doppler effect; so slight that nothing can be certainly noticeable in any observation which we can make at present. The results obtained by this theory and by Stoke's ether-drag theory will be very similar. I cannot conceive of any possible experimental evidence along these lines to distinguish between the two. It seems merely a question of which is the simpler should further experiments strengthen the belief that one or the other is true.

Even though I had the ability and the requisite knowledge of electro-magnetic theory, and of radioactivity, which I do not, to pursue the results of this hypothesis in these fields, the war, probably lasting through several years, would leave me almost no time for research. If this very preliminary work will start any other investigator in what seems to me a most important field, I shall indeed feel most happy over the result.

DINSMORE ALTER,
Captain, C. A. N. A.

FORT WINFIELD SCOTT, CALIFORNIA.
March 26, 1918.

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A SPECTROSCOPIC STUDY OF *β CANIS MAJORIS*

The star *β Canis Majoris* ($\alpha = 6^h 18^m3$, $\delta = -17° 55'$, vis. mag. = 2.0, Class B1) was discovered to be a spectroscopic binary of very short period by Dr. Albrecht at the Lick Observatory in 1908.[1] Albrecht's approximate elements from unpublished data were given in Campbell's *Second Catalogue of Spectroscopic Binary Stars*. The period, $0^d25\pm$, places this star in a class with those like *β Cephei* and *σ Scorpii*, whose periods are known to be less than one third of a day. The period of *β Cephei* is 0.190[2] and that of *σ Scorpii* 0.247.[3] The three spectra are all of Class B1 and resemble one another greatly. For the three stars, the values of $a \sin i$ are very small.

A number of three-prism spectrograms of *β Canis Majoris* were obtained at the Lick Observatory in 1904, 1907, 1908, 1909, and 1910, of which only one series, that of March 1, 1909, was taken in succession through one continuous period of the star. A great number of spectrograms were secured by me in 1917 and 1918, taking care to have continuous series of plates covering the same revolutions of the star around its center of mass. Most of these plates up to the end of 1908 were measured by Albrecht and Miss Hobe, but all of those obtained since 1909 Feb. 22 have been measured again by myself. The spectra are good and admit of accurate measurement. My measures have been made with a Hartmann spectrocomparator, using fourteen sections on each plate, in the direct and reverse positions. As the standard reference plate, a good spectrogram of *β Canis Majoris* itself (1909 Feb. 22, 2418360.760 J.D.) was used. It was measured in the usual way by Miss Hobe, who obtained a plate displacement of +60.80 km., and by me with the result +61.78 km. The use of the value +60.80 gave a series of radial velocities for the observations of 1909 March 1 which agree very well with the velocities obtained by other observers. In figure 1 my velocities are plotted as circles, while the velocities from direct measurements by other observers are plotted as crosses. The velocity curve resulting from my measures agrees with the curve from other measures. This curve and the plotted observations give us an idea of the accuracy obtained in the present measurements. The value +60.80 km. was then adopted for the standard-plate displacement. The following table gives the Greenwich mean time of each exposure, the corresponding Julian day and the observed radial velocity.

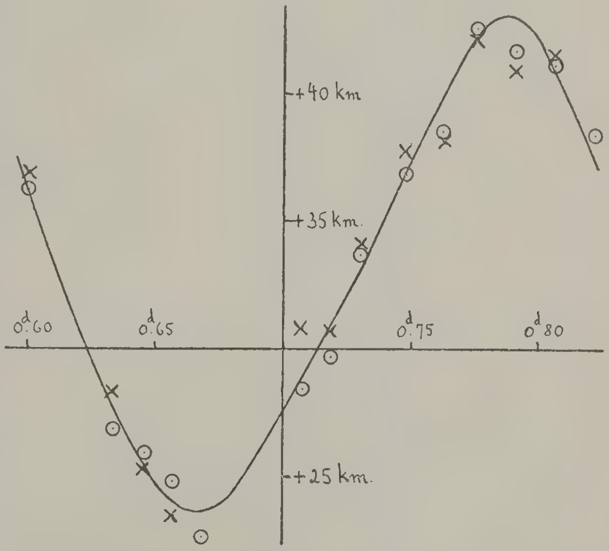

Fig. 1. Velocity Curve of *β Canis Majoris*, March 1, 1909.

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[1] *Lick Obs. Bull.*, 5, 62, 1908.

[2] *Detroit Obs. Publ.*, 2, 144, 1916.

[3] *Revista de la Soc. Astr. de España y America*, 6, 41, 1916.

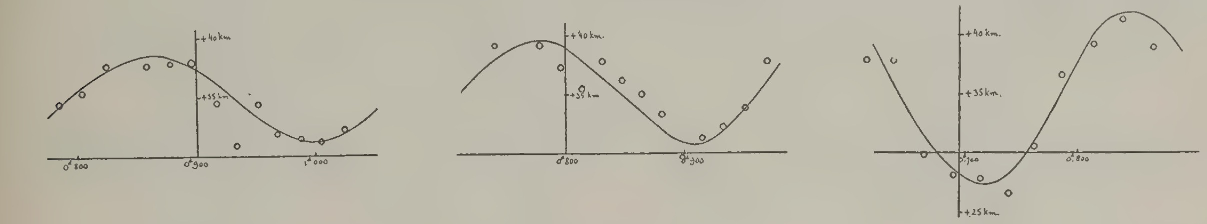
TABLE I

Observed Radial Velocities

Date	J. D. 2410000+	Observed Velocity km.	Date	J. D. 2410000+	Observed Velocity km.	Date	J. D. 2410000+	Observed Velocity km.
1904 Jan. 13	6493.808	+39.83	1909 Feb. 22	8360.675	+25.68	1909 Mar. 16	8382.642	+35.73
1904 Feb. 1	6512.731	+30.56		8360.695	+29.10	Mar. 17	8383.744	+37.00
1907 Nov. 1	7881.014	+39.86		8360.760	+41.60		8383.763	+37.24
1907 Dec. 17	7927.856	+28.21		8360.778	+42.71	Apr. 1	8398.619	+29.28
1908 Jan. 11	7952.805	+28.19	Mar. 1	8367.600	+36.09		8398.636	+25.61
Feb. 24	7996.669	+33.05		8367.634	+26.96		8398.653	+28.23
Sept. 11	8196.042	+31.93		8367.646	+25.96	1910 Feb. 11	8714.697	+23.14
Oct. 5	8220.051	+40.24		8367.657	+24.85	Apr. 15	8777.629	+32.44
9	8224.062	+40.04		8367.668	+22.53		8777.640	+32.91
Oct. 13	8228.061	+37.41		8367.706	+28.23		8777.655	+33.06
Nov. 6	8252.049	+36.48		8367.718	+29.79	17	8779.637	+33.54
10	2856.096	+26.96		8367.730	+33.75		8779.649	+35.71
19	8265.003	+40.89		8367.746	+36.77		8779.663	+34.00
28	8274.962	+28.67		8367.761	+38.34	20	8782.656	+29.24
Nov. 29	8275.906	+27.01		8367.775	+42.33		8782.674	+33.26
Dec. 5	8281.910	+34.09		8367.790	+41.88	21	8783.643	+32.42
11	8287.811	+33.30		8367.805	+41.10		8783.656	+32.53
17	8293.892	+30.53		8367.822	+38.48		8783.685	+29.63
20	8296.819	+31.48	11	8377.613	+35.94	Apr. 22	8784.640	+35.24
	8296.969	+38.59		8377.629	+31.75		8784.651	+32.47
Dec. 24	8300.777	+39.94	12	8378.611	+34.74		8784.663	+29.80
	8300.943	+35.05	15	8381.635	+36.32			

Date	J. D. 2420000+	Observed Velocity km.	Date	J. D. 2420000+	Observed Velocity km.	Date	J. D. 2420000+	Observed Velocity km.
1917 Nov. 29	1562.781	+35.35	1917 Dec. 21	1584.848	+36.14	1918 Jan. 30	1624.798	+38.74
	1562.806	+41.48		1584.865	+35.07		1624.823	+40.31
	1562.826	+43.10		1584.882	+33.25		1624.863	+35.32
	1562.922	+33.43		1584.899	+29.63		1624.890	+34.01
	1562.940	+31.03		1854.915	+31.24	Feb. 11	1636.615	+32.50
	1562.958	+32.65		1854.933	+32.21		1636.632	+28.92
Nov. 30	1563.058	+40.82		1854.951	+33.75		1636.651	+28.12
Dec. 2	1565.776	+36.24		1854.970	+37.73		1636.667	+27.62
	1565.794	+40.67	1918 Jan. 17	1611.695	+31.36		1636.682	+27.72
	1565.811	+43.75		1611.712	+27.43		1636.696	+31.77
3	1566.763	+36.44	18	1612.623	+37.94		1636.713	+33.73
	1566.781	+36.87		1612.647	+37.91		1636.749	+41.68
	1566.799	+39.04		1612.672	+29.94		1636.769	+43.72
	1566.817	+41.99		1612.696	+28.09		1636.788	+46.88
	1566.850	+39.98		1612.719	+27.93		1636.811	+41.23
	1566.960	+32.18		1612.742	+26.55		1636.833	+41.75
	1566.998	+33.94		1612.764	+30.52	26	1651.648	+34.08
4	1567.815	+37.16		1612.788	+36.55		1651.667	+30.03
5	1568.784	+34.31		1612.815	+39.12		1651.686	+25.20
	1568.803	+35.28		1612.840	+41.25		1651.706	+25.72
	1568.803	+35.28		1612.865	+38.97		1651.803	+42.76
	1568.823	+37.53	28	1622.646	+38.07	27	1652.638	+30.08
	1568.858	+37.66		1622.671	+37.21		1652.657	+27.52
	1568.878	+37.94		1622.693	+34.01		1652.679	+24.43
	1568.896	+37.83		1622.719	+35.25		1652.704	+21.85
	1568.917	+34.46		1622.741	+35.49		1652.735	+26.31
	1568.934	+30.88		1622.767	+35.47	Feb. 28	1653.604	+44.07
	1568.952	+34.45		1622.795	+35.88		1653.619	+39.63
	1568.969	+31.94		1622.822	+36.62		1653.672	+27.94
	1568.988	+31.54		1622.845	+35.29		1653.691	+26.95
Dec. 6	1569.005	+31.15		1622.870	+37.45	Mar. 8	1661.708	+32.22
	1569.023	+32.16	Jan. 30	1624.644	+30.80	9	1662.605	+42.78
Dec. 21	1584.741	+39.26		1624.669	+31.76	13	1663.721	+29.34
	1584.779	+39.15		1624.699	+31.68	Mar. 14	1667.611	+39.86
	1584.797	+37.37		1624.725	+33.53		1667.721	+33.60
	1584.814	+35.56		1624.749	+34.90		1667.745	+32.73
	1584.831	+37.84		1624.773	+35.76			

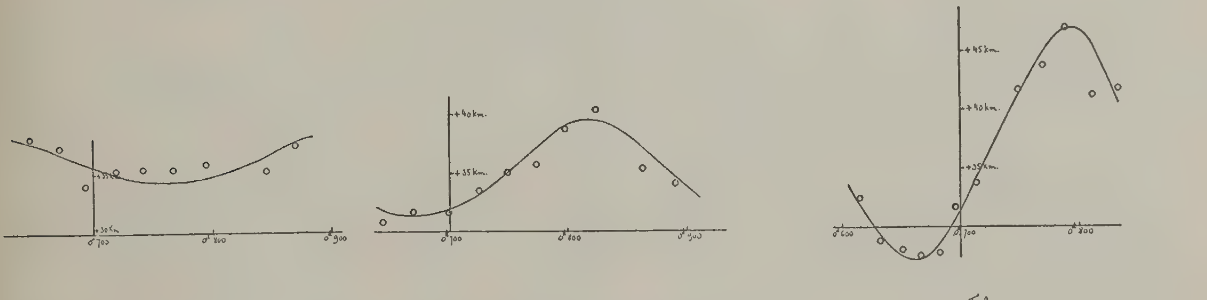
After a careful examination of the plates I found that the aspect of the spectrum changes. Sometimes the lines are narrower, sometimes wider and more diffuse than the general average condition. The maximum as well as the minimum widths are attained approximately once in each period of velocity variation, and the maximum is separated from the minimum by about half a period. The maximum width of the lines in different periods of velocity variation occurs at different places on the corresponding velocity curves, showing a slightly different period for the variation of widths than for the variation of velocity. There also seems to be a variation of the intensity of the lines. It does not seem possible that these variations of width are due to the combination of two spectra; they are most likely due to physical changes in one body only. No trace of a second spectrum has ever been found on the spectrograms.



1917 December 5

1917 December 21

1918 January 18



1918 January 28

1918 January 30

1918 February 11

Fig. 2. Velocity Curves of β *Canis Majoris.*

The period of the velocity variation being about $0^{d}25$, I was able to get six complete series of plates covering each an entire period, in addition to the one taken on March 1, 1909. The seven velocity curves show a considerable difference in range, so that it is impossible to combine all the observations into a single curve and treat them by a least squares adjustment. The different velocity curves on the six dates in 1917 and 1918 are given in figure 2, while the curve of March 1, 1909, is given in figure 1.

Dr. Paddock measured the best two lines on at least two spectrograms in each of my six series illustrated in figure 2, and his results confirm mine as to the amount and direction of velocity variation between the times when the two spectrograms in each series were made.

The mean velocities derived from the seven velocity curves are given in the following table:

1909 March 1	+33 km.
1917 December 5	+35 km.
1917 December 21	+35 km.
1918 January 18	+34 km.
1918 January 28	+36 km.
1918 January 30	+35 km.
1918 February 11	+37 km.

The mean velocity for each curve is practically constant within the limits of errors. If these mean velocities are considered as the center-of-mass velocities, then the center of mass of the system has probably a constant velocity. The center of mass might however have a great variation of velocity, if the period of this variation would be equal or nearly equal to the period of the velocity curve. In the case of near equality it would produce variations of amplitude in the observed velocity curve, without however changing its mean velocity very much, as will be shown later.

The considerable variation of range is a most interesting fact and is now well established for stars of this type. It is analogous to the results suspected by Dr. R. K. Young for 12 *Lacertae,*[4] and by Dr. C. C. Crump for β *Cephei.*[5] On 1918 January 28 the velocity curve was almost a straight line, having a variation of about 3 km. It seems to change very rapidly in only a few days. It will be shown later that the *variation of range is not periodic,* or, at least, that it does not show a simple periodicity. This change of amplitude appears to be a very complicated function, but does not seem to be a discontinuous function.

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[4] *Publ. Dominion Obs., Ottawa,* 3, 65, 1915.

[5] *Loc. cit.*

Another important result which seems to be confirmed by a great number of observations is that there is no period that connects and represents the different minima of velocities, but there is a period which sometimes gives a *maximum,* and at other times a *minimum,* in each cycle of velocity variation.

The approximate elements of an orbit under the supposition that the motion is elliptic (which in this case is doubtful), as deduced from the curve for March 1, 1909, are.

$P = 0^{d}.25$
$e = 0.04$
$\omega = 225^{\circ}$
$K = 9.10$ km.
$T = 2418367.700$ J.D.
$\gamma = +33.06$ km.
$a \sin i = 31{,}300$ km.

Owing to the great and non-periodic change of range in the velocity curve, it is impossible to make use of all the separate observations (not falling within the same period) that have been obtained previous to 1917, unless a law of variation (probably very complicated) should be known.

Measures of the widths of fourteen lines on a great many spectrograms have been made by me. The unit of width used is $\frac{1}{2000}$ of a millimeter. These measures are given in the following tables, in which the first column indicates the date of the exposure and the corresponding Julian day, and the last column the sum of the widths of the fourteen lines for each plate.

Each set of plates has been measured in an irregular order, so as not to be influenced psychologically by the preceding or the following set of measures on each plate. They have afterwards been put in their chronological order. The measures for the separate lines are given only for December 21; to save space in printing, these figures have not been given for the other dates; only the sums of the widths are printed.

TABLE II

Observed Widths of Spectral Lines

	O	O	O	O	O	He	O	O	He	He	Mg	Si	Si	Si	
1917 Dec. 21	4345.677	4348.134	4349.541	4351.495	4367.012	4388.100	4415.076	4417.121	4437.718	4471.676	4481.400	4552.636	4567.897	4574.791	Sur
J.D. 2421584.741	223	242	212	253	258	269	199	188	205	259	122	157	171	140	289
.779	278	265	220	198	255	319	242	210	268	272	201	190	200	188	330
.797	263	290	286	269	250	272	232	242	240	248	178	222	200	175	336
.814	262	292	270	249	220	270	248	237	227	269	164	162	190	198	325
.831	243	296	268	253	227	285	222	204	190	245	195	198	195	157	317
.848	210	234	215	200	153	247	198	152	186	272	201	158	148	141	271
.865	198	259	179	152	193	219	166	158	162	236	169	150	128	128	249
.882	182	188	185	190	183	227	162	160	164	190	161	147	152	120	241
.899	136	98	171	139	176	206	150	137	163	193	142	151	133	120	211
.915	155	149	170	168	131	204	143	148	160	232	140	137	120	127	218
.933	138	142	171	160	169	260	153	168	148	209	176	138	136	140	230
.951	183	192	155	183	154	270	191	187	198	205	180	138	148	175	255
.970	228	243	208	205	237	263	188	201	218	251	188	169	178	184	296

TABLE III

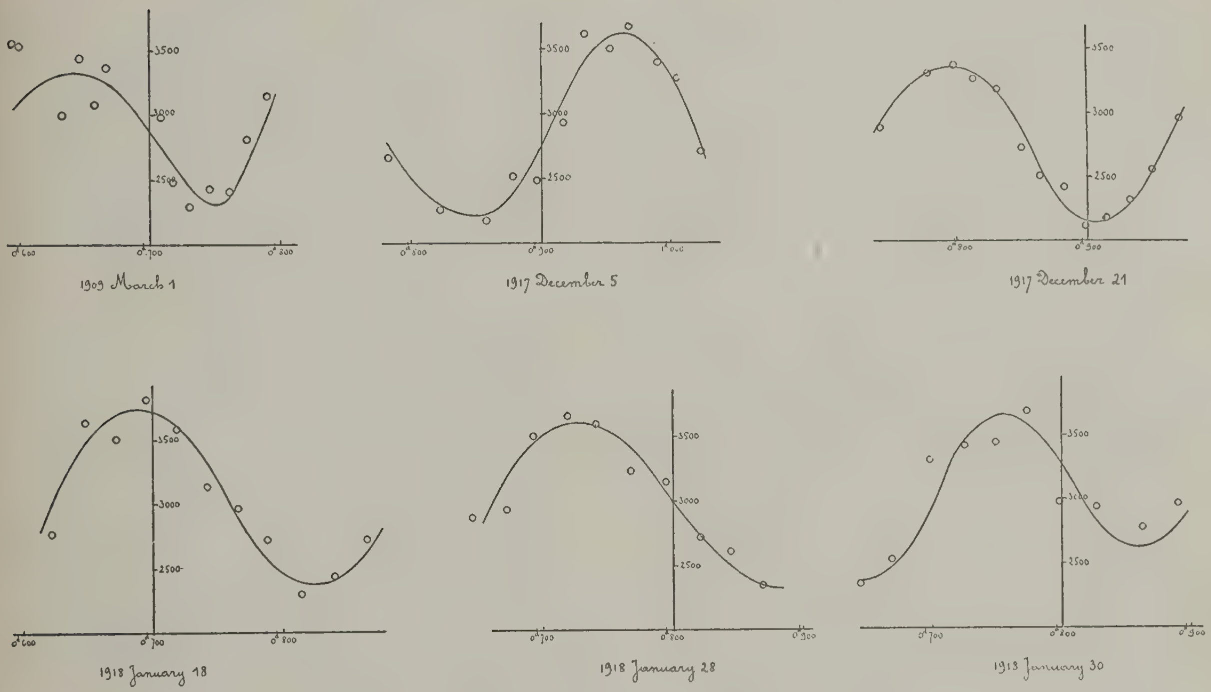
Sum of the Widths for the Other Dates

Date	J.D. 2410000+	Sum of the Widths	Date	J.D. 2420000+	Sum of the Widths	Date	J.D. 2420000+	Sum of the Widths
1909 Mar. 1	8367.600	3529	1917 Nov. 29	1562.781	2376	1917 Dec. 5	1568.784	2665
	8367.634	2998		1562.806	2093		1568.823	2265
	8367.646	3438		1562.826	2045		1568.858	2178
	8367.657	3083		1562.922	3048		1568.878	2526
	8367.668	3367		1562.940	3461		1568.896	2460
	8367.706	2909		1562.958	3149		1568.917	2954
	8367.718	2471	Nov. 30	1563.058	2185		1568.934	3624
	8367.730	2287	Dec. 3	1566.763	2879		1568.952	3497
	8367.746	2427		1566.781	2736		1568.969	3679
	8367.761	2406		1566.799	2513		1568.988	3403
	8367.775	2806		1566.817	2437	Dec. 6	1569.005	3263
	8367.790	3143		1566.850	2394		1569.023	2696
Apr. 1	8398.619	3030		1566.960	3436	1918 Jan. 17	1611.695	3481
	8398.636	3153		1566.998	2983		1611.712	3321
	8398.653	3312						

TABLE II—(*Continued*)

Observed Widths of Specral Lines

Date	J.D. 2420000+	Sum of the Widths	Date	J.D. 2420000+	Sum of the Widths	Date	J.D. 2420000+	Sum of the Widths
1918 Jan. 18	1612.623	2779	1918 Jan. 28	1622.741	3602	1918 Jan. 30	1624.863	2953
	1612.647	3630		1622.767	3339		1624.890	2968
	1612.672	3532		1622.795	3247	Feb. 11	1636.615	3196
	1612.696	3809		1622.822	2728		1636.632	3108
	1612.719	3578		1622.845	2605		1636.651	2840
	1612.742	3141		1622.870	2351		1636.667	2218
	1612.764	2976	Jan. 30	1624.644	2353		1636.682	2494
	1612.788	2716		1624.669	2532		1636.696	2526
	1612.815	2299		1624.699	3314		1636.713	2590
	1612.840	2420		1624.725	3410		1636.749	2689
	1612.865	2717		1624.749	3431		1636.769	3004
Jan. 28	1622.646	2889		1624.773	3681		1636.788	3200
	1622.671	2935		1624.798	2979		1636.811	3102
	1622.693	3517		1624.823	2953		1636.833	3874
	1622.719	3648						



1909 March 1

1917 December 5

1917 December 21

1918 January 18

1918 January 28

1918 January 30

Fig. 3. Variation of Line Width in β *Canis Majoris.*

From these measures curves of variation of line-width, figure 3, have been obtained for the six periods corresponding to the velocity curves in figure 2. The time has been taken as abscissa and the sum (which is fourteen times the mean) of the widths has been taken as ordinate. All these curves indicate a similar variation of width.

Period of Velocity Variation of the Star and Period of the Variation of Width.—The epochs of maxima and minima of velocity derived from the velocity curves are (expressed in Julian days):

	Max. 2420000+	Min. 2420000+
1917 Nov. 29	1562$^{\mathrm{d}}$848	1562$^{\mathrm{d}}$984
Dec. 3	1566.831	1566.973
5	1568.864	1568.999
1917 Dec. 21	1584.778	1584.910
1918 Jan. 18	1612.847	1612.721
28	1622.894	1622.760
Jan. 30	1624.816	1624.680
1918 Feb. 11	1636.794	1636.658

It was then found, if we take the epoch 2421612.721 (for January 18) as origin, that *no period* could connect the series of minima (the use of isolated observations on December 4 and January 17 being of great help in ascertaining this fact), but that only the underlined epochs could be rather well connected and that the period connecting them was approximately 0^{d}25714. Starting from 2421612.721 we obtain by adding or subtracting a certain number of periods, the following numbers:

Epoch 2420000+	Difference	Number of Periods
1562.835	—0^{d}013	—194
1566.950	—0.023	—178
1569.007	+0.008	—170
1584.949	+0.039	—108
1612.721	0	0
1622.749	—0.011	+39
1624.806	—0.010	+47
1636.635	—0.023	+93

Taking the epochs of maximum and minimum for January 18 as origin and the period as 0^{d}25714, an ephemeris was then computed for every night between 1918 February 26 and 1918 March 24, giving the times of maxima and minima. Spectrograms were then secured between these dates and near the predicted times whenever it was possible. The exigencies of the programme, and especially the bad weather at this season of the year, made it very hard to secure a great many of them. Every time, however, I obtained a maximum (or a minimum) as predicted.

The maxima of width are also connected by a period which is 0^{d}25130. If we take indeed the series of epochs for these maxima, the epochs computed by taking J.D.2421612.693 as origin and subtracting or adding a certain number of periods we have the following table:

Epoch Observed J.D. 2420000+	Epoch Computed J.D. 2420000+	Difference	Number of Periods
1562.940	1562.936	—0^{d}004	—198
1566.956	1566.956	0	—182
1568.967	1568.966	—0.001	—174
1584.795	1584.799	+0.004	—111
1612.693	1612.693	0	0
1622.743	1622.745	+0.002	+40
1624.755	1624.755	0	+48
1636.824	1636.818	—0.006	+96

The period of variation of width, 0^{d}25130, is then a little shorter than the period of velocity variation, which is 0^{d}25714.

The Change of Amplitude.—An inspection of the different velocity curves indicates that the amplitude of the velocity variation changes very rapidly and that the interval between two successive positive maxima is approximately eleven days. I shall call the amplitude positive when we get a maximum when we expect a maximum, and negative when we get a minimum when we expect a maximum.

From all the data obtained the following amplitudes of velocity variation have been derived:

Date	Approximate J.D. 2420000+	Amplitude km.
1917 Nov. 29	1562.8	—14.0
Dec. 3	1566.8	+15.0
5	1568.8	+ 7.4
Dec. 21	1584.8	+ 8.7
1918 Jan. 18	1612.8	+14.2
28	1622.8	+ 3.0
Jan. 30	1624.8	— 8.4
Feb. 11	1636.8	+19.8
26	1651.8	—18.0
27	1652.7	—20.0
Feb. 28	1653.7	—16.5
Mar. 8	1661.7	— 8.0
9	1662.6	—14.3
13	1666.7	+14.0
1918 Mar. 14	1667.7	+ 7.0

Taking the first eight observations it was found by applying a graphical method that no simple period could connect them. These eight observations do not depend on the period 0^{d}25714, since they have been taken directly from the velocity curves. If we now take the remaining observations we find that the change in amplitude is not at all *simple-periodic.* The values of the maxima and minima of amplitude seem to be +20 km. and —20 km.

Variation of Light.—Through the kindness of Professor Joel Stebbins, Director of the Observatory of the University of Illinois, *β Canis Majoris* was investigated by the use of an electric cell photometer. He finds that the star does not vary more than a tenth of a magnitude and suspects that it varies on the basis of the period 0^{d}25130 which is the period of change of width. According to him it looks, however, as though irregular changes are taking place, as his best observations are discordant by nearly a tenth of a magnitude. If these irregular changes are real they may correspond to variation of amplitude in the velocity curve. Professor Stebbins intends to make further studies of this star.

A Possible Explanation of the Phenomena Observed.—At first sight the change of amplitude in the velocity curve would suggest that the inclination of the plane of the orbit of the spectroscopic binary varies, this plane being sometimes inclined toward us and at other times away from us. It does not seem, however, that such a hypothesis could explain everything. Another explanation of the change of amplitude could be found in the fact that if two sinusoidal or periodic variations of approximately the same constant amplitude and very nearly the same period exist, their combination may give us a curve of variable amplitude in which a nearly constant period connects a series of maxima with a series of minima, as shown in figure 4. If in this figure the dotted curve represents one periodic function, and the curve of dashes another periodic function, their combined effect might be represented by the full-line curve of varying amplitude and period. If now slight variations occur in the period of one or both component curves, the resulting curve, although having a very nearly constant period oscillating slightly around a mean value, will show a very complicated and rather erratic change of amplitude. One of the component curves might, for instance, be due to the motion of the body around a center of mass in a binary system, while the other component curve might be due to the variation of this center of mass under the influence of some unknown cause, such as possibly the action of a third body. The variation of the center of mass in this case ought to have a period very nearly the same as that of the first-mentioned motion of the body, since there is no apparent great change in the mean velocity derived from each velocity curve of figures 1 and 2. The influence of a third body is, however, very problematic. It would require an enormous amount of labor to prove mathematically that such a three-body system could exist. Moreover, the shortness of the periods and the variation of width (although possibly due to tidal action), raise difficulties.

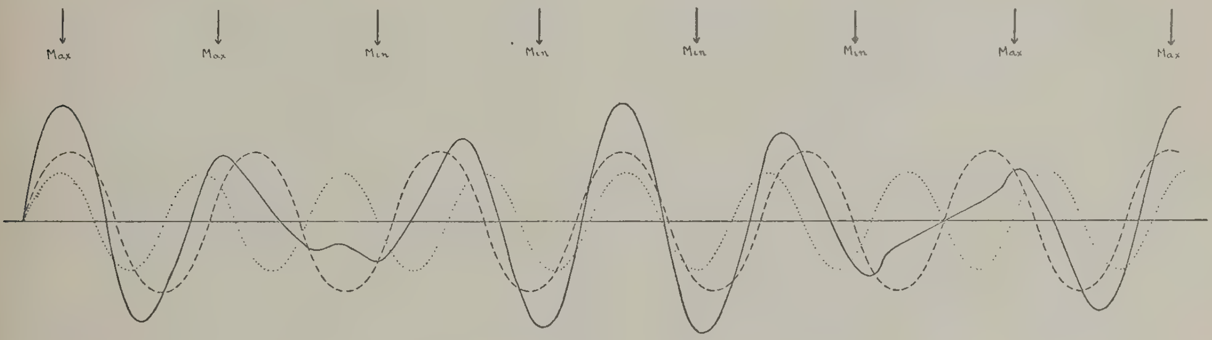


Fig. 4.

In addition, the parallax deduced by Mitchell at the McCormick Observatory for β *Canis Majoris* is:

$$\pi_{\mathrm{rel}} = +0''.009,$$

with a probable error of ±0''.009.

This would make the absolute brightness of the star about 1020 times that of the Sun. The probabilities tude is subject to variations for which no law has been found. A period, 0ᵈ25714, starting from a given maxi- are then that β *Canis Majoris* is much larger than our Sun. This idea of the enormous size of a star in the case of a Cepheid variable, combined with a very small value of $a \sin i$ (and β *Canis Majoris,* like β *Cephei*[6] and 12 *Lacertae*[7] could very well be classified among the Cepheid variables) has made Shapley doubt that these stars are binary systems.[8]

Summary of the Results.—β *Canis Majoris* seems to be a very bright giant star, if we accept the value of the parallax given by Mitchell, +0''.009. The star has a periodic velocity curve in which, however, the ampli- mum, corresponds always to a maximum or a minimum of the observed velocity curve. The spectral lines undergo a periodic change of width, the amplitude of this variation being always approximately the same and its period 0ᵈ25130.

A possible explanation of the observed phenomena is sought for, but it is very hard to explain them all.

The writer wishes to acknowledge the valuable help given by Director Campbell, Dr. Moore, and Dr. Paddock throughout the work.

F. HENROTEAU.

MOUNT HAMILTON, May 4, 1918.
Issued June 13, 1918.

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[6] *Astr. Nach.,* 205, 103, 1917.

[7] J. Stebbins (unpublished).

[8] *Ap. Jour.,* 40, 448, 1914.

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THE VISUAL MAGNITUDE SCALE AND THE COUNTS OF STARS

The relation between the magnitudes and the number of stars to any limit of magnitude has been expressed commonly by the formula: $\Sigma_m = ab^m$, where Σ is the total number of stars to the limiting grade of magnitude, m, and the factors a and b are determined by actual counts of the stars. For the counts, we have the Uranometries for bright stars alone, and the Durchmusterung for more extended lists. The Uranometry magnitudes, based upon naked-eye estimates, are undoubtedly more precise than the Durchmusterung estimates for the bright stars, made with small telescopes. At Cordoba, the Uranometry magnitudes of all stars brighter than the seventh were adopted for the visual Durchmusterung Catalogue, though we made estimates of their brightness at the time of observation. The Uranometries give us the complete count to the sixth magnitude for the whole sky, and the Argentine Uranometry extends the count to the seventh, for the southern sky.

The northern Durchmusterung of Argelander gives the complete count to the ninth magnitude, and some inferences can be drawn from the incomplete counts of fainter stars, down to 9.5. Schönfeld's extension to —23° was designed to be complete to 9.5, and includes partial counts of fainter grades, down to 10. The Cordoba extension was designed to be complete to tenth magnitude, and the lowest grade, listed as 10 in the first three volumes, includes stars much below the limiting grade. In the fourth volume, of ten degrees in width, the fainter stars have been estimated in part down to twelfth magnitude, which was close to the limit of visibility with the five-inch telescope.

The Photographic Durchmusterung of the southern sky, made at the Cape of Good Hope Observatory, was based upon the visual magnitudes of Gould's Cordoba Zones, and is complete for a grade slightly below the ninth magnitude; with widely varying numbers of stars of various faint classes, nowhere definitely complete for a limit much below ninth.

The assumption upon which the above expression for the count is based is the fairly uniform distribution in space of all classes of stars. This assumption has never been considered as rigorously true, the Milky Way and the great clusters of faint stars being evident exceptions to uniform distribution. But, from the entire area of the sky, the differential conclusions to be drawn may be approximations to the truth. The expression must finally break down, as the faint grades of stars are counted, as otherwise the count would rapidly approach such an aggregation of stars that the sky would be nearly completely filled with the faintest grades.

The expression for the count can be used to test the scale of magnitudes, and to determine the light ratio at different points of the scale. The factor b can be placed equal to $\delta^{3/2}$, when δ is the light ratio of two classes of stars that differ in distance from us by the amount necessary to reduce the apparent brightness by one unit of magnitude. The numbers of stars in two concentric spheres will vary directly as the cubes of the radii, and the brightness varies inversely to the squares of the distances. It will be convenient in the following computations to expresss the light ratio of two stars, or of two classes of stars, by L. Then the light ratio of two stars that differ by one magnitude can be expressed by L_1; and for stars that differ by two and three magnitudes, by L_2 and L_3, respectively. For the ratio of the counts of stars at any two grades we can use the letter N. Then the ratio of the counts at grades differing one magnitude can be expressed by N_1; and for the ratios of counts at grades differing two and three magnitudes, N_2 and N_3 can be used, respectively. It is evident that the ratios of the counts of stars at various grades are independent of the factor a, in the expression above for the total count at any grade. And the unit light ratio can be determined from $\delta = b^{2/3}$, when b is derived from the counts at two grades differing by one unit of magnitude.

Table I gives the theoretical ratios of the summations of the stars for four assumed scales of magnitude; based upon unit light ratios of 2.7, 2.5, 2.3, and 2.1, respectively. The unit light ratio $L_1 = 2.5$ corresponds to the normal scale, [0.40]; the ratio $L_1 = 2.3$ corresponds to the historic scale, [0.36]; and the other two ratios correspond to assumed scales, which will be referred to here as the lower and upper test scales, [0.32], and [0.44], respectively. The table gives the ratio of the summations for differences of one tenth of a unit of magnitude, up to two full units, and for full units up to five. The four assumed scales have been used in a previous discussion of screen values.[1]

TABLE I

RATIO OF SUMMATIONS. N

Diff. Mag.	Upper Test	Normal	Historic	Lower Test
0.1	1.16	1.15	1.13	1.12
0.2	1.36	1.32	1.28	1.25
0.3	1.58	1.51	1.45	1.39
0.4	1.84	1.74	1.64	1.56
0.5	2.14	2.00	1.86	1.74
0.6	2.49	2.29	2.11	1.94
0.7	2.90	2.63	2.39	2.17
0.8	3.37	3.02	2.70	2.42
0.9	3.93	3.47	3.06	2.70
1.0	4.57	3.98	3.47	3.02
1.1	5.3	4.6	3.9	3.4
1.2	6.2	5.2	4.4	3.8
1.3	7.2	6.0	5.0	4.2
1.4	8.4	6.9	5.7	4.7
1.5	9.8	7.9	6.5	5.2
1.6	11.4	9.1	7.3	5.9
1.7	13.2	10.5	8.3	6.5
1.8	15.4	12.0	9.4	7.3
1.9	17.9	13.8	10.6	8.2
2.0	20.9	15.8	12.0	9.1
3.0	95	63	42	27
4.0	437	251	145	83
5.0	1996	1000	501	251

The first illustration of the use of the counts of stars will be that of the northern Durchmusterung, down to seventh magnitude, the count of fainter stars being given separately later on. The count was made by Littrow, and was published in the *Sitzungsberichte* of the Vienna Academy of Sciences, in 1869. It was reprinted by Dr. Gould, in the Cordoba Uranometry, from which the tabulation following has been condensed. In all of the tabulations here of counts of stars, the upper limit has been taken at second magnitude, since the very small number of stars brighter than that grade may introduce too much uncertainty in the computed ratios by the inclusion or exclusion of a few stars only. In all tabulated counts there will be noted an excess at the full grades of magnitude, and to some extent this will be true of the half grades. There is also a recognized tendency to include an excess of stars at the lowest grade of any plan designed to be complete. In the Durchmusterung estimates, since two or more observations were made of every star, the combination of the original observations to half magnitudes has resulted in a better distribution of the tenths than would be found in single estimates. In the first twenty degrees of the Cordoba Durchmusterung we made an average of over three observations per star, owing to overlapping of zones and to repetition, and the tenths are well distributed in the means of the estimates, originally made to quarters of a magnitude. The tendency to excess at the full grades probably appears as a percentage, nearly the same in amount for all grades that are similar. Thus 8.0 and 9.0 may both contain an excess of stars in the counts, but the percentage is likely to be nearly the same in both. And 7.9 and 8.9 would exhibit deficits in the count, too large a proportion of these grades being included in the full grades adjoining; but the percentages, represented by the deficits in numbers, would be nearly the same. Computations of b probably do not suffer much loss of precision due to this condition, but the definite computation of a is made rather uncertain.

In Table II following, which has been given with full detail in order that it may serve as a sample computation for the other catalogues, a value of N has been derived by comparing every full grade, and every half grade, with all the fainter grades following at intervals of one unit. Thus, in the vertical columns we have computations of N_1, N_2, N_3, etc. based upon the counts of the stars to half grades in the northern sky. For each vertical column a mean value of N_1 has been derived. These will be seen to increase progressively, with fair uniformity, from the N_1 of the brightest grade, to the N_1 of the faintest grade. Simple means have been taken in all cases, since it is evident that no arbitrary system of weighting would be an improvement, and might obscure the evidence of variation in N_1.

The vertical columns of the lower portion of the following table show a progression downwards, that is a result of the progressive variation in light ratios from the bright to the faint grades of the naked eye stars. The influence of the tendency to an excess in the lowest limiting grade of any catalogue does not appear here, since the Durchmusterung includes all stars to ninth magnitude. The northern Durchmusterung has an

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[1] *Lick Obs. Bull.*, 307.

TABLE II

ARGELANDER'S DURCHMUSTERUNG. N

To Magnitude	Count	N								
2.0	28	1.0								
2.5	45		1.0							
3.0	80	2.86		1.0						
3.5	144		3.20		1.0					
4.0	230	8.21		2.88		1.0				
4.5	365		8.11		2.53		1.0			
5.0	680	24.3		8.50		2.96		1.0		
5.5	1120		24.9		7.77		3.07		1.0	
6.0	2108	75.3		26.3		9.17		3.10		1.0
6.5	4122		91.6		28.6		11.3		3.68	
7.0	8017	286		100		34.9		11.8		3.81
Mean values of N_1		2.94	3.01	2.98	2.79	3.09	3.21	3.27	3.68	3.81

Average N_1 to sixth magnitude, 3.04; $L_1 = 2.10$
Average N_1 to seventh magnitude, 3.20; $L_1 = 2.17$

COMBINATION

	N_1	N_2	N_3	N_4	N_5
	2.86				
	3.20	8.21			
	2.88	8.11	24.3		
	2.53	8.50	24.9	75.3	
	2.96	7.77	26.3	91.6	286
	3.07	9.17	28.6	100	
	3.10	11.3	34.9		
	3.68	11.8			
	3.80				
Means	3.12	9.26	27.8	89.0	
N_1	3.12	3.04	3.03	3.07	3.10

Mean $N_1 = 3.07$; $L_1 = 2.11$

excess at 9.5, and the Cordoba work to 42° has a large excess at 10, these being the lowest estimates made. These lowest grades were, however, intended to include stars fainter than the recorded estimates. The light ratio above for the naked eye stars appears to be that of the lower test scale, used as a comparison scale here, especially if the summation be concluded at the sixth magnitude.

TABLE III

COUNTS AND LIGHT RATIOS FOR BRIGHT STARS

Mag.	Heis Uran.	Arg. Uran.	Arg. B.D.	B.D. Adjusted	North. sky Gould	Uran. Argent. South. sky
2.0	29	29	28	24	22	19
2.5	53	52	45		43	46
3.0	84	88	80	72	78	85
3.5	166	174	144		142	152
4.0	257	270	230	221	228	251
4.5	446	443	365		380	387
5.0	721	799	680	626	640	572
5.5	1289	1087	1120		1115	943
6.0	2363	2352	2108	1958	2055	1810
6.5			4122		3903	3204
7.0			8017	7455	7800	6694
N_1	2.96	2.93	3.07	3.12	3.13	3.05
N_1	(2.0 to 6.0)		2.93	3.00	3.04	2.98
L_1	2.06	2.05	2.11	2.13	2.14	2.10
L_1	(2.0 to 6.0)		2.05	2.08	2.10	2.07

	2.0 to 6.0	6.0 to 7.0
N_1 Northern sky	2.0 to 6.0 = 2.96	6.0 to 7.0 = 3.80
N_1 Southern sky	2.98	3.70
N_1 Mean	2.97	3.75
L_1 Whole sky	2.07	2.41

Table III gives the count of the bright stars by grades from various authorities, with some limitations, which may be briefly explained. The two standard Uranometries of the northern sky, those of Heis and Argelander, give estimates only to thirds of a magnitude, so that each grade tabulated below probably includes the summation of stars to a limit one sixth of a magnitude fainter. Argelander's Durchmusterung exhibits an evident excess at the full grades, and an adjustment has been made of the tenths, by assigning two thirds of the sum of the counts at 4.9, 5.0 and 5.1, for example, to the grade 5.0. This adjustment is the fourth count below. Dr. B. A. Gould made an adjustment of the counts of the northern sky, by allowing for the excess of one sixth at the various grades in the Uranometries, and then combining the results with the count of the Durchmusterung. This is the fifth count below. The sixth count is that for the southern sky, from Gould's Uranometry. His catalogue also included a belt of stars north of the equator. For each catalogue and adjusted count the mean value of N_1 has been computed, as in the detailed example preceding. From N_1 the value of L_1 is derived, and the general average has been taken, at the foot of the table.

The unit light ratio above, for stars as bright as sixth magnitude, averages close to that of the lower test scale, $L_1 = 2.1$. From sixth to seventh magnitude the ratio, 2.4, is half way between those of the historic and normal scales.

Table IV gives the count of the northern Durchmusterung, by individual tenths, from sixth magnitude to the lowest limit, 9.5. This census of the stars was designed to be complete only to the ninth magnitude. The lowest grade, 9.5, included stars that were recognized as likely to be fainter than 9½. While the grades 9.1, 9.2, 9.3, and 9.4 are not complete, the summations at these grades show such an excess that one begins to perceive the change of scale, or of light ratio, that must have begun at ninth magnitude. The ratios below have been derived, first with 6.0 as a basis; then with 7.0, 8.0, and 9.0, successively. Finally, values of N_1 have been derived from the ratios at similar tenths throughout; 6.1 to 7.1, 7.2 to 8.2, etc.

TABLE IV

TELESCOPIC STARS OF ARGELANDER'S DURCHMUSTERUNG

Mag.	Count	N	Mag.	Count	N	N	N	N_1
6.0	2108	1.0	7.0	....	1.0			3.81
6.1	2214	1.05	7.1	....	1.04			3.78
6.2	2507	1.19	7.2	....	1.17			3.72
6.3	2782	1.32	7.3	....	1.34			3.85
6.4	2883	1.37	7.4	....	1.40			3.90
6.5	4122	1.95	7.5	....	1.76			3.41
6.6	4281	2.03	7.6	....	1.83			3.43
6.7	4738	2.25	7.7	....	2.02			3.42
6.8	5639	2.67	7.8	....	2.34			3.32
6.9	5876	2.79	7.9	....	2.46			3.35
7.0	8017	3.81	8.0	....	3.16	1.0		3.16
7.1	8362	4.0	8.1	27099	3.4	1.07		3.24
7.2	9346	4.4	8.2	30749	3.8	1.21		3.29
7.3	10702	5.1	8.3	35358	4.4	1.40		3.30
7.4	11218	5.3	8.4	38459	4.8	1.52		3.43
7.5	14078	6.7	8.5	48247	6.0	1.90		3.43
7.6	14687	7.0	8.6	52436	6.5	2.07		3.57
7.7	16224	7.8	8.7	59235	7.4	2.34		3.65
7.8	18708	8.9	8.8	70198	8.7	2.77		3.75
7.9	19699	9.4	8.9	77794	9.7	3.07		3.95
8.0	25321	12.0	9.0	101071	12.6	3.96	1.0	3.96
			(9.1)	116686		4.6	1.15	4.31
			(9.2)	137420		5.4	1.36	4.46
			(9.3)	168698		6.5	1.67	4.77
			(9.4)	203649		8.0	2.01	5.29
			(9.5)	314925		12.4	3.12	6.52

Average N_1 from mag. 7.0 to 9.0 = 3.56

The above counts show distinct variations at individual tenths from a uniformly progressive increase, especially in the brighter grades. This is due to the tendency to favor certain estimates more often than others of nearly the same grade. Excess in the counts is evident at the half grades and the full grades. The excesses and deficits are similar in percentages for similar grades, as is evident from the individual values of N_1.

With 6.0 as a basis, the summation at seventh magnitude is close to that of the normal scale, Table I, and that at eighth magnitude is close to the historic scale. With 7.0 as a basis, the summation at eighth magnitude is close to that of the lower test scale, and that of the ninth magnitude is close to the historic scale. With 8.0 as a basis, the summation at the ninth magnitude is quite close to the normal scale. These indications are confirmed by the values of N_1. The transition from naked eye stars to telescopic stars is marked by an increase in the ratio of the summations, that corresponds to an increase in the unit light ratio. For the telescopic stars from seventh to eighth there is a relative decrease in unit light ratio, followed by an increase from eighth to ninth magnitude. The counts at grades 9.1 to 9.4 show excess above any one of the theoretical summations, and the count at 9.5 is much in excess of theory.

The mean values of the ratios of counts for single units of the magnitude scale, and the corresponding mean unit light ratios, are as follows:

Range			N_1	L_1	
7.0	to	7.4 B.D. Mag.	3.8	2.44	at 6.7 Mag.
7.5		7.9	3.4	2.26	7.2
8.0		8.4	3.3	2.22	7.7
8.5		8.9	3.7	2.39	8.2
9.0		(9.4)	(4.6)	(2.77)	8.7

The incomplete count at 9.4 has been used to indicate the minimum value of the corresponding light ratio. With 9.0 as a basis, the incomplete count at 9.4 is equivalent to the theoretical summation at 9.5 of the normal scale, and, allowing for the deficit in stars, it is undoubtedly as faint as 9.5 on the upper test scale. The 9.5, which was designed to include many stars fainter than 9½, has a summation equivalent to the 9.8 of the normal scale, or to 9.75 of the upper test scale. The unit light ratio of the northern Durchmusterung has undoubtedly increased below the ninth magnitude, for the grades 9.1 to 9.4 are actual estimates in that catalogue. The 9.5 is a flexible grade, not necessarily limited to stars estimated on the same scale as stars of ninth magnitude. The ninth magnitude of Argelander has been estimated with a unit light ratio greater than that of the upper test scale.

No complete count of faint stars has been made for the full area of the southern sky by individual grades, but a count of 37,000 stars in representative parts of the first ten degrees of the Cordoba Durchmusterung, made by the writer, gives the percentages for an extensive area. The table of percentages was first printed in the *Astrophysical Journal* for May, 1898. The treatment of the count in the present form has not been attempted before, however. The second ten degrees of this catalogue would give similar percentages, since the observations of the first two volumes, of ten degrees each, are entirely homogeneous. These two volumes include the observations made by Dr. Thome and the writer, in the proportion of two-fifths and three-fifths respectively. In the third volume Dr. Thome adopted some illumination of the field of the telescope for his own work, to increase the precision of the observations of position, somewhat as Schönfeld had done for his southern zone of twenty degrees. Either from this cause, or from some other conditions, the scale of estimates was changed, and from 42° to 52° the stars between seventh and ninth magnitude were evidently estimated fully half a magnitude fainter than in the preceding volumes. The magnitudes agree very closely with those of Gould's Cordoba Zones, for stars included in both catalogues, while the estimates of the first twenty degrees were from a quarter to a half magnitude brighter than those of the Zones.

TABLE V

CORDOBA DURCHMUSTERUNG. FIRST TEN DEGREES

Mag.	Per cent	N	Mag.	Per cent	N	N	Mag.	N_1
7.0	.008	1.0	8.0	.028	1.0		8.0	3.5
7.1	.009	1.1	8.1	.032	1.1		8.1	3.6
7.2	.010	1.2	8.2	.037	1.3		8.2	3.7
7.3	.011	1.4	8.3	.043	1.5		8.3	3.9
7.4	.012	1.5	8.4	.047	1.7		8.4	3.9
7.5	.015	1.9	8.5	.057	2.0		8.5	3.8
7.6	.016	2.0	8.6	.065	2.3		8.6	4.1
7.7	.019	2.4	8.7	.076	2.7		8.7	4.0
7.8	.021	2.6	8.8	.086	3.1		8.8	4.1
7.9	.023	2.9	8.9	.097	3.5		8.9	4.2
8.0	.028	3.5	9.0	.121	4.3	1.0	9.0	4.3
8.1	.032	4.0	9.1	.144	5.1	1.2	9.1	4.5
8.2	.037	4.6	9.2	.170	6.1	1.4	9.2	4.6
8.3	.043	5.4	9.3	.197	7.0	1.6	9.3	4.6
8.4	.047	5.9	9.4	.230	8.2	1.9	9.4	4.9
8.5	.057	7.1	9.5	.284	10.0	2.4	9.5	5.0
8.6	.065	8.1	9.6	.347	12.4	2.9	9.6	5.3
8.7	.076	9.5	9.7	.434	15.5	3.6	9.7	5.7
8.8	.086	10.7	9.8	.534	19.1	4.4	9.8	6.2
8.9	.097	12.1	9.9	.620	22.1	5.1	9.9	6.4
9.0	.121	15.1	(10)	1.000	(35.7)	(8.3)	(10)	(8.3)

In Table V, similar to that for the northern Durchmusterung, the values of N are first computed from seventh to ninth magnitude, with a basis of the percentage at seventh. Values are also computed from eighth to tenth magnitude, with a basis of the percentage at eighth; and from ninth to tenth, with a basis at ninth. The values tabulated for N_1 are derived from the percentages at each tenth of a unit; for 8.0 from 7.0, 9.1 from 8.1, etc. The estimate 9.9 is the lowest definite grade, just as 9.4 is the lowest definite grade for Argelander.

Our scale for the Cordoba Durchmusterung was based upon practice comparisons with observed areas of the northern sky, in addition to considerable previous experience with meridian instruments. Perhaps because the actual observing was done at much smaller zenith distances than the practice work, the comparison of percentages of ninth magnitude stars per square degree indicates that our estimates at this grade were about 0.2 brighter than those of Argelander.

In the preceding tabulation of N, based upon the percentages at 7.0, 8.0, and 9.0, successively, the following changes of light scale are apparent, from the comparison with Table I. The ratios from seventh to eighth magnitude are in close agreement with the historic scale. The ratios from eighth to ninth are in agreement with the normal scale, and in accord with those of the northern Durchmusterung. At ninth magnitude the change of scale becomes pronounced in character, and the 9.5 of the catalogue is evidently more than 0.1 too bright on the light scale of the ninth magnitude stars. This variation becomes more marked for the fainter grades, and the 9¾ should evidently be tenth magnitude, on the light scale of the ninth magnitude. The summation of the flexible grade 10 is equivalent to the computed summation at 10.5, normal scale.

The mean values of the ratios of counts, for single units of magnitude by tenths, and the corresponding unit light ratios, are as follows:

Range, Cordoba			N_1	L_1		
8.0	to	8.4 Mag.	3.7	2.4	at	7.7 Mag.
8.5		8.9	4.0	2.5		8.2
9.0		9.4	4.6	2.8		8.7
9.5		9.9	5.7	3.2		9.2

The unit light ratio increases as the brightness diminishes. It is but slightly greater than that of Argelander, up to ninth magnitude; and L_1 equals 3.0, approximately, for both authorities, at the ninth magnitude.

TABLE VI

ADJUSTMENT OF N_1 AND L_1 FOR B.D.

Mag.	Stars	N_1	Adjusted N_1.	O–C	Adjusted L_1
2.0	28	......	2.60	....	
2.5	45	......	2.68	....	1.9
3.0	80	2.86	2.76	+0.10	
3.5	144	3.20	2.84	+0.36	2.0
4.0	230	2.88	2.92	—0.05	
4.5	365	2.53	3.00	—0.47	2.1
5.0	680	2.96	3.08	—0.12	
5.5	1120	3.07	3.16	—0.09	2.2
6.0	2108	3.10	3.24	—0.14	
6.5	4122	3.68	3.32	+0.36	2.2
7.0	8017	3.80	3.40	+0.40	
7.5	14078	3.42	3.48	—0.06	2.3
8.0	25321	3.16	3.56	—0.40	
8.5	48247	3.43	3.64	—0.21	2.4
9.0	101071	3.96	3.72	+0.24	
Means		3.24		±0.23	
Average residual		±0.32			
		$L_1 = 2.19$			

The preceding computed variation in light scale through the whole range of the northern Durchmusterung has been condensed in the following table, with a computed adjustment of the variation by a progressive rate. Each value of N_1 has been derived from the summation at one full unit preceding.

The mean of the observed values of N_1 above corresponds to a light scale half way between the historic and the lower test scale. The historic scale is represented at seventh magnitude, and the normal scale at ninth magnitude. The average O–C is less than three quarters of the average residual from the mean. Some improvement would be introduced in the agreement of observed and computed values of N_1 by adopting a mean value of 2.94 from third to sixth magnitude, and a mean value of 3.51 from sixth to ninth. The light scale is certainly not the same for both bright and faint stars, and an abrupt division at the limit of naked eye visibility is perhaps justified. Such a division into two parts would be represented by unit light ratios of 2.0 and 2.3, respectively, one below that of the lower test scale, and one equal to that of the historic scale.

Numerical values of the factor a in the summation expression have been calculated below, from the count of stars in the northern Durchmusterung, with the mean value $b = 3.24$ from Table VI. Separate values of a evidently would be represented by a curve, indicated by the size and the algebraic signs of the residuals. From the mean value, $a = 2.16$, the summations have been computed, and the column O–C indicates the deviation of the counts from the computed numbers. This deviation is expressed in percentages, in the last column.

TABLE VII

Mag.	Count	bm	a	Computed number	O–C	Percentage
2.0	28	10	2.80	22	+ 6	+0.21
2.5	45	19	2.37	41	+ 4	+0.09
3.0	80	34	2.34	73	+ 7	+0.09
3.5	144	61	2.36	132	+ 12	+0.08
4.0	230	110	2.09	238	— 8	—0.03
4.5	365	198	1.84	428	— 63	—0.17
5.0	680	357	1.91	771	— 91	—0.13
5.5	1120	643	1.74	1389	— 269	—0.24
6.0	2108	1157	1.82	2499	— 391	—0.19
6.5	4122	2082	1.98	4497	— 377	—0.09
7.0	8017	3748	2.14	8096	— 79	—0.01
7.5	14078	6747	2.09	14573	— 495	—0.04
8.0	25321	12145	2.09	26233	— 912	—0.04
8.5	48247	21861	2.21	47220	+ 1027	+0.02
9.0	101071	39351	2.57	84998	+16073	+0.16
Means			2.16			±0.10
Average residual			±0.23			

The expression computed for the summation of the stars to any grade of magnitude, in the northern sky, would be $2.16\,(3.24)^m$, and the separate grades agree within an average of ten per cent of the count. The value of a is larger than hitherto found usually in computations of this character. This is due to the relatively smaller value of b derived above. The larger values of b, which have generally been computed, were perhaps due in part to the inclusion of the stars brighter than second magnitude, too few in number to give reliable results. Also excessive weight has probably been given to the count of the fainter classes. The numerical deviations of the faintest classes appear large, but, expressed as percentages of the total summations, they are not far from the mean percentage of deviation.

The ratios of the summations make an effective test of the variations in screen values, and in the reductions of magnitude by diaphragms, which have recently been under discussion here.[2] The five sets of results for the hand screen in use here give the following tabulated average variation in screen values. Two sets are included in the tabulation of the cap screen. The San Luis screen and diaphragm and the thin screens have one set each. The B.D. counts are tabulated for the magnitude of the bright stars, and for the magnitude that corresponds to the screened image. The ratio of these two counts follows, with the corresponding values of the light ratio of the screen.

Hand Screen, 1896 to 1914

Mag.	B.D. Count	Screen Value	Reduced	B.D. Count	N	L
2.5	45	4.0 Mag.	6.5 Mag.	4122	92	20
3.5	144	3.8	7.3	10702	74	18
4.5	365	3.8	8.3	35358	97	21
5.5	1120	3.4	8.9	77794	70	17
6.3	2782	3.2	(9.5)	....		
Means		3.8			83	19
			Cap Screen, 1911 to 1917			
2.5	45	4.9	7.4	11218	249	40
3.5	144	4.7	8.2	30749	214	36
4.5	365	4.4	8.9	77794	213	36
5.2	702	4.4	(9.6)	....		
Means		4.6			225	37
			San Luis Screen, 1910 to 1911			
2.5	45	4.2	6.7	4738	105	22
3.5	144	4.0	7.5	14078	98	21
4.0	230	3.8	7.8	18708	81	19
Means		4.0			95	21
			San Luis Diaphragm			
6.7	4738	1.8	8.5	48247	10.2	4.7
7.2	9346	1.6	8.8	70198	7.5	3.8
7.6	14687	1.4	9.0	101071	6.9	3.6
Means		1.6			8.2	4.0
			Thin Screens, 1918			
6.4	2883	1.8	8.2	30749	10.6	4.8
7.4	11218	1.6	9.0	101071	9.0	4.3
Means		1.7			9.8	4.55

The average residual of L in the five combinations preceding is seven per cent of the mean values. With a constant screen value for each combination, the average residual would be twelve per cent. The hand screen gives a mean unit light ratio of 2.17 at magnitude 5.9. The cap screen gives 2.19 at magnitude 6.2. The San Luis screen gives 2.14 at 5.3. The diaphragm gives 2.38 at 8.0. The thin screens give $L_1 = 2.44$ at magnitude 8.75. The two dense screens transmit $\frac{1}{19}$ and $\frac{1}{37}$ of the light respectively, and the thin screens transmit twenty-two per cent of the light. The San Luis screen transmits $\frac{1}{21}$, and the diaphragm reduces the light to $\frac{1}{4}$.

The diaphragm observations, made here in 1917, give the following results, from the similar comparison of the summations at the bright and faint grades of magnitude, corresponding to the bright stars and the reduced images.

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[2] *Lick Obs. Bull.*, 307.

Diaphragms, 1917

Aperture	Mag.	Reduced	L	L_1		
½ inch	3.5	8.7 Mag.	55	2.16	at	6.1 Mag.
1	4.0	8.6	37	2.20		6.3
2	6.0	8.0	5.2	2.29		7.0
3	6.2	7.6	3.2	2.32		6.9
3	8.0	9.0	2.5	2.52		8.5

There is a regular progression in the values of the unit light ratio for the various grades of stars, thus tested by counts, from that of the lower test scale to that of the normal scale.

The relation between two scales of magnitude can be established by the direct comparison of a sufficient number of stars, included in catalogues representing both scales. In volume 72, *Annals of the Harvard College Observatory,* very extensive comparisons have been tabulated, for the various volumes of the Durchmusterung and the Harvard Photometry. A condensed summary is made here of some of the results, representing very inadequately the great amount of work in the original comparisons. The photometric magnitudes are given below, for three volumes of the northern Durchmusterung, from south to north; and for three volumes of the Cordoba Duchmusterung, from north to south. The fourth Cordoba volume had not been published at that time.

TABLE VIII

Comparisons with the Harvard Photometry

Argelander	Photometer			Mean
7.0	6.9	7.1	7.1	7.0 Mag.
8.0	7.9	8.1	8.1	8.0
9.0	9.2	9.4	9.3	9.3
9.1	9.4	9.5	9.5	9.5
9.2	9.6	9.8	9.6	9.7
9.3	9.9	10.0	9.8	9.9
9.4	10.2	10.0	10.0	10.1
(9.5)	10.6	10.7	10.3	10.5

Schönfeld	Photometer	Cordoba	Photometer				
7.0	7.0	7.0	7.0	7.0	6.9	Mean of three	7.0
8.0	8.1	8.0	8.1	8.1	7.9		8.0
9.0	9.3	9.0	9.7	9.5	9.1	First two	9.6
9.1	9.6	9.1	9.9	9.7	9.2		9.8
9.2	9.8	9.2	10.2	9.9	9.3		10.0
9.3	10.0	9.3	10.4	10.1	9.5		10.2
9.4	10.2	9.4	10.6	10.3	9.7		10.4
9.5	10.5	9.5	10.9	10.5	9.9		10.7
9.6	10.6	9.6	11.1	10.8	10.1		10.9
9.7	10.7	9.7	11.4	11.0	10.3		11.2
9.8	10.9	9.8	11.6	11.2	10.6		11.4
9.9	11.0	9.9	11.8	11.5	11.0		11.6
10.0	11.1	(10.0)	12.0	11.9	11.6		12.0

The original comparison gave the results for each hour of right ascension for all these catalogues. But the mean results tabulated here are sufficient to show the relation of the critical part of the visual scale, from ninth to tenth magnitude, to the photometric scale. The difference between the visual scales of the separate catalogues can also be as well defined, probably, by means of the intermediate photometric scale, as if the same stars could have been directly compared. The photometric scale is in agreement with the northern Durchmusterung at seventh and eighth magnitudes. At ninth, the photometric estimates are 0.3 fainter; and at 9.4, the lowest definite grade of Argelander, the photometric is 0.7 fainter. A similar description applies to the difference between Schönfeld and the photometric scale, the estimates of the latter being a full unit fainter than Schönfeld from 9½ to 10. The photometric estimates are 0.6 fainter than the first two volumes of Cordoba, at ninth magnitude, and but 0.1 fainter than the third volume. This illustrates the change that had been made in the Cordoba scale of magnitudes, evident for all grades from nine to ten. At 9.9 of the first two Cordoba volumes, the lowest definite grade, since 10 included the fainter stars, the difference is 1¾. There is a striking increase in the differences, in progressing downwards through the faint telescopic grades.

One and a half units of Argelander's scale, from 8 to 9.4, are represented by 2.1 units of the photometric scale. If the Harvard photometric magnitudes have retained the light ratio of the normal scale, the unit light ratio of Argelander's scale would be 3.7, between these grades; and it would be 6.4 for stars fainter than ninth magnitude.

Two units of Schönfeld, from 8 to 10, are represented by three full units of the photometric scale. The mean unit light ratio of Schönfeld, between these limits, would be 4.0; or 5.3 from 9 to 10.

Two units of Cordoba, in the first two volumes, from 8 to 9.9, are represented by 3.5 units photometric, and the unit light ratio of Cordoba would be 4.3 between these limits; or 7.7 from 9 to 9.9. In the third volume the range from 8 to 9.9 would be represented by unit light ratio of 4.5; and from 9 to 9.9 by 7.0. The three Cordoba volumes do not vary greatly differentially in unit light ratios, for these grades of magnitude, though the estimates of ninth magnitude are for stars fully half a magnitude fainter in the third volume, as compared with the estimates of the first two. If, on the other hand, the visual unit light ratio be assumed at 3.0 for the ninth magnitude, the unit light ratio of the photometric scale at this point would be 2.0, from the direct comparison of scales, between eighth and tenth magnitudes.

Another medium of testing the visual scale for the southern sky is furnished by the magnitudes of the Cape Photographic Durchmusterung. These were based upon comparison with Gould's Cordoba Zones. An extensive comparison of the visual and photographic Durchmusterung magnitudes was made by

Kapteyn. An independent comparison for the first ten degrees of Cordoba made by the writer, was published in the *Astrophysical Journal* for May, 1908. A similar comparison for the fourth ten degrees has also been printed in *Popular Astronomy* for July, 1917. Reference to these publications will show that the differences from the photometric scale are confirmed, for the standard type of ninth magnitude. The indirect testimony of the two comparisons shows that Gould's magnitudes were closely those of Argelander, down to the magnitude 9½ of the former. The estimates of 10, however, appear to have belonged to stars quite as bright as the 9½ of the Zones. A systematic difference between the visual Durchmusterung estimates for areas within the Milky Way, and outside, noted also by Kapteyn, is probably to be accounted for by the bright background, which might influence the estimates of stars in a dark field, while it should have no perceptible effect upon the estimates in the illuminated field of a meridian circle.

The average distribution of the stars offers another test of standard grades. Taking the ninth magnitude as the critical grade, the complete count of Argelander gives 4.90 as the average per square degree for the northern sky. Schönfeld has 4.65 in his twenty degrees of the southern sky, and his standard ninth magnitude should be slightly brighter than that of Argelander. The Photographic Durchmusterung has an average of 5.85 for the area from —19° to the south pole, and should accordingly include stars 0.1 fainter than Argelander's B.D. at ninth magnitude. The respective figures for the four volumes of the Cordoba Durchmusterung are tabulated here, with the corresponding figures for the photographic scheme. The resulting values of N, the ratio of the photographic to the visual, show the relation of the scales at ninth magnitude, and the change introduced between the observations of the second and third Cordoba volumes.

Vol. I and II	Cordoba = 6.7	Cape = 5.2	$N = 0.78$
Vol. III	3.9	6.4	1.64
Vol. IV	5.2	7.7	1.48

Comparing these ratios with Table I, the visual estimates of ninth magnitude in the first two volumes include stars 0.2 fainter than the photographic standard. The photographic ninth includes stars 0.4 fainter than the visual estimates of the third volume and stars 0.3 fainter than the visual estimates of the fourth volume. The ninth of the first two volumes should include stars 0.2 fainter than B.D. The B.D. ninth should include stars 0.2 fainter than the third volume, and the fourth volume should be nearly in accord with B.D.

The northern A.G. Zones have an average of 6.6 stars to a square degree. All the B.D. 9.0 stars have been included. Compared with the average of B.D. ninth, these zones must include an equivalent of the stars to the grade B.D. 9.2. Not all stars of 9.1 and 9.2 have been observed, but enough stars fainter than 9.2 have been included to bring the total up to the equivalent of the summation at that grade. A recent count of B.D. within the limits of the Harvard A.G. Zone gives the ratio of B.D. 9.0 to the latter catalogue; 5.63 : 7.39 = 1.31, which by Table I is equivalent to the summation at 9.2, normal scale.

Gould's Zones have quite exactly six stars to the square degree, and while his catalogue is not absolutely complete to any definite grade of magnitude, this average is equivalent to a summation fainter than B.D. 9.1. Enough stars fainter than ninth magnitude were observed in the zones to more than make up for the omission of stars brighter than ninth magnitude.

A zone of stars from —22° to —27° has been observed at Cordoba, upon lines similar to those of the A.G. system. While the details of observation and of reduction do not conform in all respects to those of the larger scheme, the published catalogue gives a list of stars which is complete to a lower grade of magnitude. It has an average of 9.7 stars to the square degree, which is equivalent to the summation of the stars of the Cordoba Durchmusterung, from which the observing list was selected, to the magnitude 9.2. This grade is close to the 9.5 of the visual scale of the northern Durchmusterung as can be seen by referring to Table I, where the ratio 2.0 for the normal scale corresponds to a difference of 0.5 magnitude. Such an extension of our catalogues of faint stars certainly bears testimony to the persistence and aptitude of the observers, who completed the work under the active direction of the late Dr. J. M. Thome. The excellent conditions of the Cordoba sky are also confirmed, since the telescope, of five inches aperture, is smaller than those of most modern meridian instruments. Other southern zones have been under observation at the La Plata Observatory, in the Argentine Republic, similar in design and character.

Such tests of the visual scale as have been made in this discussion will be necessary for fixing the limit of stars of the visual grade 9.5. Thus Argelander's 9.4 can be quite safely taken as establishing this visual limit for the northern sky. The 9.3 of Schönfeld is about the same grade of brightness. For the first two volumes of Cordoba the limit should be 9.2. For the third and fourth volumes the grade 9.5 could be adopted. The 9.3 of the Photographic Durchmusterung would be a similar limit. These grades are all estimated at tenth magnitude on the photometric scale.

R. H. Tucker.

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NOVA MONOCEROTIS

Nova Monocerotis, found by Wolf[1] at Heidelberg on February 4, 1918, to have a spectrum of nova type and a magnitude of eight and a half, seems to have passed its maximum before that date. The Harvard photographs[1] show its magnitude to have been as follows, including meridian circle estimates by Mr. Tucker, at Mount Hamilton:

1917 Dec. 22, fainter than 9.8 and not seen.
1918 Jan. 1, 5.4 photographic.
Feb. 4, 8.9 photographic.
Feb. 17, 9.0 photographic.
1918 March 1, 15, 20, 9½ to 9¾, visual, meridian circle.

Four determinations of position have been made and published:

R.A. (1918)	Dec. (1918)	Observer	Instrument	Observatory	Published
7h 22m 46s.93	—6° 30′ 34″.7	Barnard		Yerkes	*H. C. O. Bull.,* 654, 1918.
7 22 47.0	—6 30 34	Van Biesbroeck		Yerkes	*H. C. O. Bull.,* 654, 1918.
7 22 47.0	—6 30 33	Thiele	Crossley Reflector	Lick	*H. C. O. Bull.,* 654, 1918.
7 22 46.989	—6 30 35.86	Tucker	Meridian Circle	Lick	*L. O. Bull.,* 9, 144, 1918.

Observations of spectrum have been made at three observatories on dates as follows:

Date	Observer	Observatory	Instrument	Reference
1918 Feb. 21	....	Harvard	Objective prism	*H. C. O. Bull.,* 654.
25	Moore and Paddock	Lick	One-prism, 16-inch camera	
Feb. 28	Adams and Joy	Mt. Wilson	One-prism, 7-inch camera	*Publ. A. S. P.,* 30, 162, April, 1918.
Mar. 1	Adams and Joy	Mt. Wilson	One-prism, 18-inch camera	*Publ. A. S. P.,* 30, 162, April, 1918.
1	Paddock	Lick	One-prism, 16-inch camera	
15	Paddock	Lick	One-prism, 16-inch camera	
1918 Mar. 27	Paddock	Lick	One-prism, 16-inch camera	

The various spectrum plates have shown the following bands of hydrogen, H, nebulium, N_1 and N_2, and others unknown: Hζ (2), Hϵ (2)*, Hδ (5), Hγ and 4363 (together 10)*, 4520 (1), 4640–50 (4)*, 4686*, Hβ (1)*, N_1 4959*, N_2 5007 (3)*, 5440 (1), 5526, 5630 (3), 5677, 5755(1), Hα (15).

Those lines which Miss Cannon found on the objective prism plates are indicated by numbers in parentheses which are the estimates of relative intensities. Those followed by a star (*) were seen on the Mount Wilson plates. Those observed at Mount Hamilton are listed and described below. It is probable that Hδ was unintentionally omitted from the Mount Wilson list and also that intensity four in the Harvard list pertains to line 4640–50 rather than 4686 which is faint.

The bands in the yellow region between N_2 and Hα are faint and diffuse, so that the above mean wave-length positions and their identifications are very uncertain. The nearest correspondences with the bands observed by Campbell and Wright in *Nova Persei* 1901 are:

Place observed	Approximate mean wave-length	Campbell and Wright's bands of *Nova Persei*	
Harvard	5440 (1)	5393–5438	"Diffuse bright band."
Lick	5526 very faint	(5603)	"Diffuse bright band" maximum.
Harvard	5630 (3)	5513–5562	"Diffuse bright band."
Lick	5677 faint	5651–5716	"Diffuse bright band."
Harvard	5750 (1)	5735–5776	Broad bright band.
Lick	5755 faint		

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[1] *H. C. O. Bull.,* 653, Feb. 21, 1918.

On looking into the camera of the spectrograph on the night of the first observation, February 25, we saw four bright bands, which we found to be Hγ and 4363 overlapping, 4650, Hβ, N_1 and N_2 overlapping. The camera was set for Hγ central and the exposure was 5¼ hours on a Seed 30 plate.[2] For the second observation a camera set for the yellow D lines central was used. An almost continuous, but extremely faint spectrum, between N_1 and D_3 could be seen. The exposure this night was 5½ hours on a Seed orthochromatic plate.

The third and fourth observations, March 15 and 27, were made with the D lines central and on Seed 27 plates stained with Wallace's pinacyanol–pinaverdol-homocol formula.[3] The exposure of March 15 was 4 hours and 40 minutes with some interference by thin clouds, and that of March 27 was 4 hours and 20 minutes with a suspicion of somewhat diminished brightness of the nova. The orthochromatic plate alone shows the faint yellow bands but no trace of the red Hα, while the stained plates show the Hα fairly strongly, but not the yellow bands. The first plate was given the iron spark comparison and the other three hydrogen and helium tube spectra. The first, second, and third plates were measured and the wave-lengths computed by Hartmann formulas. For bands between Hδ and N_2 the data are derived from the first plate, for the yellow bands from the orthchromatic plate, and for Hα from the stained third plate. Corrections for the Earth's motion with respect to the Sun amount to only about three-tenths of an angstrom and do not affect the results here presented. The collected datad, which must be taken as only approximate, are given in the two tables below and consist of the derived wave-lengths of emission and absorption bands, their widths, their apparent displacements, and descriptive notes.

WAVE-LENGTHS AND WIDTHS OF BANDS

	Approximate wave-length	Width of Bright band	Width of Absorption band	
Hδ edge	4080 A	40 A		
Middle absorption band	4101		7 A	
Hδ edge	4120			
Hγ edge	4322	66		
Middle absorption band	4341		9	
Middle overlapping bright parts	4350			Width 10 A, very strong.
Middle absorption band	4361		10	
4363 edge	4388			
464 edge	4613	56		
Middle absorption band	4638		15	
464 edge	4669			
4686 edge	4680	26		Faint.
4686 edge	4706			
Hβ edge	4840	43		
Middle absorption band	4859		15	
Hβ edge	4883			
N_2 edge	4940	50(+)		N_2 is faint.
Middle absorption band	4966		14	
Edges overlapping		40		
N_1 violet maximum	4995			Width 5 A, strong.
Middle absorption band	5008		21	
N_1 edge	5032			
Unknown band	5526±			Very faint.
Unknown band	5677±			Faint.
Unknown band	5756±			Faint.
Hα edge	6545	43		Absorbing band faint.
Hα edge	6588			

DISPLACEMENTS OF BANDS

Band	Natural wave-lengths	Bright bands Middle	Bright bands Displ.	Absorption bands Middle	Absorption bands Displ.
Hδ	4101.9 A	4100 A	—2 A	4101 A	—1
Hγ	4340.6			4341	+0
	4363.4			4361	—2
	4686.0	4693	+7		
Hβ	4861.5	4862	+0	4859	—2
N_2	4959.1			4966	+7
N_1	5007.0			5008	+1
Hα	6563.0	6566	+3		
Mean displacements			+2		+0.5

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[2] See *Publ. Astron. Soc. Pac.*, 30, 189, 1918, for a reproduction of this plate.

[3] *Ap. Jour.*, 26, 317, 1907, and *Lick Obs. Bull.*, 5, 151, 1909.

A slit width of .06 mm. was used for the first plate, but it was made a quarter millimeter for the other three. The measures of all bands except the faint yellow bands and Hα depend on the narrower slit.

Particular points noted with respect to the bands are the following: Some of them give indication of being complex. In general, a faint broad absorption band lies over the middle of every emission band. It may not be a uniform band, but may be a group of close, narrow absorption bands or lines. The full widths of the absorption bands range from 6 to 21 angstroms and give indication of increase in width with increasing wave-length. A range of 40 to 60 angstroms is found for the widths of the bright bands, but with no appearance of regularity. The bands of hydrogen and 4640–50 are very nearly symmetrical, the intensity curves consisting of a gradual rise from the violet side, a slight depression due to the absorption, and a perhaps more gradual decline to the red side. But the two nebulium bands are distinctly unsymmetrical, the bright parts being stronger on the violet side of the absorption band than on the red side. Adams and Joy mention this feature, noting that it is the reverse of that seen in the bands of *Nova Geminorum* No. 2, 1912.

The individual displacements of the bands are erratic and must not be given any quantitative weights. Those of the bright bands are obtained by averaging the measures of the edges of the bands, and are too few to be significant, although the average of +2 angstroms is similar to other cases of displaced bright bands. The average of only a half angstrom for the superposed absorption bands probably indicates that they are in general normally placed. There are no narrow lines available on these plates for determining radial velocity.

It is obvious from the spectra here described that this nova at the times of the observations had reached the nebular stage of the usual cycle of spectra presented by novae.

G. F. Paddock.

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A SPECTROGRAPHIC STUDY OF σ *SCORPII*

The star σ *Scorpii* ($\alpha = 16^h\ 15^m.1$, $\delta = -25° \ 21'$, vis. mag. = 3.1, spectral class B1) was shown to have a variable radial velocity by Dr. V. M. Slipher at the Lowell Observatory in 1904.[1] He suspected first that its period was about 30 days and reported the fact that the H and K lines of calcium yield a constant velocity. A study of this star was made by M. Selga, S.J., at the Lowell and the Lick Observatories. σ *Scorpii* was found by him to be a very short period binary[2] ($P = 0^d.246829$). Father Selga also suspected a variation of the velocity of the center of mass of the system.

σ *Scorpii* thus belongs to a class of short-period binary stars, of the helium type, which seem to present a great interest in astrophysics. To this class belong β *Canis Majoris*, β *Cephei* and 12 *Locertae*. It may be questioned whether these stars are really spectroscopic binaries in the strict sense of the word.

The very short period of these stars, only a few hours, is amazing. They are by no means simple systems. Possibly a third body, or something which transforms the motion considerably, is present. β *Canis Majoris* was found by me to have a velocity curve of variable amplitude;[3] Dr. R. K. Young suspected the existence of such a variation of amplitude for 12 *Lacertae*;[4] Dr. C. C. Crump has shown that β *Cephei*[5] has a variable velocity of the center of mass, and a series of three-prism plates taken by me also show this variation very well.

A great number of single-prism spectrograms of σ *Scorpii* were obtained by Father Selga at the Lick Observatory in 1914 and 1915 and were all measured by him for radial velocities. As I shall make use of his measures in order to improve the value found for the short period variation, and shall also use them in a determination of the period of variation of the velocity of the center of mass of the binary system, most of his data are reproduced in the two following tables. The first table gives the velocities obtained by Selga from Slipher's spectrograms at the Lowell Observatory and the second table gives Selga's velocities from spectrograms which he secured at the Lick Observatory. All these data, together with the available measures made at other observatories, have already been given in Selga's paper.[2] The dates of observations are given in Greenwich mean time.

TABLE I

SLIPHER'S SPECTROGRAMS OBTAINED IN 1909

Date	J.D. 2410000+	Observed velocity, km.	Computed velocity, km.	Residual O—C, km.
1909 June 22	8480.748	—31	—43	+12
23	8481.776	+ 5	— 5	+10
25	8483.785	+60	+41	+19
28	8486.778	+84	+76	+ 8
July 8	8496.748	—34	—25	— 9
10	8498.748	—51	—41	—10
12	8500.748	—43	—36	— 7
14	8502.763	—12	—14	+ 2
15	8503.738	—17	—26	+ 9
22	8510.697	—10	— 6	— 4
27	8515.678	+67	+30	+36
28	8516.659	+ 3	+46	—33
29	8517.702	+13	+ 4	+ 9
30	8518.683	+20	+16	+ 4

TABLE II

SELGA'S RADIAL VELOCITIES IN 1915

Date	J.D.	Observed velocity, km.	Computed velocity, km.	Residual O—C, km.
1915 Jan. 23	0521.083	—49.3	—39	—10
March 2	0559.047	—46.5	—13	—34
	0559.089	—28.0	+ 3	—31
3	0560.039	—36.9	—10	—27
	0560.066	—31.4	+ 1	—30
7	0564.079	+45.1	+45	0
8	0565.067	+24.0	+44	—20
	0565.084	+53.1	+57	— 4
11	0568.028	+31.7	+33	— 1

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[1] *Lowell Obs. Bull.*, 1, 57, 1904, and 2, 1, 1909.

[2] *Revista de la Soc. Astr. de España y América*, 6, 41, 1916.

[3] *Lick Obs. Bull.*, 9, 155, 1918.

[4] *Publ. Dominion Obs.*, 3, 65, 1915.

[5] *Detroit Obs. Publ.*, 2, 144, 1916.

Date	J.D. 2420000+	Observed velocity, km.	Computed velocity, km.	Residual O—C, km.
1915 March 11	0568.047	+35.7	+50	—14
16	0573.066	— 3.6	—11	+ 7
18	0575.969	+26.5	+33	— 6
19	0576.070	—24.3	—43	+19
21	0578.055	—39.2	—51	+11
22	0579.073	—27.9	—42	+14
24	0581.058	—40.8	—42	+ 1
April 10	0598.867	+32.6	+29	+ 4
11	0599.042	—22.0	—14	+ 8
	0599.904	+31.7	+63	—31
13	0601.874	+30.9	+57	—26
	0601.914	— 2.9	+29	—26
14	0602.003	—25.0	—23	— 2
	0602.869	+39.2	+52	—13
	0602.928	— 5.0	+ 8	—13
15	0603.014	—15.0	—15	0
	0603.882	+52.1	+33	+19
	0603.927	—14.6	— 5	—10
	0603.963	—30.9	—26	— 5
16	0604.002	—18.1	—16	— 2
	0604.038	— 3.6	+ 7	—11
	0604.981	—23.0	—25	+ 2
17	0605.012	— 7.7	— 6	— 2
	0605.037	+ 1.3	+11	—10
18	0606.941	—27.7	—34	+ 6
	0606.986	+ 4.0	— 9	+13
19	0607.024	+21.1	+19	+ 2
	0607.945	—20.5	—31	+10
	0607.993	— 0.8	+ 1	— 2
May 25	0643.763	—21.5	—20	— 2
	0643.784	—11.6	— 7	— 5
	0643.802	+ 9.8	+ 7	+ 3
	0643.823	+25.2	+27	— 2
26	0644.707	—42.8	—48	+ 5
	0644.730	—34.0	—36	+ 2
	0644.749	—16.4	—26	+10
	0644.764	—22.6	—15	— 8
	0644.782	+ 5.2	— 1	+ 6
	0644.806	+22.9	+21	+ 2
	0644.836	+35.3	+32	+ 4
	0644.856	+ 8.1	+12	— 4
	0644.893	—11.2	—20	+ 9
	0644.930	—32.1	—42	+10
	0644.963	—41.7	—45	+ 3
June 7	0656.867	—14.3	—21	+ 7
	0656.881	— 0.7	— 9	+ 8
	0656.893	+ 4.3	0	+ 4
	0656.907	+ 9.1	+13	— 4
	0656.923	+12.3	+21	— 9
	0656.940	+18.2	+12	+ 6
22	0671.684	+20.1	+18	+ 2
	0671.699	+30.3	+31	— 1
	0671.713	+34.1	+42	— 8
	0671.727	+41.6	+53	—11
	0671.742	+41.5	+52	—11
	0671.756	+27.4	+40	—12
	0671.768	+11.9	+28	—16
	0671.780	+ 6.1	+16	—10
	0671.794	+ 4.5	+ 5	0
	0671.807	— 7.8	— 4	— 4
	0671.823	—19.2	—14	— 5
	0671.836	—26.3	—21	— 5
	0671.851	—39.1	—26	—13
1915 June 22	0671.866	—24.3	—23	— 1
	0671.880	—24.9	—17	— 8
	0671.895	— 7.7	— 7	— 1
	0671.911	— 0.1	+ 4	— 4
25	0674.664	+30.0	+23	+ 7
	0674.677	+38.2	+36	+ 2
	0674.691	+41.3	+44	— 3
	0674.703	+36.8	+43	— 6
	0674.718	+24.5	+30	— 5
	0674.754	— 2.4	— 4	+ 2
	0674.766	—14.8	—12	— 3
	0674.785	—32.4	—24	— 8
	0674.797	—45.3	—30	—15
	0674.810	—41.8	—36	— 5
	0674.824	—32.4	—35	+ 3
	0674.837	—28.9	—29	0
July 7	0686.783	+ 6.9	+ 9	— 2
	0686.792	+10.9	+11	0
	0686.801	+13.9	+ 6	+ 8
	0686.810	+ 3.0	+ 1	+ 2
22	0701.668	+ 6.6	+12	— 5
	0701.678	+ 5.8	+ 5	+ 1
	0701.688	+ 0.4	— 1	+ 1
	0701.699	— 6.2	— 6	0
	0701.711	—21.6	—11	—11
	0701.722	—31.3	—14	—17
	0701.730	—19.3	—12	— 7
	0701.740	— 9.2	— 7	— 2
	0701.750	—10.0	— 3	— 7
	0701.764	— 3.7	+ 7	—11
	0701.774	+ 2.4	+14	—12
	0701.784	+10.9	+21	—10
	0701.795	+ 9.5	+28	—18
	0701.805	+16.2	+36	—20
	0701.814	+27.4	+44	—17
	0701.825	+27.3	+54	—27

Several series of three-prism spectrograms were secured by me in 1918, each series covering nearly a complete revolution of the star around its center of mass. These spectrograms, taken on Seed 27 plates (average exposure 45 minutes), are good, but not nearly so good as those obtained for *β Canis Majoris*. The average probable error for each plate was computed to be ± 1 km., although certain plates, on which the lines are not so well defined, give probable errors as high as ± 1.5 km. My measures have been made with a Hartmann spectrocomparator, utilizing fourteen sections on each plate, in the direct and reversed positions. As the standard reference plate, a good spectrogram of *β Canis Majoris* (1909, Feb. 22, 2418360.760 J.D.) was used. This plate had already been used in my measures of radial velocities of *β Canis Majoris*. The following table gives the Greenwich mean time of each exposure, the corresponding Julian day, the observed radial velocity, the radial velocity derived from the computed orbits and the residual difference of the observed and computed velocities.

TABLE III

Radial Velocities in 1918 (Three-Prism)

Date 1918	J.D. 2420000+	Observed velocity, km.	Computed velocity, km.	Residual O—C, km.	Sum of the widths of 5 spectral lines[6]
April 3	1687.869	+20.5	+14	+ 6	1127
	1687.909	+47.4	+43	+ 4	1056
	1687.943	+76.3	+71	+ 5	1319
	1687.977	+57.7	+52	+ 6	1877
4	1688.010	+22.4	+23	— 1	1033
	1688.041	+ 3.6	+ 3	+ 1	1417
	1688.868	+31.0	+21	+10	1142
	1688.902	+46.7	+46	+ 1	1139
	1688.935	+69.7	+70	0	1352
	1688.970	+44.6	+44	+ 1	1755
5	1689.004	+16.7	+16	+ 1	1310
	1689.043	— 8.3	— 7	— 1	1218
11	1695.854	+44.1	+48	— 4	——
	1695.892	+ 6.9	+14	— 7	1499
	1695.934	—12.7	—18	+ 5	1352
	1695.970	—31.5	—33	+ 1	1186
12	1696.002	—14.1	—18	+ 4	1252
	1696.032	+ 1.9	+ 2	0	1153
15	1699.836	+ 7.3	+ 6	+ 1	1811
	1699.871	—21.0	—22	+ 1	1365
	1699.903	—33.0	—40	+ 7	1093
	1699.935	—39.3	—38	— 1	1206
	1699.969	—20.8	—19	— 2	1290
16	1700.003	+ 0.6	+ 6	— 5	1170
17	1701.844	—24.6	—25	0	1482
	1701.876	—38.0	—46	+ 8	1329
	1701.907	—48.2	—47	— 1	——
	1701.939	—27.8	—28	0	1147
	1701.969	— 8.4	— 7	— 1	1249
18	1702.004	+25.1	+23	+ 2	——
21	1705.810	—51.7	—49	+ 3	1212
	1705.866	—50.3	—52	+ 2	929
	1705.897	—32.5	—33	+ 1	1024
	1705.929	—10.3	—10	0	939
23	1707.840	—57.5	—59	+ 1	1016
	1707.876	—38.1	—35	— 3	933
	1707.906	—16.0	—13	— 3	985
	1707.935	+ 4.0	+12	— 8	1133
	1707.967	+ 4.9	— 1	+ 6	1678
	1707.998	—29.1	—31	+ 2	1523
26	1710.845	—32.5	—35	+ 3	——
29	1713.823	— 0.2	— 8	+ 8	1066
	1713.856	+19.3	+21	— 2	1280
	1713.886	+ 4.1	+15	—11	1539
	1713.917	—17.4	—15	— 2	1463
	1713.951	—36.1	—39	+ 3	1234
	1713.987	—51.0	—55	+ 4	1342
May 2	1716.826	+59.1	+56	+ 3	1077
	1716.857	+55.8	+37	+19	1983
	1716.895	+ 8.3	+ 3	+ 5	1201
	1716.925	— 8.3	—15	+ 7	1158
	1716.957	—16.6	—21	+ 4	1301
	1716.990	— 4.8	— 3	— 2	1009
5	1719.850	+22.5	+24	— 2	1129
	1719.875	+11.9	+ 8	+ 4	1287
	1719.904	— 7.2	— 6	— 1	1224
	1719.934	— 1.8	+ 2	— 4	1239
	1719.970	+23.8	+25	— 1	1006
10	1724.816	— 6.0	— 4	— 2	961
May 10	1724.845	—17.5	—16	— 2	1092
	1724.875	— 9.1	— 5	— 4	999
	1724.906	+ 8.5	+15	— 7	834
	1724.938	+37.0	+39	— 2	996
	1724.970	+60.6	+64	— 3	1033
12	1726.790	— 9.3	—10	+ 1	1286
	1726.820	—23.4	—23	0	1284
	1726.851	—11.6	—11	— 1	1009
	1726.881	+ 9.0	+ 9	0	1101
	1726.922	+39.2	+41	— 2	1152
	1726.953	+58.6	+57	+ 2	1345
29	1743.765	—19.9	—17	— 3	——
	1743.797	—46.7	—44	— 3	——
	1743.834	—67.8	—66	— 2	——
	1743.880	—57.9	—63	+ 5	——
	1743.916	—32.7	—38	+ 5	——

Period of velocity variation. From the velocities observed in 1918, thirteen nearly complete velocity curves were obtained, each giving a different center-of-mass velocity. All the epochs of maxima or of minima of these curves as well as of four nearly complete velocity curves obtained from Selga's 1915 observations are found to be well connected by the period $0^d246834$. Using this period we have the following table.

TABLE IV

Epochs of Maxima and Minima of Velocity Variation

	Observed		Computed	
Date	Maximum J.D. 2420000+	Minimum J.D. 2420000+	Maximum J.D. 2420000+	Minimum J.D. 2420000+
1915				
May 26	0644.829		0644.830	
June 22	0671.728	0671.853	0671.734	0671.854
25	0675.688	0675.810	0675.696	0675.816
July 22		0701.720		0701.721
1918				
April 3	1687.950		1687.950	
4	1688.933		1688.937	
11		1695.968		1695.968
15		1699.918		1699.917
17		1701.900		1701.892
21		1705.840		1705.841
23	1707.950		1707.944	
29	1713.860		1713.867	
May 2		1716.955		1716.948
5		1719.916		1719.911
10		1724.850		1724.848
12		1726.821		1726.822

The period $0^d246834$ is in very close agreement with that obtained by Selga, $0^d246829$.

The short period orbit. From all the velocity curves obtained we find that although the velocity of the center of mass is greatly variable, the amplitude of each velocity curve may be considered as constant. We do not find here, as in the case of β *Canis Majoris,* variations of amplitude. Besides the constancy of amplitude, we also find that the different velocity curves are connected by a well determined period. Taking account of the mean probable errors of observation, we may superpose the different velocity curves, so that all of them coincide exactly and have the same, but undetermined mean velocity. We then obtain the curve represented in Fig. 1, in which having taken care that all the computed maxima (or minima) fall at the same point, we may consider the superposition as very accurate. From this velocity curve and assuming the orbit as elliptic, we obtain the following approximate elements of the orbit.

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[6] The unit of width, in the last column, is 1/2000 of a millimeter.

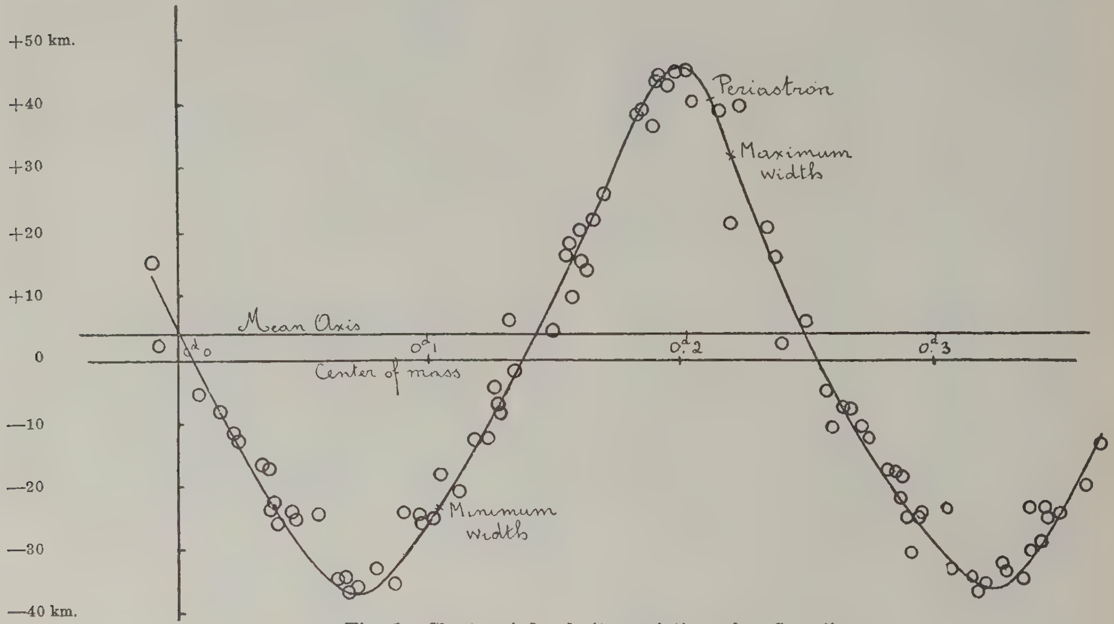

Fig. 1. Short-period velocity variation of σ *Scorpii.*

Approximate Elements for the Short–Period Orbit

$k = 41.2$ km.
$e = 0.11$
$P = 0\overset{d}{.}246834$
$\omega = 15°$
$T = 2421687.972$ J.D.
γ variable
$a \sin i = 138{,}500$ km.
$$\frac{m_1^3 \sin^3 i}{(m + m_1)^2} = 0.00176$$

It is interesting to compare these elements with those given in Selga's paper, for the determination of which he utilized the combined observations of the two dates 1915 June 22 and 1915 June 25. In doing so, however, he did not take account of the difference of about 10 km. which exists between the center of mass velocities of these two dates. He reduced these two sets of observations to the same phase and treated them as if the center of mass velocity was the same for both. Selga's elements are:

Selga's Elements

$k = 39.0$ km.
$e = 0.05$
$P = 0\overset{d}{.}246829$
$\omega = 110°$
$T = 2420671.800$ J.D.
$\gamma = +2.0$
$a \sin i = 128{,}800$ km.
$$\frac{m_1^3 \sin^3 i}{(m + m_1)^2} = 0.0014$$

The center of mass velocity variation. If we now superpose the mean velocity curve obtained above with its center of mass velocity line on each of the separate velocity curves, we can obtain for each of them a rather accurate value of the center of mass velocity. We find moreover that these different velocities are fairly well connected by the period $34\overset{d}{.}08$. The following table gives the Greenwich date, the corresponding Julian days, the observed radial velocities of the center of mass of the system, and their residuals from their curve of variation.

TABLE V

VALUES OF CENTER OF MASS VELOCITY

Number	Date	J.D. 2420000+	Phase	Velocity, km.	Residual, km.
	1915				
1	April 14	0602	$8^{\mathrm{d}}6$	+ 2.0	—10
2	15	0603	9.6	+ 5.0	— 3
3	May 26	0644	16.5	— 6.0	+ 6
4	June 7	0656	28.5	—26.0	— 2
5	22	0671	9.4	+ 1.6	— 8
6	25	0674	12.4	— 2.2	— 1
7	July 7	0686	24.4	—32.0	+ 2
8	22	0701	5.3	+14.0	— 7
	1918				
9	April 3	1687	3	+33.0	+ 5
10	4	1688	4	+24.8	— 1
11	11	1695	11	+ 4.0	0
12	15	1699	15	— 8.0	0
13	17	1701	17	—13.5	0
14	21	1705	21	—25.6	0
15	23	1707	23	—32.2	— 1
16	29	1713	29	—17.0	0
17	May 2	1716	32	+16.4	+ 2
18	5	1719	35	+28.0	— 1
19	10	1724	5.9	+17.1	— 3
20	12	1726	7.9	+13.5	0
21	29	1743	25	—37.5	— 1

The above values of the velocity of the system are plotted against their respective phases in Fig. 2. From this velocity curve, it is found that the orbit described by the center of mass is an ellipse. The approximate elements of this orbit are as follows:

ELEMENTS OF ORBIT DESCRIBED BY THE CENTER OF MASS

$$k = 33 \text{ km.}$$
$$e = 0.33$$
$$P = 34^{\mathrm{d}}08$$
$$\omega = 270^\circ$$
$$T = 2421715.35 \text{ J.D.}$$
$$\gamma = -3.2 \text{ km.}$$
$$a \sin i = 14{,}600{,}000 \text{ km.}$$
$$\frac{m_1^3 \sin^3 i}{(m + m_1)^2} = 0.107$$

The observations made on 1918 May 29, group number 21, verify very well the occurrence of the minimum. The velocities of the center of mass, which are derived from Selga's one-prism observations, are subject to a greater probable error, but they follow the curve rather well, as shown by the points marked 1 to 8. The computed velocities for all the observation dates have been derived from the velocity curves of the combined orbits. For Selga's observations and mine, the phases for the two curves have been computed by taking a 1918 epoch as origin. The phases for Slipher's observations were computed by taking the epoch 2418486.778 J.D. as origin. This epoch is very nearly that of the coincidence of two maxima of the two velocity curves.

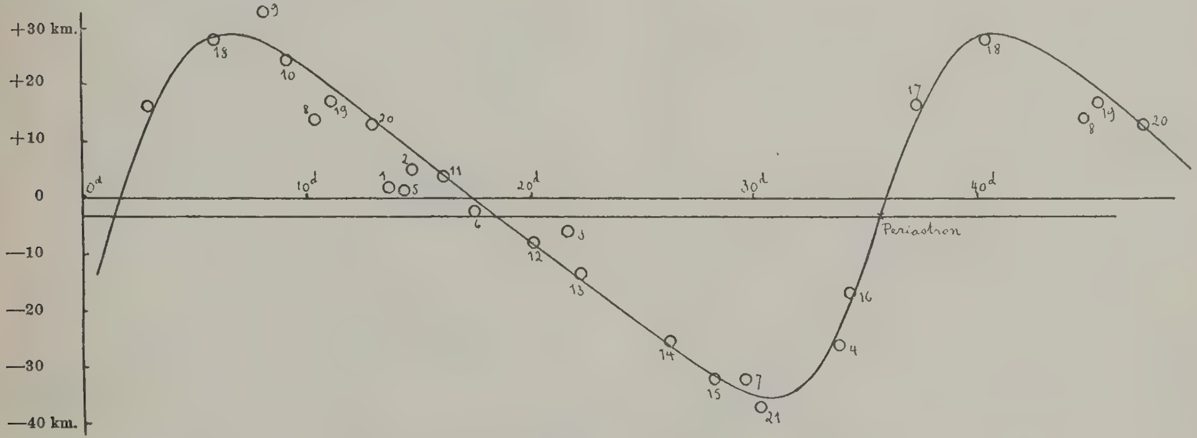

Fig. 2. Variation of center of mass velocity of *σ Scorpii*.

The short period $0^{\mathrm{d}}246834$ can be used for all the observations from 1909 to 1918 without interruption, for, subtracting 12969 periods from the epoch of maximum 2421687.950 J.D. we obtain the epoch 2418486.760 J.D., which is very near the epoch of maximum of Slipher's plates, 2418486.778 J.D.

For the longer variation, the period of 34.04 days seems to connect the maximum of Slipher's plates with the maximum of 1918. Although the residuals computed seem to be within the limits of probable errors for the majority of the plates, we have not enough well distributed observations to determine the last decimal place of the long period with certainty. Moreover it is not impossible that this period varies slightly as well as the amplitude of the velocity curve. Inasmuch as we have observed only one complete cycle with three-prism dispersion, and the one-prism observations have a rather high probable error, the more accurate determination of the long period will have to wait for further observations in the next few years.

Change of line widths in the spectrum. A careful examination of the spectrograms of *σ Scorpii* show, as in the case of *β Canis Majoris*, that sometimes the lines are narrower, and sometimes wider and more diffuse than the general average condition. The spectrograms of *σ Scorpii* are not so good as those of *β Canis Majoris*, however, but on three-prism plates the change of width is so marked that one cannot avoid noticing it. The five best lines on each of a great number of spectrograms (the two lines of oxygen λ4415.076 and λ4417.121 and the three lines of silicon, λ4552.636, λ4567.897, and λ4574.791), have been measured by me. The unit of width used is $1/2000$ of a millimeter. The sum of the five line-widths for each plate is given in the last column of Table III.

An examination of these sums shows us that the period of variation of width is the same as the short period of velocity variation. Moreover, by putting on the velocity curve the points of maximum and minimum widths, we arrive at the interesting result, that *the maximum line-width occurs near periastron passage, and the minimum, near apastron passage.* This result seems to throw some light on the phenomena occurring in *β Canis Majoris* and to support the hypothesis that the variation of amplitude in its velocity curve is due to the combination of two motions, and also that the changes of width are due to physical causes.

A very interesting fact is that Barnard's beautiful photograph of the region of *σ Scorpii* in the Milky Way, Plate 36, *Publications of the Lick Observatory*, **11**, 1913, shows apparent nebulosity around this star. This nebulosity, Barnard says, is largely real.

It is not illogical to suppose that *σ Scorpii* is located in this nebulosity, which probably plays an important part in the phenomena observed.

F. Henroteau.

June, 1918.

THE RADIAL VELOCITIES OF THE STARS ε AND γ *ORIONIS*

I. ε *ORIONIS*

The star ε *Orionis* ($\alpha = 5^h\ 31^m.1$, $\delta = -1°\ 16'$, vis. mag. = 1.75, Class B) has been observed for radial velocity by several spectroscopists. The physical appearance of its spectrum is very poor; on three-prism plates, the lines are broad and hazy. The early Potsdam radial velocities by Vogel and Scheiner give a mean of +26.7 km. for the epoch 1889. Four three-prism plates by Frost and Adams in 1901 and 1902 give a practically constant velocity, with a mean value of +27.6 km. Seven three-prism plates obtained with the Mills spectrograph at Mt. Hamilton in 1902–11, listed in the table below, yield a mean velocity of +24.8 km., with no suggestion that the velocity varies. Fourteen single-prism plates measured by Lee at the Yerkes Observatory give a mean velocity of + 27.8 km.[7] Some of his plates seem to give velocities sufficiently different to suggest that the star is a spectroscopic binary; it is however for only three plates that the residuals from the mean are rather high, amounting to +9.2, —10.8 and —11.8 km., respectively, and I think they might be explained, considering the poor type of spectrum, as accidental errors.

Working on the assumption made by Frost that stars of Class B with wide and diffuse hydrogen and helium lines and very sharp H and K calcium lines are likely to be spectroscopic binaries, Professor R. H. Curtiss[8] and Father Selga[9] undertook, but in vain, to find the period of variable velocity of this star. Curtiss, basing his research on thirty spectrograms taken at Ann Arbor, found that the period could be either in the neighborhood of 100 days or only a fraction of a day. He concluded that more observations must be available to determine this period.

In order to see whether the radial velocity of ε *Orionis* really varies, and especially to find if a short period exists, I measured three series of one-prism plates taken by Dr. F. J. Neubauer at the Lick Observatory in 1917, and two series of three-prism plates taken by me in 1918. Each series was obtained in the course of a single night.

The following table contains the Gr. M.T., the corresponding Julian days and the radial velocities of the Mt. Hamilton observations:

Date		2410000+ J.D.	Velocity, km.	Residual from the mean
3-prism				
1902 Dec.	1	6085.835	+25 km.	—0.5 km.
1904 Jan.	26	6506.792	+26.9	+1.3
1907 Dec.	17	7927.819	+18.6	—6.9
1910 Dec.	21	9027.887	+27.8	+2.3
	22	9028.736	+22.1	—3.4
1911 Feb.	23	9091.660	+25.7	+0.2
Sept.	27	9307.936	+27.4	+1.9
1-prism		2420000+		
1917 Feb.	13	1273.643	+29.6	+4.1
		1273.730	+27.1	+1.6
		1273.739	+30.6	+5.1
		1273.762	+28.4	+2.9
		1273.769	+27.3	+1.8
		1273.821	+21.7	—3.8
		1273.838	+27.4	+1.9
	14	1274.622	+26.9	+1.4
		1274.630	+16.8	—8.7
		1274.655	+23.8	—1.7
		1274.661	+29.2	+3.7

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[7] *Ap. Jour.*, 29, 235, 1909.

[8] *Pub. Detroit Observatory*, **1**, 132, 1915.

[9] *Revista de la soc. astr. de España y América*, **6**, 81, 1916.

Date		J.D. 2410000+	Velocity, km.	Residual from the mean
1917 Feb.	14	1274.686	+28.4	+2.9
		1274.722	+23.6	—1.9
		1274.730	+27.0	+1.5
		1274.762	+26.1	+0.6
		1274.769	+23.8	—1.7
		1274.809	+24.7	—0.8
		1274.817	+27.1	+1.6
	28	1288.629	+21.3	—4.2
		1288.635	+23.6	—1.9
		1288.665	+25.9	+0.4
		1288.672	+30.2	+4.7
		1288.696	+31.3	+5.8
		1288.702	+28.5	+3.0
		1288.726	+32.1	+6.6
		1288.765	+24.8	—0.7
		1288.777	+31.3	+5.8
3-prism				
1918 Jan.	2	1596.699	+21.0	—4.5
		1596.713	+19.4	—6.1
		1596.764	+18.8	—6.7
		1596.776	+27.0	+1.5
		1596.844	+25.2	—0.3
		1596.857	+23.8	—1.7
		1596.916	+25.7	+0.2
		1596.931	+22.1	—3.4
	24	1618.592	+19.5	—6.0
		1618.606	+23.3	—2.2
		1618.619	+23.3	—2.2
		1618.635	+28.0	+2.5
		1618.651	+26.1	+0.6
		1618.692	+20.2	—5.3
		1618.707	+27.2	+1.7
		1618.737	+25.0	—0.5
		1618.774	+30.9	+5.4
		1618.790	+28.3	+2.8
		1618.822	+20.6	—4.9
		1618.837	+30.7	+5.2
		1618.853	+23.6	—1.9

A frequency curve was constructed (Fig. 3), following the method outlined by Schlesinger,[10] separating all the above velocities into groups for which the intervals are two kilometers. The curve obtained is similar to that of the curve of error, so that the above velocities, unless the actual variation of velocity is exceedingly small, would not indicate that the star is a binary. The constant velocity of the star, corresponding to the point of maximum frequency, would be about + 25.5 kilometers, which agrees very well with former measures. This value +25.5 km. is also exactly the mean of the above velocities. The equation of the probability curve is

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2x^2}$$

If we take h as unit of abscissae this equation becomes

$$y = \frac{1}{\sqrt{\pi}} e^{-x^2}$$

and it is found that, adopting the proper units for x and y, the frequency curve above can be represented by this equation.

If we now utilize the well known formula giving the probable error of a single observation, we obtain

$$r = \pm 2.46 \text{ km.}$$

According to recent results which I have obtained for the stars *β Canis Majoris* and *σ Scorpii*, which have early Class B spectra and are very short-period spectroscopic binaries, it seems probable that for any such stars having a period of a few hours, the lines would show a considerable change of widths. This was sought for in the case of *ε Orionis*. For the two series of three-prism plates (1918 January 2 and January 24) the widths of the two best defined lines He 4471.676 and Si 4552.636 were measured on all the plates, but the apparent variations in widths seem to be of the order of the accidental errors of observations.

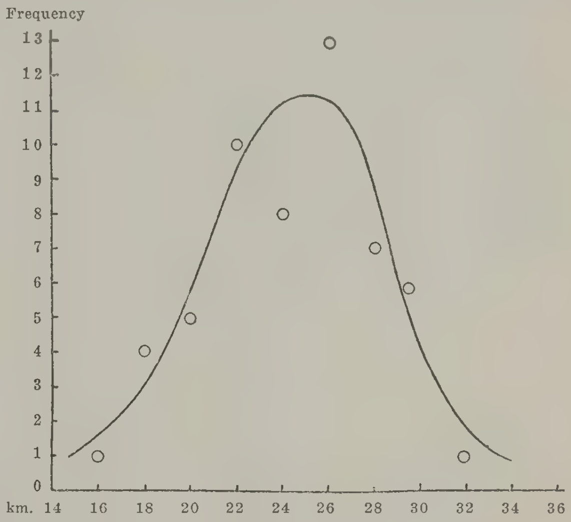

Fig. 3. Frequency curve for *ε Orionis.*

Summary. The values of the radial velocities of *ε Orionis* obtained from the Lick Observatory spectrograms, do not indicate the presence of the short-period variation suspected by Curtiss. Furthermore, an examination of the distribution of these velocities fails to indicate that the apparent variations in velocity are periodic. This investigation does not, however, exclude the possibility of a long-period variation of velocity with a very small range.

II. γ *ORIONIS*

The star γ *Orionis* ($\alpha = 5^{\mathrm{h}}\,19^{\mathrm{m}}8$, $\delta = +6^\circ\,16'$, vis. mag. = 1.70, Class B2) was announced by Frost to be a spectroscopic binary star.[11] Five three-prism plates of it were obtained with the Mills spectrograph at Mt. Hamilton in 1902–11. They are listed in the table below. A spectrographic study of this star was made by Father Selga,[12] with the conclusion that its velocity variation was problematic.

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[10] *Ap. Jour.*, **41**, 162, 1915.

[11] *Ap. Jour.*, **31**, 432, 1910.

Three series of one-prism plates taken at the Lick Observatory by Dr. F. J. Neubauer on three different nights in 1917 and one series of three-prism plates taken by me in 1918, were measured by me, in order to obtain further evidence on the subject of a variable radial velocity for this star. These velocities, together with the older Lick Observatory velocities, are given in the following table:

Date	J.D. 2410000+	Velocity, km.
1902 Oct. 6	6029.949	+16.4
1904 Jan. 26	6506.821	+20.6
1906 Sept. 10	7464.044	+16.5
1908 Dec. 11	8287.782	+17.1
1911 Nov. 19	9360.890	+15.4
	2420000+	
1917 Feb. 13	1273.615	+14.5
	1273.621	+12.5
	1273.682	+14.5
	1273.688	+14.3
	1273.748	+12.9
	1273.753	+12.4
	1273.778	+17.0
	1273.788	+18.5
14	1274.605	+21.4
	1274.611	+12.4
	1274.638	+16.5
	1274.645	+14.0
	1274.671	+18.5
	1274.678	+12.9
	1274.705	+16.7
	1274.713	+19.1
	1274.745	+18.7
	1274.752	+18.8
	1274.779	+20.5
	1274.787	+20.6
	1274.830	+20.0
	1274.838	+12.3
28	1288.611	+16.7
	1288.617	+19.0
	1288.644	+21.7
	1288.649	+16.3
	1288.680	+20.1
	1288.712	+21.3
	1288.718	+20.1
	1288.742	+27.0
	1288.750	+18.0
	1288.787	+20.8
	1288.800	+22.2
	1288.816	+22.1
1918 Jan. 2	1596.666	+13.77
	1596.678	+16.01
	1596.731	+24.58
	1596.744	+25.68
	1596.812	+21.33
	1596.826	+21.15
	1596.874	+20.73
	1596.888	+18.63

From these velocities, a frequency curve was constructed (Fig. 4), as in the case of ϵ *Orionis*. This frequency curve does not resemble the error curve at all and is not opposed to the hypothesis that γ *Orionis* is a spectroscopic binary with a range of only a few kilometers. It does not seem to have a very short period.

Measures of my series of three-prism plates do not disclose any variation in the width of the lines. Moreover, a faint line on the violet side of 4472 A. does not seem to be a component of 4472, as was thought by Professor Frost, who found this line in some of his single-prism plates. This faint line occurs also on several good spectrograms of other class B stars.

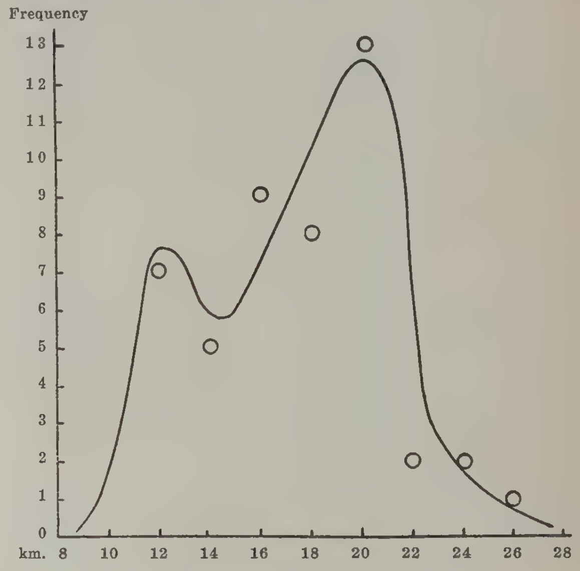

Fig. 4. Frequency curve for γ *Orionis.*

It seems possible that γ *Orionis* is a spectroscopic binary, but the determination of its orbit would be very difficult, owing to the small range of radial velocities, and would necessitate an enormous number of spectrograms.

At my departure from Mt. Hamilton, where, owing to the generosity of Dr. Campbell, I have been able to use the wonderful equipment of the observatory, I wish to express to him my gratitude for all the help and advice he has given me. I also appreciate greatly the kindness and the help of Dr. Moore, Dr. Paddock and other members of the staff.

F. HENROTEAU.

MT. HAMILTON, June 30, 1918.

Issued September 7, 1918.

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[12] *Revista de la soc. astr. de España y América,* 6, 65, 1916.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 315

THE ORBIT OF THE SPECTROSCOPIC BINARY *p VELORUM*

The binary character of *p Velorum* (1900.0: $\alpha = 10^{h}\ 33^{m}1$, $\delta = 48^{\circ}\ 42'$; mag. 4.1; Class F2) was announced by Wright, in *Lick Observatory Bulletin*, **3**, 111, 1905. In all there are now available thirty-seven spectrograms taken at the Observatory of the D. O. Mills Expedition, of which the first nine were made with a dispersion of three prisms and the last twenty-eight with a dispersion of two prisms. The former group was obtained in the interval 1903–07, the latter in the interval 1913–17. The 3-prism plates were measured with a Toepfer engine, and the 2-prism plates with a Hartmann spectrocomparator using a standard-velocity plate. It has been necessary to apply a systematic correction of —2.84 km./sec. to all 2-prism velocities to bring them into accord with those taken with 3-prism dispersion. This correction has already been applied to the velocities of the following table.

Observations of *p Velorum*

Gr. M.T.	Phase	Primary Obs. Vel. km./sec.	Primary O–C km./sec.	Secondary Obs. Vel. km./sec.	Secondary O–C km./sec.	Measurer
1903 Dec. 14.845	$2^{d}939$	+35.34	—0.21	— 4	— 4	Wr.
1904 Feb. 6.853	5.892	+38.40	+0.17	—10	— 7	Wr.
Dec. 31.797	8.086	+21.41	—2.92			Wr.
1905 Jan. 25.744	2.400	+25.38	—6.54			Wr.
Jan. 26.701	3.357	+39.23	+1.86	— 6	— 4	Wr.
1907 Jan. 29.861	1.328	+13.65	—1.89			Pa.
Feb. 4.810	7.277	+33.30	+1.18			Pa.
Feb. 6.714	9.181	+ 0.50	+0.87			Pa.
Mar. 19.716	9.339	— 5.50	+1.39			Pa.
1913 Dec. 18.837	4.409	+39.84	+0.36			Sa.
Dec. 19.853	5.425	+36.42	—2.69			Sa.
Dec. 20.851	6.423	+35.75	—0.85			Sa.
Dec. 22.819	8.391	+24.42	+4.63			Sa.
Dec. 26.804	2.165	+28.95	—0.69			Sa.
1914 Apr. 29.567	3.397	+37.85	+0.34	—11	— 9	Sa.
May 2.609	6.439	+37.81	+1.27			Sa.
May 4.465	8.295	+23.54	+2.19			Sa.
May 7.668	1.287	+17.39	+2.93			Sa.
May 8.671	2.290	+30.23	—0.67			Sa.
May 12.572	6.191	+38.51	+1.11			Sa.
May 13.612	7.231	+28.00	—4.44			Sa.
May 14.572	8.191	+22.01	—0.89			Sa.
May 17.596	1.004	+ 5.61	+0.19			Sa.
May 18.628	2.036	+23.20	—4.97			Sa.
May 19.506	2.914	+37.31	+1.89			Sa.
June 10.432	4.418	+41.44	+1.96			Sa.
June 12.522	6.508	+35.49	—0.78			Sa.
June 24.424	8.199	+24.99	+2.20			Sa.

Observations of *p Velorum*—(*Continued*)

Gr. M.T.	Phase	Primary Obs. Vel. km./sec.	Primary O–C km./sec.	Secondary Obs. Vel. km./sec.	Secondary O–C km./sec.	Measurer
June 25.451	9.226	— 4.61	—2.52	+60	+14	Sa.
July 26.473	9.615	—20.14	+0.58	+62	— 7	Wi. and Sa.
Aug. 4.469	8.400	+20.78	+1.14			Sa.
Aug. 5.473	9.404	— 9.78	+0.01	+61	+ 7	Sa.
Aug. 6.462	0.182	—39.47	+0.10	+80	—13	Sa.
1917 Jan. 31.832	0.777	— 3.36	+1.08			Wi. and Sc.
Feb. 20.760	0.283	—36.78	—2.72	+84	— 2	Wi. and Sc.
Mar. 2.719	0.032	—43.26	+1.30	+102	+ 3	Wi. and Sc.
Apr. 2.662	0.342	—32.03	—1.67	+82	+ 1	Wi. and Sc.

The table contains the data for the thirty-seven spectrograms. For each plate are given the Greenwich Mean Time of mid-exposure, the phase and, under "Primary," the radial velocity of the principal star in km./sec., and the residual velocities based upon the final elements. The measurer is indicated in the last column, wherein Wr. signifies W. H. Wright; Pa., G. F. Paddock; Wi., R. E. Wilson; Sc., A. A. Scott; and Sa., R. F. Sanford. On eleven of the spectrograms the velocities of the secondary star have been measured. Under the heading "Secondary" are to be found the corresponding velocities and the residuals based upon the final elements. The measures for the radial velocities of the secondary star on the last three spectrograms were made by R. E. Wilson only.

Preliminary elements, derived by the graphical method of Lehmann-Filhés, were made the basis of successive least-squares solutions for corrections to the elements.[1] The final solution gave the following corrections to the preliminary elements: $\delta\gamma = 0.00$ km./sec., $\delta K = +0.11$ km./sec., $\delta\omega = +0^{\circ}.10$, $\delta e = -0.005$, $\delta T = +0.0002$ days, $\delta P = -0.000045$ days. When they were applied and the residuals for the observations computed, no material reduction in the sum of the squares of the residuals was found. These residuals are given in the table as (O–C) under "Primary."

The following elements, which resulted from the application of the above corrections, are taken as defining the orbit that best satisfies the observations.

$$P = 10.210955 \pm 0.000062 \text{ days}$$
$$T = \text{J. D. } 2420259.381 \pm 0.00015 \text{ days}$$
$$e = 0.541 \pm 0.004$$
$$\omega = 184^{\circ}.62 \pm 0^{\circ}.59$$
$$K = 42.34 \pm 0.36 \text{ km./sec.}$$
$$\gamma = +19.25 \text{ km./sec.}$$
$$a \sin i = 4{,}981{,}000 \text{ km.}$$
$$m \sin^3 i = 0.281\odot$$

For the primary star, the probable error of a single observation of velocity of weight unity is ±1.4 km./sec.

Measures of the velocities of the secondary star were possible on eleven spectrograms. Because these velocities are not only very much less accurate but also less numerous than those for the primary, no attempt has been made to obtain all the elements from them. From the adopted orbit of the primary five elements are immediately available, if the change of 180° in ω is made, and K alone need be solved for. A rough velocity curve for the secondary has served as a basis for obtaining a value of K (52 km./sec.) which rendered the sums of the squares of the residuals a minimum.

With a value of K equal to about 70 km./sec. the residuals for the measures near the secondary's minimum velocity can be made very small, but in this case the residuals near its maximum velocity are far larger than can be accounted for on any reasonable assumption. The elements adopted for the secondary are:

$$P = 10.210955 \text{ days}$$
$$T = \text{J. D. } 2420259.381$$
$$e = 0.541$$
$$\omega = 4^{\circ}.62$$
$$K_1 = 52 \text{ km./sec.}$$
$$\gamma = +19.25 \text{ km./sec.}$$
$$a_1 \sin i = 6{,}107{,}000 \text{ km.}$$
$$m_1 \sin^3 i = 0.235\odot$$

With K and K_1 both determined, the mass factors for both primary and secondary are found to be as given with the elements of each. The mass ratio ($m/m_1 = K_1/K$) is 1.23. For Class F stars Russell[2] obtains an average mass of about three times that of the Sun. On the assumption that the primary has this mass, the inclination of the orbit plane would be 27°. With such an inclination the values of a and a_1 prove to be 10,880,000 km. and 13,340,000 km. respectively.

The velocity curves have been constructed from the final elements. Open circles represent the measured velocities of the primary in the interval 1913–1917. Barred circles correspond to the early 3-prism velocities of the primary. The radius of these circles is equal to the probable error of a single observation of

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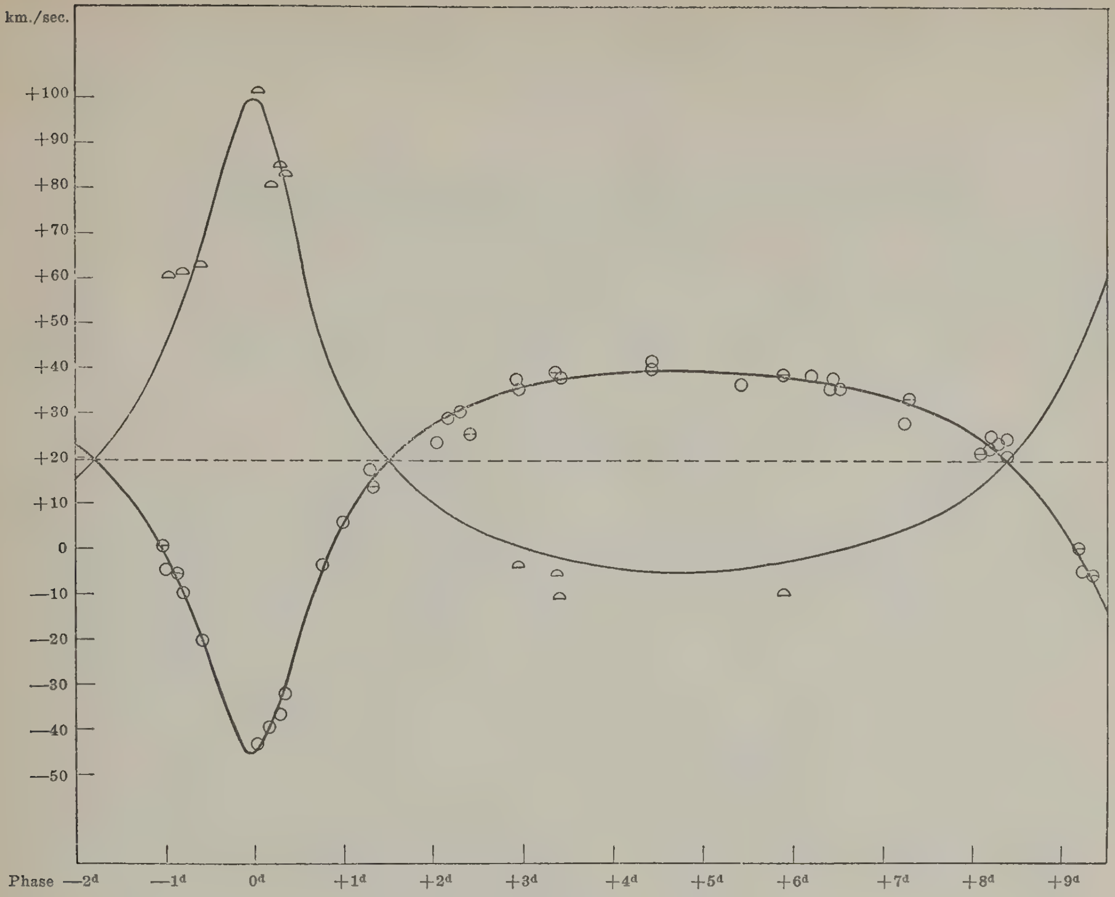
[1] The observation of 1905, Jan. 25 was not used in making the least-squares solutions because of the uncertainty of its velocity determination. See footnote, *Publ. Lick Obs.*, 9, 196.

[2] *Publ. A. A. S.*, 3, 327, 1917.



Radial Velocity Curve of *p Velorum*

weight unity. Observed velocities of the secondary are represented by semicircles.

Some of the best of the spectrograms of *p Velorum* have been examined by Adams and Joy for the purpose of estimating its absolute magnitude. Spectrograms were chosen near the time of the primary's maximum positive velocity. The mean of the determinations gives +1.9 for the absolute visual magnitude. The apparent visual magnitude is +4.1. Therefore the parallax resulting from the formula[3] $\log \pi = \frac{M - m}{5} - 1$ is +0″.036.

See[4] found *p Velorum* to be a visual binary star in 1897 (λ 119, magnitude 4.5, 5.0) and it has since been measured also by J. Voûte[5] with the 9-inch Grubb refractor of the Union Observatory, Johannesburg, South Africa. The two sets of measures are:

1897.06	267°.0	0″.52	4n	See and Cogshall
1915.33	253.8	0.42	4n	Voûte

According to Innes[6] the two components have the common P.M. of 0″.177 is 262°.9.

R. F. Sanford.

July, 1918.
Issued September 7, 1918.

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[3] π = parallax, M absolute visual magnitude, m apparent visual magnitude.

[4] *Astron. Jour.*, **18**, 195, 1898.

[5] *Union Obs. Circ.*, **27**, 210, 1915.

[6] *Reference Catalogue of Souhern Double Stars.*

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 316

THE ORBIT OF *SIRIUS*

The most recent orbit of the binary star *Sirius* is Lohse's,[1] computed in 1906 when the observed arc described by the companion covered about 331°. The available arc is now 13° in excess of a complete revolution and this fact, together with the systematic and slowly increasing deviation of the observed path of the companion from the one predicted by Lohse's elements, has led me to undertake a new investigation of the orbit.

The first step consisted in a careful examination of the micrometric measures as originally published and as given in later publications (some measures are repeated in three or more periodicals or volumes); and the collection of later, unpublished measures. In this connection I am under great obligations to Messrs. Barnard, Comstock, Mitchell, Olivier and Van Biesbroeck, for their kindness in sending me results of their observations, including those made early in the present year. The excellent list of measures printed in Burnham's *General Catalogue* served as my guide to the measures in years preceding 1906; but in examining the original publications I found a number of corrections that should be made to this list. These are as follows (retaining Burnham's form):

1863

The measure of Dawes should be deleted. It is given correctly under the date 1864.23.

1867

Eastman: *For*	1867.27	74°9	9″92	1n,
read	1867.28	74°9	9″46	2n.

1875

Add,	1875.28	56°4	11″08	4n	Hall

1879

Howe: *For*	1879.17	49°8	10″32	2n,
read	1879.16	50°0	10″44	3n.

1881

Holden: *For* 43°3 10″83 *read* 46°7 10″74.
Frisby: *For* 46°9 *read* 43°2.

1885

Add,	1855.27	34°7	8″06	6n	Hall

1886

Add,	1886.21	31°2	7″38	6n	Hall

1897

The measures should read:

1896.83	189°1	3″69	2n	Aitken
1896.85	189.2	3.73	4n	Schaeberle
1897.16	185.4	3.88	3n	Aitken
1897.21	186.6	3.78	1n	Hussey

1898

Add,	1898.27	168°9	4″79	3n	See
	1898.27	170.7	4.86	3n	Boothroyd

1902

Add,	1902.24	127°7	5″49	3n	Aitken

1903

Barnard: *For*	1903.09	119°8	6″08	4n,
read	1903.11	120°0	6″10	5n.

1904

Add,	1903.81	115°4	6″32	2n	Barnard
	1904.85	111.1	6.99	3n	Barnard
and, *for*	1904.90	109°6	6″76	2n	Aitken
read	1904.98	109°6	6″85	3n	Aitken

The measures credited to Bond in 1863.27 and in 1864.22 are not given in any of the references cited by Burnham and I have been unable to find them elsewhere; also, Gledhill's name is omitted from the reference list and I have not succeeded in finding his original measures.

For the convenience of other investigators, the later measures which I have collected are here given in the form adopted in Burnham's list, but I have recorded separately the number of angle and the number of distance measures, when these numbers differ.

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[1] *Publ. Astroph. Obs. Potsdam,* 20, 87, 1908.

1906

1906.08	103°8	7″42	2n	Aitken
1906.11	104.7	6.95	3n	Olivier

1907

1906.79	(96.7)	....	1n	Lohse
1906.96	100.9	7.85	3n	Barnard
1907.00	100.0	7.83	3n	Aitken
1907.20	99.4	7.74	2n	Comstock
1907.23	99.8	....	4n	Lohse

1908

1908.11	95.5	(6.88)	6, 1n	Lohse
1908.17	96.1	8.52	6n	Barnard
1908.20	95.9	8.38	2n	Aitken
1908.26	95.1	8.44	2n	Comstock

1909

1909.06	93.3	8.62	4n	Neff
1909.09	92.0	8.56	4n	Olivier
1909.14	92.5	8.75	10n	Barnard
1909.17	94.4	8.67	2n	Fox
1909.25	91.6	8.70	2n	Aitken

1910

1910.10	91.5	9.45	6, 5n	Fox
1910.11	89.1	9.07	8n	Barnard
1910.13	88.9	9.05	2n	Aitken

1911

1911.00	89.6	9.65	7n	Fox
1911.10	87.1	(8.76)	4n	Jonckheere
1911.13	87.7	9.46	7n	Barnard
1911.21	85.2	9.38	3n	Aitken

1912

1911.92	86.9	9.84	5n	Fox
1912.07	84.8	9.90	5n	Barnard
1912.08	83.4	9.58	2n	Aitken
1912.24	85.4	9.39	2n	Comstock

1913

1913.05	82.3	9.96	15, 14n	Barnard
1913.06	83.4	10.27	4n	Fox
1913.18	81.7	10.02	2n	Aitken
1913.25	81.6	(9.22)	2n	Jonckheere

1914

1913.99	80.1	10.10	6n	Mitchell
1914.02	79.6	10.24	4n	Voûte
1914.05	80.6	10.35	2n	Fox
1914.15	79.1	10.25	5n	Barnard
1914.24	80.6	10.12	1n	Jonckheere

1915

1914.90	77.4	10.21	1n	Mitchell
1914.91	78.0	10.48	2n	Olivier
1915.03	78.8	9.98	10, 7n	Jonckheere
1915.13	76.2	10.70	3n	Aitken
1915.16	76.8	10.63	2n	Comstock

1916

1915.92	75.1	10.46	2n	Olivier
1915.96	76.8	10.73	5n	Van Biesbroeck
1915.99	75.1	10.66	16, 15n	Barnard
1916.11	74.6	10.57	6n	Jonckheere
1916.19	76.5	10.78	3, 2n	Comstock
1916.21	75.5	10.81	4n	Innes

1917

1916.88	74.2	10.60	11, 10n	Olivier
1916.95	73.8	10.80	20, 18n	Barnard
1917.18	73.4	(11.24)	4n	Innes
1917.19	73.7	10.90	3, 2n	Comstock
1917.23	73.9	10.61	3n	Jonckheere

1918

1917.90	72°2	11″05	5n	Van Biesbroeck
1918.04	71.0	11.15	4n	Aitken
1918.07	71.6	10.96	3n	Olivier
1918.08	70.6	10.99	8n	Barnard

REFERENCES

Aitken: *Publ. Lick Obs.*, **12**, 46.

Barnard: *Astron. Jour.*, **23**, 132; **26**, 143; **27**, 108, 193; **29**, 38; **30**, 24, 182; *Astron. Nach.*, **182**, 14; Letter.

Comstock: Letter.

Fox: *Annals Dearborn Obs.*, **1**.

Innes: *South African Union Obs. Circ.*, **37**, 41.

Jonckheere: *Astron. Jour.*, **31**, 4.

Lohse: *Publ. Potsdam Astroph. Obs.*, **20**, 23.

Mitchell: Letter.

Neff: *Astron. Nach.*, **182**, 258.

Olivier: *Astron. Nach.*, **174**, 212; **182**, 258; *Astron. Jour.*, **30**, 67, 161; Letter.

Van Biesbroeck: Letter.

Voûte: *Mon. Not. R. A. S.*, **75**, 432.

A few very discordant measures (e.g., Secchi's in 1863, Pechüle's in 1871, Otto Struve's in 1886, etc.) were rejected, and annual means formed from the rest by giving weight ½ to single night's measures, weight 1 to the mean of two to four nights, weight 2 to the mean of five to eight nights, and weight 3 to the mean of a larger number of nights, provided the measures were accordant. If specially inconsistent (e.g., Peirce's measures on twelve nights in 1870, which have a range of 18°2 in angle and of 1″25 in distance), a lower weight was assigned. These annual means were compared with an accurate ephemeris based on Lohse's elements and curves were drawn to represent the mean corrections to the predicted motion. The individual measures were then compared with these corrected interpolating curves and the average residual for each observer who had complete measures in at least three different years was regarded as his systematic error of measure. Corresponding corrections were applied to the measures and new annual means formed and reduced to the epoch 1900.0. These means were finally combined into fourteen normal places, the weight of each normal being the combined weight of the annual means comprised in it, rounded off to perfect squares.

The comparison of the measures from 1912 to 1918 with Lohse's ephemeris and with the measures in the years 1862 to 1868 clearly indicated that the actual revolution period was very close to 50.0 years, instead of 49.32 years as given by Lohse.[2] The first and fourteenth normal places were therefore combined on the assumption that the period was precisely 50.0 years and this value was used, in conjunction with Lohse's values for the other elements, in computing the differential coefficients.

The observed distance of the companion from *Sirius* varies from less than 4″ to more than 11″; and this variation in the radius vector must be taken into account in the least squares solution based upon the observed position angles by expressing the quantities (O–C) in the equations in terms of circular rather than of angular measure. As it was also my plan to make an independent solution based upon the observed distances, I adopted Comstock's convenient formulae[3] in computing the coefficients. It is not necessary to print the detailed results of these computations, the equations of condition, nor even the normal equations, since they offer no peculiarities of any kind. It will suffice to say that, as anticipated, the solution based upon the observed distances proved to be of small value because of the great discordances in the measures. The corrections given by the solution based upon the observed position angles were therefore adopted for all of the elements except the semi-major axis. For this element, the value best satisfying the normal distances, in conjunction with the final values of the other elements, was adopted.

The elements with their probable errors are:

	LOHSE
$P = 50.04$ years ± 0.09 years	49.32 years
$T = 1894.133 \pm 0.011$ years	1894.138
$e = 0.5945 \pm 0.0023$	0.5904
$a = 7''570$	7″55
$i = +43^{\circ}31 \pm 0^{\circ}25$	44°548
$\omega = 145.69 \pm 0.38$	146.275
$\Omega = 42.71 \pm 0.33$	43.266

Angles decreasing.

Lohse's elements have been added for comparison.

EPHEMERIS

1919.24	68°9	11″14	1935.24	33°1	8″09	
1921.24	65.1	11.30	1936.24	29.2	7.48	
1923.24	61.4	11.36	1937.24	24.4	6.79	
1925.24	57.6	11.23	1938.24	18.5	6.03	
1927.24	53.7	10.97	1939.24	10.8	5.19	
1929.24	49.5	10.55	1940.24	360.0	4.30	
1931.24	44.9	9.94	1941.24	346.6	3.43	
1932.24	42.4	9.57	1944.173	349.1	2.83	Periastron
1933.24	39.6	9.14	1946.173	202.9	3.61	
1934.24	36.6	8.65	1947.173	185.6	3.87	

Various attempts were made to effect improvements in these elements but without success, and I believe that we must wait until the companion star has passed the point of maximum elongation which it is now approaching and has again advanced well toward its periastron before any real improvement becomes possible. The sums of the squares of the residuals for the annual means (*not the normal places*), when the angle residuals are expressed in arc, are:

LOHSE'S ELEMENTS		REVISED ELEMENTS	
Angle	Distance	Angle	Distance
0.9147	1.4276	0.2723	1.1468

The largest residual in distance, for each orbit, is that for the discovery year, 1862, when this coördinate was certainly considerably smaller than the measured value. Omitting this residual, the sums of the squares of the distance residuals are reduced to 1.0676 and 1.0243 for Lohse and Aitken respectively. In estimating the improvement in the general representation of the observations effected by the new orbit, it must be remember that a different method of weighting the measures when combining them to form the annual means would modify the relative sizes of the residuals. Personally, I regard the correction to the period as the essential improvement effected.

Table I gives the annual means reduced to the epoch 1900.0, *but not corrected for systematic errors of observation*, and the residuals derived by comparison with the ephemeris computed from the adopted elements. I have computed the residuals for every observer as well, but consider it unnecessary to record them here, for unless a given observer has measures in a number of different years and these show a somewhat systematic deviation from the computed path the residuals can hardly be regarded as satisfactory material for the determination of personal equation. When, for example, two of five residuals in angle are positive, three negative, and the range is 10°, we can merely infer that the measures were made under unfavorable conditions. Some series of measures, however, give very definite data for personal equation. For example, eighteen of the twenty-one residuals in angle for my own measures are negative; all six of Fox's are positive. Table II is a summary of the data for the thirteen observers who have complete sets of measures in at least six years. The columns give, in order, the name of the observer, the number of sets of measures, the mean $\Delta\theta$ in terms of arc displacement, the range in $\Delta\theta$, which will serve as a guide to the consistency of the measures, the $\Delta\rho$ and the range in $\Delta\rho$.

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[2] See my note in *Publ. A. S. P.*, 30, 194, 1918. Jonckheere has a note in *Mon. Not. R. A. S.*, 78, 480, 1918, a copy of which has just reached me, to the same effect.

[3] *Astron. Jour.*, 31, 1918.

TABLE I

Annual Means and Their Residuals

Date	θ_0	ρ_0	O–C $\Delta\theta$		$\Delta\rho$
			Angle	Arc	
1862.22	84°46	10″01	+0°21	+0″04	+0″35
1863.21	82.02	9.74	+0.25	+0.04	—0.21
1864.20	78.49	10.42	—0.92	—0.16	+0.20
1865.21	76.63	10.15	—0.52	—0.10	—0.31
1866.20	75.16	10.54	+0.16	+0.03	—0.14
1867.19	73.11	10.80	+0.15	+0.03	—0.06
1868.20	70.53	11.13	—0.38	—0.07	+0.12
1869.17	69.24	11.08	+0.26	+0.05	—0.06
1870.17	66.65	11.18	—0.43	—0.08	—0.06
1871.21	64.51	11.04	—0.59	—0.12	—0.26
1872.19	63.26	11.36	±0.00	±0.00	+0.03
1873.22	61.53	11.06	+0.19	+0.04	—0.30
1874.18	59.19	11.37	—0.32	—0.06	+0.07
1875.18	56.63	11.28	—0.99	—0.19	+0.05
1876.14	55.35	11.40	—0.42	—0.07	+0.27
1877.19	53.37	11.08	—0.32	—0.06	+0.11
1878.13	51.66	10.85	—0.12	—0.02	+0.06
1879.13	49.64	10.58	—0.02	—0.00	+0.01
1880.13	47.94	10.20	+0.50	+0.09	—0.09
1881.18	45.26	9.82	+0.29	+0.05	—0.13
1882.12	42.61	9.41	±0.00	±0.00	—0.19
1883.16	39.54	9.11	—0.22	—0.03	—0.05
1884.18	36.44	8.57	—0.22	—0.03	—0.09
1885.21	33.58	8.00	+0.50	+0.07	—0.08
1886.15	29.49	7.25	+0.11	+0.01	—0.26
1887.19	24.18	6.79	—0.30	—0.03	—0.01
1888.97	13.84	5.27	+1.01	+0.10	—0.12
1890.27	359.63	4.19	+0.50	+0.04	—0.05
1891 to 1895 companion invisible.					
			Angle	Arc	
1896.86	189°03	3″71	—1°08	—0″07	—0″11
1897.18	185.78	3.85	+0.89	+0.06	—0.03
1897.84	174.47	4.00	—0.31	—0.02	—0.05
1898.23	170.12	4.22	+1.21	+0.09	+0.06
1898.76	162.04	4.19	—0.03	—0.00	—0.13
1899.26	154.80	4.42	—1.01	—0.08	—0.06
1899.88	148.05	4.65	—0.65	—0.05	—0.05
1900.24	144.70	4.68	—0.18	—0.02	—0.17
1901.08	136.38	5.01	—0.35	—0.03	—0.18
1902.07	128.67	5.50	+0.22	+0.02	—0.09
1903.11	120.19	5.98	—0.88	—0.09	—0.06
1903.81	115.42	6.32	—1.21	—0.14	—0.03
1904.89	109.89	6.92	—0.74	—0.09	+0.09
1906.10	104.28	7.18	—0.59	—0.08	—0.18
1907.10	100.06	7.81	—0.54	—0.07	+0.02
1908.17	95.72	8.46	—0.86	—0.12	+0.24
1909.14	92.77	8.67	—0.49	—0.07	+0.08
1910.11	90.08	9.22	—0.15	—0.02	+0.27
1911.11	87.46	9.50	+0.13	+0.02	+0.21
1912.08	85.29	9.67	+0.58	+0.10	+0.06
1913.11	82.34	10.03	+0.23	+0.04	+0.12
1914.07	80.01	10.22	+0.19	+0.03	+0.05
1915.04	77.77	10.33	+0.17	+0.03	—0.09
1916.04	75.57	10.67	+0.13	+0.02	+0.03
1917.04	73.96	10.72	+0.63	+0.12	—0.11
1918.02	71.45	11.04	+0.10	+0.02	+0.05

TABLE II

Systematic Errors of Measure

Observer	n	$\Delta\theta$ (arc)	Range in $\Delta\theta$	$\Delta\rho$	Range in $\Delta\rho$
Otto Struve	18, 19	+0″04	+0″52 to —0″61	+0″15	+0″73 to —0″73
Hall to 1877	6	—0.10	+0.18 to —0.26		
Hall, 1878–87	10	+0.10	+0.26 to —0.08		
Hall	18			—0.05	+0.17 to —0.48
Holden	7	—0.19	—0.06 to —0.30	+0.66	+0.18 to +0.77
Burnham to 1884	8	+0.05	+0.21 to —0.33	—0.23	—0.14 to —0.32
Hough to 1884	4	+0.04	+0.09 to —0.03		
Hough, 1885–87	3	—0.08	—0.06 to —0.10		
Hough	8			—0.22	—0.02 to —0.41
C. A. Young	6	+0.05	+0.30 to —0.15	+0.06	+0.25 to —0.16
Aitken	21	—0.11	+0.05 to —0.30		
Aitken to 1905	11			—0.08	+0.02 to —0.17
Aitken, 1906–18	10			+0.10	+0.26 to —0.03
Hussey to 1898	3	+0.10	+0.15 to +0.04		
Hussey, 1899–1904	6	—0.09	—0.01 to —0.24		
Hussey	9			—0.10	+0.13 to —0.24
Barnard	16	—0.04	+0.11 to —0.18	+0.10	+0.30 to —0.03
Olivier	6	—0.02	+0.10 to —0.21	—0.12	+0.09 to —0.41
Comstock	6	+0.03	+0.26 to —0.17	+0.03	+0.19 to —0.27
Fox	6	+0.25	+0.45 to +0.13	+0.30	+0.50 to +0.06
Jonckheere	6	+0.07	+0.21 to —0.13	—0.36	—0.08 to —0.73

In forming this table, Otto Struve's "corrected" measures as printed in Volume X of the *Pulkowa Observations* have been used. One or two very discordant measures have been left out of account. An occasional erroneous measure by other observers has also been omitted and so have Burnham's two sets of measures with the 36-inch refractor. The conditions under which these were made differed so much from the conditions for the earlier measures that we cannot be sure that the observing habit did not change. It will be noted that the average $\Delta\rho$, disregarding algebraic sign, is fully twice as large as the average $\Delta\theta$. Slight changes in the orbit elements would, of course, effect changes in the residuals in certain years; but no permissible variation in the elements would remove, or even appreciably modify, such a personal equation as that shown, for example, by the measures of Holden and Burnham. As soon as the data permit, it will be a matter of interest to compute an orbit based entirely upon the observations to which systematic corrections may fairly be applied.

In *Lick Observatory Bulletin,* **3**, 80, 1905, Dr. Campbell discussed the variable radial velocity of the bright star in the system of *Sirius* and came to the conclusion that it could be accounted for very satisfactorily by the motion of the star in the orbit derived from the micrometer measures of the visual binary, assuming that the mass ratio of the two components as given by Auwers is substantially correct. Certain irregularities in the progression of the velocities were noted but it was impossible at that time to say whether these were real and due to unrecognized disturbing forces in the system or purely accidental. In 1910, W. Münch[4] made a similar investigation based upon spectrograms secured at the Potsdam Astrophysical Observatory in the years 1901–1910 and concluded that the marked irregularities he found (several times greater than those in the Lick series of spectrograms) were due to unknown systematic "Einflüsse" in the measures which might possibly be produced by a short-period variation in the motion of *Sirius.*

TABLE III

OBSERVED RADIAL VELOCITIES OF *Sirius*

Place	Date	n	V_0 km.	V_1(A) km.	V_1(B) km.	$(V_0 - V_1) - V_m$ A km.	B km.
Paris	1891.17	1	(—1.2)	+0.3	+0.2	(+5.9)	(+5.8)
Paris	1895.21	1	(—4.1)	+4.9	+3.4	(—1.6)	(—0.3)
Lick	1896.97	8	—3.2	+4.0	+2.8	+0.2	+1.2
Lick	1898.07	1	—5.9	+3.3	+2.3	—1.8	±0.0
Lick	1898.74	4	—3.6	+2.9	+2.0	+0.9	+1.6
Lick	1899.92	6	—4.8	+2.3	+1.6	+0.3	+0.8
Lick	1901.93	1	—4.8	+1.5	+1.0	+1.1	+1.4
Yerkes	1902.06	10	—6.9	+1.4	+1.0	—0.9	—0.7
Lick	1903.07	9	—6.9	+1.1	+0.8	—0.6	—0.5
Cape of Good Hope	1903.89	3	—7.0	+0.9	+0.6	—0.5	—0.4
Cape of Good Hope	1904.06	3	—7.1	+0.9	+0.6	—0.6	—0.5
Lick	1904.95	2	—5.4	+0.7	+0.5	+1.3	+1.3
Lick	1905.14	5	—7.9	+0.6	+0.4	—1.1	—1.1
Lick	1906.07	3	—7.7	+0.4	+0.3	—0.7	—0.8
Cape of Good Hope	1907.92	9	—7.3	+0.1	+0.1	±0.0	—0.2
Cape of Good Hope	1908.12	7	—7.4	+0.1	+0.1	+0.1	—0.3
Cape of Good Hope	1908.25	14	—7.6	+0.1	+0.0	—0.1	—0.4
Lick	1909.29	4	—7.4	—0.1	—0.1	+0.1	—0.1
Lick	1909.95	2	—7.3	—0.2	—0.1	+0.3	±0.0
Mount Wilson	1909.96	4	—7.8	—0.2	—0.1	—0.2	—0.5
Lick	1910.13	3	—8.0	—0.2	—0.2	—0.4	—0.6
Lick	1910.29	13	—7.5	—0.2	—0.2	+0.1	—0.1
Lick	1910.79	3	—5.9	—0.3	—0.2	+1.8	+1.5
Lick	1911.01	1	—7.9	—0.3	—0.2	—0.2	—0.5
Lick	1911.98	2	—7.9	—0.4	—0.3	—0.1	—0.4
Lick	1913.23	4	—8.8	—0.6	—0.4	—0.8	—1.2
Santiago, Chile	1913.73	2	—6.7	—0.6	—0.4	+1.3	+0.9
Lick	1914.20	2	—9.2	—0.7	—0.5	—1.1	—1.5
Santiago, Chile	1916.87	2	—6.5	—0.9	—0.6	+1.8	+1.3
						±0.68	±0.73

$V_m = -7.37$ km., Auwers's mass ratio.
$V_m = -7.23$ km., Boss's mass ratio.

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[4] *Astron. Nach.,* **186**, 225, 1910.

It has seemed desirable to repeat this investigation and I have therefore collected all measures of the radial velocity of *Sirius* which have been published since the appearance of Campbell's paper and give them, together with the earlier data taken from his paper and the later unpublished measures made at the Lick Observatory,[5] in Tables III and IV. The first four columns of Table III give, in order, the place of observation, the mean date, the number of separate measures which have been combined, and V_0, the mean observed radial velocity. The measures combined into a single mean, in all cases, cover only a very short time interval. The fifth and sixth columns give V_1, the radial component of the orbital velocity of *Sirius,* computed from my elements and the mass ratios as determined by Auwers and by Boss. These ratios are, respectively, $1.04/2.20 = 0.47$, and $0.74/2.56 = 0.29$. Each combination of V_0 with the corresponding V_1 gives a value of V_m, the radial velocity of the center of mass of the system. The question at issue is whether these values show any variations which cannot be accounted for by errors of observation.

Omitting, in Table III, the two single measures made at Paris, and giving half-weight to the single measures made at the Lick Observatory in 1898.07, 1901.93 and 1011.01, I find $V_m = -7.37$ km., if the mass ratio of Auwers is used, and $V_m = -7.23$ km., if the value given by Boss is adopted. Rounding these values to —7.4 and —7.2 respectively, and comparing them with $(V_0 - V_1)$ we have the residuals entered in the last two columns of the table. These residuals represent the departures from undisturbed motion in the visual orbit, so far as they can be determined from the present data. It appears that the general progression in the observed radial velocities from 1895 to 1916 agrees very well with the progression in the computed radial component of the orbital motion. The maximum residuals, omitting Paris, are +1.8 km. and —1.8 km. in the seventh column, and +1.6 km. and —1.5 km. in the last, and there is no clear evidence in either column that the irregularities in the progression of the velocities are of other than accidental origin. The average residuals without regard to sign are ±0.68 km. and ±0.73 km. in the two columns respectively. The differences in the residuals in the two columns afford no basis for discriminating between the two values of the mass ratio.

TABLE IV

Radial Velocities of *Sirius* Observed at Potsdam

Date	n	V_0 km.	$V_1(A)$ km.	$V_1(B)$ km.	$(V_0 - V_1) - V_m$ A km.	$(V_0 - V_1) - V_m$ B km.
1888.99	3	—13.9	—0.9	—0.6	+1.5	+1.2
1890.09	3	—17.0	—0.5	—0.3	—2.0	—2.2
1891.20	4	—14.9	+0.3	+0.2	—0.7	—0.6
1906.26	2	—13.9	+0.4	+0.3	+0.2	+0.3
1908.23	3	—13.3	+0.1	±0.0	+1.1	+1.2
					±1.10	±1.10
$V_m = -14.48$ km., Auwers's mass ratio.						
$V_m = -14.52$ km., Boss's mass ratio.						
1901.15	7	— 8.8	+1.8	+1.2	—0.8	—0.4
1902.26	3	— 9.4	+1.4	+1.0	—1.0	—0.8
1903.09	7	— 8.4	+1.1	+0.8	+0.3	+0.4
1904.23	2	— 9.2	+0.8	+0.6	—0.2	—0.2
1905.21	4	— 8.3	+0.6	+0.4	+0.9	+0.9
1907.21	2	— 8.0	+0.2	+0.2	+1.6	+1.4
1910.17	3	—10.8	—0.2	—0.2	—0.8	—1.0
					±0.80	±0.73
$V_m = -9.80$ km., Auwers's mass ratio.						
$V_m = -9.56$ km., Boss's mass ratio.						

The Potsdam measures differ so greatly from the rest that they have been entered in a separate table (IV), similar in form to Table III, except that the first column has been omitted. The measures fall into two groups, as indicated in the table, which give for V_m the values —14.48 km. and —9.80 km., respectively, on the basis of Auwers's mass ratio. Obviously, the two sets cannot at present be combined either with each other or with the measures made elsewhere. The hypothesis that the differences between them are due to disturbances in the orbital motion of *Sirius* seems to be negatived by the measures made at the other observatories in the same period of time.

The adopted method of investigation will not reveal a variation having a period of only a few days or a few weeks; and the observations available are themselves not well suited to the detection of such a short period variation. I have, however, examined in detail the two longest series of measures, those made at the Cape of Good Hope in 1907–1908, and those made at Mount Hamilton in 1909–1910, and present the results in Table V. The columns give, in order, the date, the observed radial velocity and the residual (O–C), the computed velocities being obtained by combining V_m and V_1 for each date, taking the mass ratio as given by Auwers.

Campbell says:[6] "The lines in this type of spectrum are rendered difficult of measurement both by underexposure and by overexposure," and I am of opinion that this fact, combined with the error of measure of even the best plates, as shown in the different results obtained from the same plate by skilled measurers, is sufficient to account for the residuals in the Lick series of observations. The extreme range is only 1.7 km., and when the residuals are plotted they exhibit no certain periodic variations.

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[5] Many of the Lick spectrograms have been measured by two or more observers. In every such case I have adopted the mean of the results.

[6] *Loc. cit.*

TABLE V

Radial Velocities of *Sirius*

Cape of Good Hope

Date	V_0	(O–C)
1907 Nov. 15.60	— 5.9 km.	+1.4 km.
15.61	— 7.5	—0.2
Dec. 3.48	— 8.2	—0.9
Dec. 5.43	— 5.5	+1.8
5.44	— 6.8	+0.5
Dec. 10.49	— 7.5	—0.2
10.51	— 7.2	+0.1
Dec. 11.46	— 8.5	—1.2
11.47	— 8.1	—0.8
1908 Jan. 19.42	—10.4	—3.1
Feb. 4.33	— 9.0	—1.7
Feb. 11.35	— 5.4	+1.9
Feb. 15.35	— 8.2	—0.9
Feb. 18.31	— 7.2	+0.1
Feb. 22.29	— 6.1	+1.2
Feb. 26.36	— 5.6	+1.7
Mar. 5.29	— 6.9	+0.4
Mar. 6.38	— 9.8	—2.5
Mar. 17.25	— 7.1	+0.2
Mar. 18.30	— 6.4	+0.9
Mar. 25.30	— 7.4	—0.1
25.31	— 8.5	—1.2
Mar. 29.27	— 8.3	—1.0
29.28	— 8.5	—1.2
Apr. 7.23	— 7.2	+0.1
Apr. 10.28	— 9.8	—2.5
Apr. 24.30	— 5.9	+1.4
Apr. 30.25	— 7.0	+0.3
May 1.27	— 7.2	+0.1
May 2.24	— 6.7	+0.6

Mount Hamilton

Date	V_0	(O–C)
1909 Dec. 11.92	—7.3 km.	+0.3 km.
11.93	—7.3	+0.3
1910 Feb. 11.72	—7.9	—0.3
11.74	—7.8	—0.2
Mar. 4.67	—8.1	—0.5
Apr. 6.67	—7.7	—0.1
6.68	—6.6	+1.0
Apr. 14.67	—7.8	—0.2
Apr. 15.67	—7.8	—0.2
15.68	—7.5	+0.1
Apr. 17.68	—7.7	—0.1
17.69	—7.2	+0.4
Apr. 20.69	—7.7	—0.1
20.70	—6.8	+0.8
Apr. 21.67^-	—7.5	+0.1
21.67^+	—8.0	—0.4
Apr. 22.62	—8.3	—0.7
22.63	—7.3	+0.3

The range in the residuals for the Cape observations is considerably larger, the extremes being —3.1 km. and +1.9 km., range 5.0 km., and the plot shows alternating maxima and minima. Taken alone, this series might perhaps be regarded as evidence for a short-period disturbance in the orbital motion of the star; considering the Lick series as well, we must conclude that the evidence is as yet not convincing. My conclusion is that up to the present time we have no definite evidence in either the micrometric or the spectrographic measures of *Sirius* that the observed motion departs from undisturbed elliptic motion.

Robert G. Aitken.

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THE ORBIT OF THE BINARY STAR A 417 = 83 *AQUARII*

The star 83 *Aquarii* (R. A. 1900.0 = $22^h\,59^m9$; Decl. = —8° 14′; magnitude 5.56; spectral class A) was noted with the 12-inch telescope on July 24, 1902, as being a close pair with components of equal brightness. Subsequent measures with the 36-inch telescope gave the position, 1902.64 61°0 0″19. According to the *Preliminary General Catalogue* of Boss, the star has an annual proper motion of 0″123 in 82°, hence it was at once obvious that the close pair was a physical system, for 83 and 84 *Aquarii* were catalogued by Struve in 1835 as the very wide pair Σ 59, App. 1. Measures in 1903 showed decided angular motion within the year and the system has therefore been measured regularly ever since. Until 1912, the angular motion appeared to be nearly uniform with but little evidence of variation in the apparent separation; but in the last few years the angular motion has been extremely rapid and the apparent distance so small as to make the pair a very difficult object to measure even with the great refractor. The observed motion now covers an arc of 270° which is sufficient to serve as the basis for the computation of a preliminary orbit.

TABLE I

MEASURES AND RESIDUALS

Date	θ_0	ρ_0	n	(O–C) $\Delta\theta$	(O–C) $\Delta\rho$
1902.64	61°0	0″19	4	—2°8	—0″04
1903.66	73.5	0.19	5	+0.3	—0.03
1904.58	84.8	0.22	2	+2.2	+0.01
1905.59	94.6	0.20	3	+0.6	0.00
1906.51	106.1	0.20	3	—0.5	+0.01
1907.70	118.0	0.20	1	—2.6	+0.01
1908.66	130.6	0.19	3	—2.3	0.00
1909.72	144.9	0.17	3	—0.8	—0.03
1910.72	156.9	0.20	3	—0.1	—0.01
1911.84	169.4	0.22	3	—1.0	0.00
1912.64	177.5	0.19	3	+1.3	—0.03
1914.60	196.0	0.18	4	+0.3	—0.03
1915.62	207.5	0.16	3	—0.9	—0.02
1916.68	230.0	0.14	3	+0.3	+0.01
1917.69	277.0	0.12	3	+0.1	+0.03
1918.56	330.9	0.13	3	—0.1	+0.03

Table I contains my measures of this pair, and, so far as my knowledge goes, these are all the measures of it that have been made. These measures were plotted and it was found possible to pass through them an ellipse which fairly represented the distances and at the same time approximately satisfied the law of areas. When the observations were compared with those computed from the elements resulting from this ellipse it was seen that the residuals exhibited a systematic run of such character that they could be improved by a least squares solution. The measures to 1912 were accordingly grouped into five normal places, as follows:

TABLE II

NORMAL PLACES TO 1912, AND RESIDUALS

Date	θ	ρ	(O–C) $\Delta\theta$	(O–C) $\Delta\rho$
1903.15	67°2	0″19	—1°2	—0″03
1905.08	89.7	0.21	+2.0	+0.01
1907.62	118.2	0.20	—1.3	+0.01
1910.22	150.9	0.19	—0.5	—0.01
1912.24	173.4	0.21	+1.0	—0.01

The annual means for the five years 1914–1918 were adopted as given in Table I and equal weights assigned to the ten positions. Equations of condition were then set up and solved in the usual manner on the basis of the residuals in position angle, the observed distances being used only to reduce these residuals to circular measure and to determine the correction to the semimajor axis. The resulting elements, with their computed probable errors are:

ELEMENTS

$P = 23.82 \pm 1.37$ years
$T = 1917.68 \pm 0.20$ years
$e = 0.404 \pm 0.016$
$a = 0''245 \pm 0''005$
$\omega = 261^\circ3 \pm 5^\circ4$
$i = \pm 56^\circ35 \pm 4^\circ1$
$\Omega = 21^\circ6 \pm 2^\circ4$
Angles increasing.

EPHEMERIS

Date	θ	ρ
1919.68	2°9	0″16
1920.68	16.8	0.20
1921.68	26.6	0.23
1922.68	34.6	0.25
1923.68	42.1	0.25
1924.68	49.5	0.25
1925.68	57.3	0.24
1926.68	65.7	0.23

The residuals resulting from the comparison between the positions computed from these elements and the observed and normal places are entered in the last two columns of tables I and II. They are all within the probable error of observation, and to this extent the elements may be regarded as satisfactory. It does not follow, however, that they may not require corrections, later, which are even larger than the computed probable errors indicate; in fact, I regard the orbit as only provisional, and publish it chiefly to call the attention of other observers to this fine pair. It will be seen from the ephemeris that the angular separation is likely to increase during the next few years, thus bringing the system within the observing power of telescopes of moderate size.

If the mass of the system is assumed to be twice that of the Sun, the elements give +0″023 for the parallax, a value which is in harmony with the magnitude, spectral class and proper motion of the star.

Attention may be called to the fact that this is the second time it has been my privilege to present the orbit of a visual binary star, all the work on which, including the discovery and measures, has been my own. The other orbit is that of the 12-year-period system, A 88. I am not aware that a similar statement can be made of any other visual binary system.

ROBERT G. AITKEN.

September 6, 1918.
Issued October 7, 1918.

END OF VOLUME IX

INDEX TO VOLUME IX

PAGE

A.G. Zone, Observed errors in the Harvard. R. H. Tucker. 146
Aitken, R. G.,
  Measures of comets in 1915 and in 1916 64
  One hundred new double stars. Twenty-fourth list 132
  The orbit of *Sirius* 184
  The orbit of the binary star A 417 = 83 *Aquarii* 191
Algol variable and spectroscopic binary, RS *Sagittarii*. G. F. Paddock 39
Alter, D.,
  Elements and ephemeris of Comet *b* 1916 (Wolf) [with R. T. Crawford] 15, 74
  On finite velocity of gravitation as a possible factor in stellar evolution 128
  On the orbit of *Aethra* 32
  On variation of velocity of waves due to motion of the source 151
Asteroid: See *Minor Planet.*
Astronomische Gesellschaft Zone stars, Modern positions of. R. H. Tucker 17
Atmospheric refraction, The diurnal variation in the. R. H. Tucker 145
β Ursae Majoris, On the radial velocity of. W. W. Campbell and Adelaide M. Hobe 22
Binary Stars,
  The orbit of *Sirius.* R. G. Aitken 184
  The orbit of the binary star A 417 = 83 *Aquarii.* R. G. Aitken 191
  See also *Double Stars* and *Spectroscopic Binaries.*
Campbell, W. W.,
  An error in radial velocity observations arising from non-uniform slit illumination 30
  On systematic errors in the measured interval between close lines 28
  On the radial velocity of *β Ursae Majoris* [with Adelaide M. Hobe] 22
  On the rotation of some planetary nebulae [with J. H. Moore] 1
  The wave-length of the green nebular lines N_1 and N_2 [with J. H. Moore] 6
Close lines, On systematic errors in the measured interval between. W. W. Campbell 28
Comet *e* 1909 (Daniel), Note on the return of. S. Einarsson and Margaret Harwood 14
Comet Faye, Search ephemeris for. H. M. Jeffers 78
Comet *e* 1915 (Taylor), Elements and ephemeris of. H. M. Jeffers and F. J. Neubauer 12
Comet *a* 1916 (Neujmin), Elements and ephemeris of. Jessica M. Young, H. M. Jeffers, and T. A. Pierce 11
Comet *b* 1916 (Wolf),
  Elements and ephemeris of. R. T. Crawford and D. Alter 15, 74
  Ephemeris of. R. T. Crawford 35, 48
Comet *a* 1917 (Mellish), Elements and ephemeris of. H. M. Jeffers and Elizabeth J. Easton 50
Comet *b* 1917 (Schaumasse), Elements and ephemeris of. Jessica M. Young and H. M. Jeffers 78
Comets,
  Measures of, in 1915 and in 1916. R. G. Aitken 64
  Obervations of. W. K. Green 67
  Photographic measures of. R. F. Sanford 65
  Photographic positions of. W. K. Green 67
Crawford, R. T.,
  Elements and ephemeris of Comet *b* (Wolf) [with D. Alter] 15, 74
  Ephemeris of Comet *b* 1916 (Wolf) 35, 48
Curtis, H. D., Three novae in spiral nebulae 108
Double stars, One hundred new. Twenty-fourth list. R. G. Aitken 132
  See also *Binary Stars.*

PAGE

Easton, Elizabeth J., Elements and ephemeris of Comet *a* 1917 (Mellish) [with H. M. Jeffers] 50
Einarsson, S., Note on the return of Comet *e* 1909 (Daniel) [with Margaret Harwood] 14
Gravitation, On finite velocity of, as a possible factor in stellar evolution. D. Alter 128
Green, W. K.,
  Observations of comets 67
  Photographic positions of comets 67
  Study of certain nebulae, A 92
Harwood, Margaret, Note on the return of Comet *e* 1909 (Daniel) [with S. Einarsson] 14
Henroteau, F.,
  Radial velocities of the stars *ε* and *γ Orionis* 178
  Spectrographic study of *κ Pegasi*, A 120
  Spectrographic study of *σ Scorpii*, A 173
  Spectroscopic study of *β Canis Majoris*, A 155
Hobe, Adelaide M., On the radial velocity of *β Ursae Majoris* [with W. W. Campbell] 22
Jeffers, H. M.,
  Elements and ephemeris of Comet *e* 1915 (Taylor) [with F. J. Neubauer] 12
  Elements and ephemeris of Comet *a* 1916 (Neujmin) [with Jessica M. Young and T. A. Pierce] 11
  Elements and ephemeris of Comet *a* 1917 (Mellish) [with Elizabeth J. Easton] 50
  Elements and ephemeris of Comet *b* 1917 (Schaumasse) [with Jessica M. Young] 78
  Elements and ephemeris of Wolf's Object, 1918 DB 147
  Search ephemeris for Faye's Comet 78
Light-Variables, The radial velocities of eleven southern. G. F. Paddock 68
Magnitude equation and the visual scale, The. R. H. Tucker 136
Minor Planets,
  Comparison stars for Asteroid (21). R. H. Tucker 145
  On the orbit of *Aethra.* D. Alter 32
  On the orbit of (718) *Erida.* C. S. Mundt 115
  Orbit and Perturbations of (716) *Berkeley.* F. J. Neubauer 111
  Elements andephemeris of Wolf's Object, 1918 D.B. H. M. Jeffers 147
Miscellaneous Meridian Circle results. R. H. Tucker 144
Moon, observed corrections to the right ascension of the. R. H. Tucker 144
Moore, J. H.,
  On the rotation of some planetary nebulae [with W. W. Campbell] 1
  Six stars whose radial velocities vary 37
  The wave-lengths of the green nebular lines N_1 and N_2 [with W. W. Campbell] 6
Mundt, C. S., On the orbit of (718) *Erida* 115
Nebulae,
  A study of certain. W. K. Green 92
  Wolf-Rayet bands in the nuclei of three, N.G.C. 6826, N.G.C. 418_2, N.G.C. 40. G. F. Paddock 29
  See also *Ring Nebula, Planetary Nebulae, Spiral Nebulae.*
Nebular lines N_1 and N_2,
  The wave-lengths of the green. W. W. Campbell and J. H. Moore 6
  The wave-lengths of the. W. H. Wright 9
Neubauer, F. J.,
  Elements and ephemeris of Comet *e* 1915 (Taylor) [with H. M. Jeffers] 12
  Orbit and perturbations of (716) *Berkeley* 111
Newkirk, B. L., Parallax of the Ring Nebula in *Lyra* 100
Nova Monocerotis. G. F. Paddock 170
  Meridian Circle observations of. R. H. Tucker 144
Novae in spiral nebulae, Three. H. D. Curtis 108

PAGE
Paddock, G. F.,
  Note on the identification of two more lines in the spectrum of υ_4 *Eridani* 47
  Notes on the spectrum of γ *Argus* 40
  Nova Monocerotis 170
  Radial velocities of eleven southern light-variables 68
  RS *Sagittarii,* Algol variable and spectroscopic binary, 39
  Spectroscopic binary h 4 *Centauri* 42
  Wolf-Rayet bands in the nuclei of three nebulae, N.G.C. 6826, N.G.C. 418$_2$, N.G.C. 40 29
Pierce, T. A., Elements and ephemeris of Comet *a* 1916 (Neujmin) [with Jessica M. Young and H. M. Jeffers] 11
Planetary Nebulae, On the rotation of some. W. W. Campbell and J. H. Moore 1
Quartz spectrograph of the Lick Observatory and some obervations recently secured with it. W. H. Wright. 52
Radial velocities of eleven southern light-variables. G. F. Paddock 68
Radial velocities of the stars ϵ and γ *Orionis.* F. Henroteau 178
  See also *Spectroscopic Binaries.*
Radial velocity observations, An error in, arising from non-uniform slit illumination. W. W. Campbell 30
Refraction at Mount Hamilton, The. R. H. Tucker 56
Ring Nebula in *Lyra*, Parallax of the. B. L. Newkirk 100
Sanford, R. F.,
  On some relations of the spiral nebulae to the Milky Way 80
  Orbit of the spectroscopic binary *p Velorum* 181
  Photographic measures of satellites and of comets 65
Satellites, Photographic measures of. R. F. Sanford 65
Sirius, The orbit of. R. G. Aitken 184
Spectrograph, The quartz, of the Lick Observatory and some observations recently secured with it. W. H. Wright 52
Spectroscopic Binaries,
  Four stars whose radial velocities are variable. R. E. Wilson 38
  Six stars whose radial velocities vary. J. H. Moore. 37
Spectroscopic Binary,
  β *Canis Majoris,* A spectroscopic study of. F. Henroteau 155
  h 4 *Centauri.* G. F. Paddock 42
  κ *Pegasi,* A spectrographic study of. F. Henroteau 120
  γ *Phoenicis,* The orbit of. R. E. Wilson 116
  σ *Puppis,* On the orbit of. R. E. Wilson 117
  RS *Sagittarii,* Algol variable and spectroscopic binary. G. F. Paddock 39
  σ *Scorpii,* A spectrographic study of. F. Henroteau 173
  p Velorum, Orbit of. R. F. Sanford 181

PAGE
Spectrum of υ_4 *Eridani,* Note on the identification of two more lines in the. G. F. Paddock 47
Spectrum of γ *Argus,* Notes on the. G. F. Paddock 40
Spiral Nebulae,
  On some relations of the, to the Milky Way. R. F. Sanford 80
  Three novae in. H. D. Curtis 108
Systematic errors in the measured interval between close lines. W. W. Campbell 28
Tucker, R. H.,
  Magnitude equation and the visual scale, The 136
  Miscellaneous Meridian Circle results:
    Observed correction to the right ascension of the Moon. Observed correction to the right ascension of *Venus.* Meridian Circle observations of *Nova Monocerotis.* Comparison stars for Asteroid (21). Observations of XZ *Cygni.* The diurnal variation in the atmospheric refraction. Observed errors in the Harvard A.G. Zone 144
  Modern positions of Astronomische Gesellschaft Zone stars 17
  The refraction at Mount Hamilton 56
  The visual magnitude scale and the counts of stars 162
Variable Star XZ *Cygni,* Observations of. R. H. Tucker. 145
Venus, Observed correction to the right ascension of. R. H. Tucker 144
Visual Binary Stars: See *Binary Stars.*
Visual magnitude scale and the counts of stars. R. H. Tucker 162
Wave-lengths of nebular lines: See *Nebular lines.*
Waves, On variation of velocity of, due to motion of the source. D. Alter 151
Wilson, R. E.,
  Four stars whose radial velocities are variable 38
  On the orbit of the spectroscopic binary σ *Puppis* 117
  The orbit of the spectroscopic binary γ *Phoenicis* 116
Wolf-Rayet bands in the nuclei of three nebulae, N.G.C. 6826, N.G.C. 418$_2$, N.G.C. 40. G. F. Paddock 29
Wright, W. H.,
  The quartz spectrograph of the Lick Observatory and some results recently secured with it 52
  The wave-lengths of the nebular lines N$_1$ and N$_2$ 9
Young, Jessica M.
  Elements and ephemeris of Comet *a* 1916 (Neujmin) [with H. M. Jeffers and T. A. Pierce] 11
  Elements and ephemeris of Comet *b* 1917 (Schaumasse) [with H. M. Jeffers] 78

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